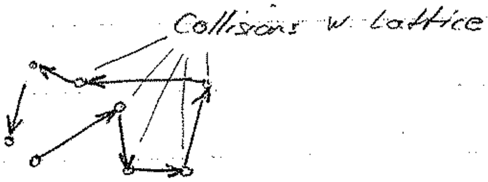
V Carrier Transport in Semiconductors

5.1 Electron/Hole Mobility

Thermal motion:

Thermal energy of an electron: $3 \times \frac{1}{2} kT = \frac{1}{2} m_n^* V_{th}^2$

3 degrees of freedom in 3-dim.



Example of thermal motion in 2-dim.

average thermal velocity

$$V_{th} = \sqrt{\frac{3kT}{m_n^*}} \cong 10^5 \frac{m}{s} \text{ (at } 300°K)$$

average distance between collisions:

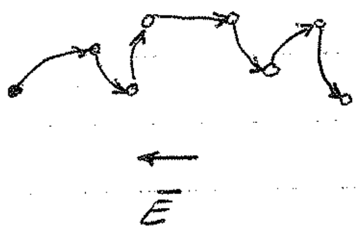
mean free path $L_c \approx 10^{-7} m$ (at $300°K$)

average time between collisions

mean free time $\tau_c \approx 10^{-12} s$ (at $300°K$)

Carrier Drift:

Motion of electron in presence of an electric field



($V_n \ll V_{th}$ assumed)

Momentum that el. gains between two collisions:

$$m_n^* \bar{V}_n = -q \bar{E} \tau_c \quad \text{(Force} \times \text{Time)}$$

so $$\bar{V}_n = -\frac{q \tau_c}{m_n^*} \bar{E}$$

e.g. $E = 10 \frac{V}{m} \Rightarrow \bar{V}_{n_{av}} \cong 2.5 \left[\frac{m}{s}\right]$

Def. The mobility of a charge carrier is given by:

electron

$$\underline{\mu_n = \frac{q\,\tau_c}{m_n^*}}$$

hole

$$\underline{\mu_p = \frac{q\,\tau_c}{m_p^*}}$$

Thus:

$$\underline{\bar{V}_n = -\mu_n \bar{E}}$$

$$\underline{\bar{V}_p = \mu_p \bar{E}}$$

The mobility describes how strongly the motion of an electron/hole is influenced by an applied el. field.

Note: The probab. of a collision taking place in unit time, $\frac{1}{\tau_c}$, is the sum of the probabilities of collisions due to the various scattering mechanisms:

$$\frac{1}{\tau_c} = \frac{1}{\tau_{c_{Lattice}}} + \frac{1}{\tau_{c_{Impur.}}}$$

or

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

$$\mu = \frac{\mu_L \cdot \mu_I}{\mu_L + \mu_I}$$

$$\mu_L \gg \mu_I \Rightarrow \underline{\mu = \mu_I}$$

$$\mu_I \gg \mu_L \Rightarrow \underline{\mu = \mu_L}$$

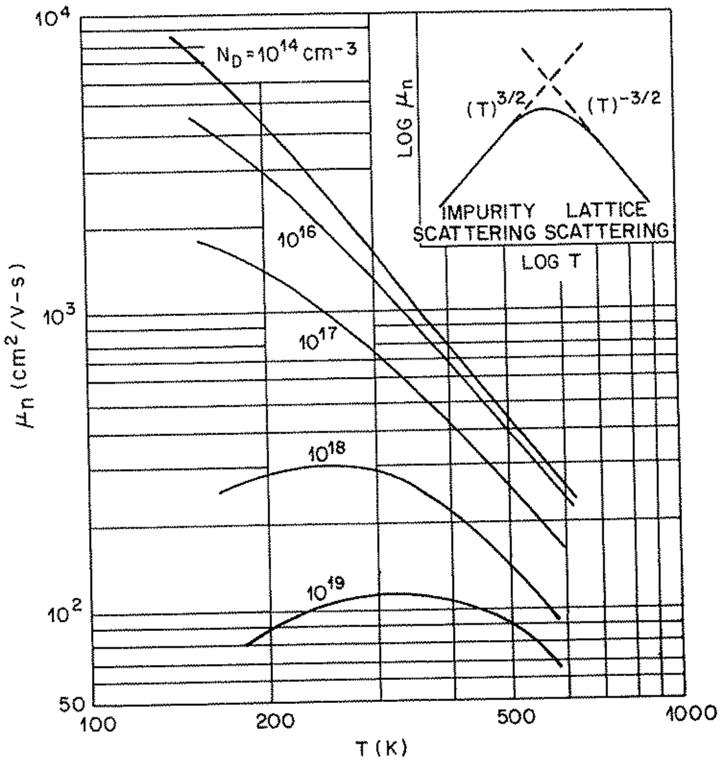
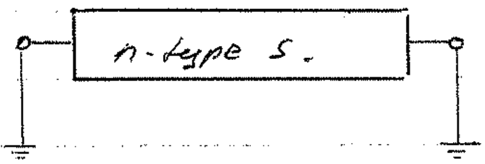


Fig. 2 Electron mobility in silicon versus temperature for various donor concentrations. Insert shows the theoretical temperature dependence of electron mobility.[4]

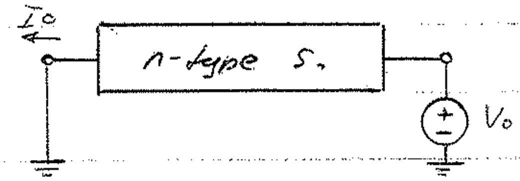
5.2 Carrier Drift (Ohm's law)

A sample of an n-doped semiconductor is considered under 2 different conditions:

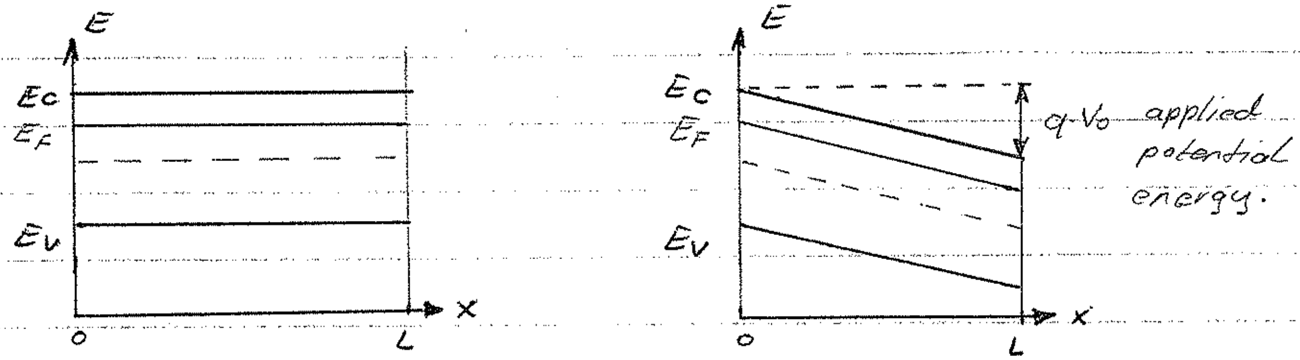
a) Equilibrium



b) Nonequilibrium



Energy-Band Diagram

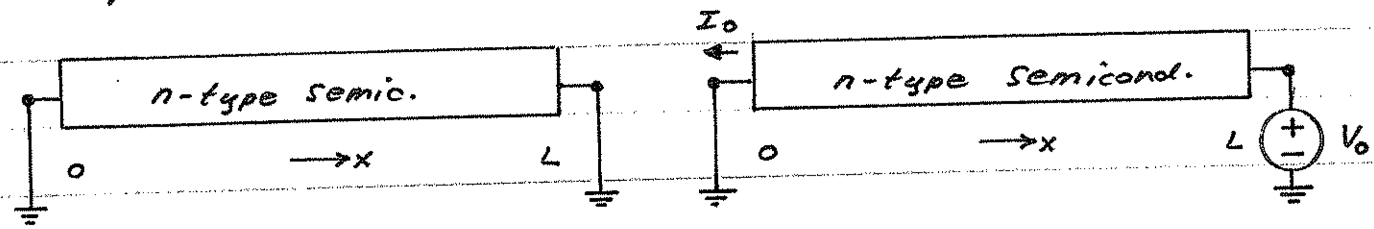


The applied el. Field $E_0 = \frac{V_0}{L}$ increases the potential energy of the electrons at the right hand side of the sample (Potential energy of el. is represented by their distance from the bottom of the conduction band).
The electrostatic potential ψ [V] is related to the el. field with $\underline{\bar{E}_{el} = -d\psi/dx}$ and the energy is related to the potential over $\underline{E_{energy} = -q\psi}$

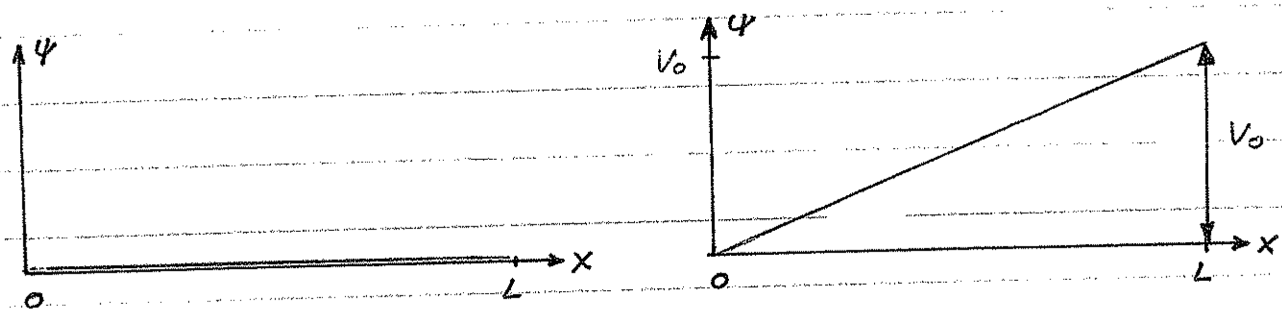
$$\Rightarrow \underline{\frac{dE_{energy}}{dx} = -q\, d\psi/dx = q\bar{E}_{el} = q\frac{V_0}{L} = \text{const}}$$

Equilibrium and Nonequilibrium state

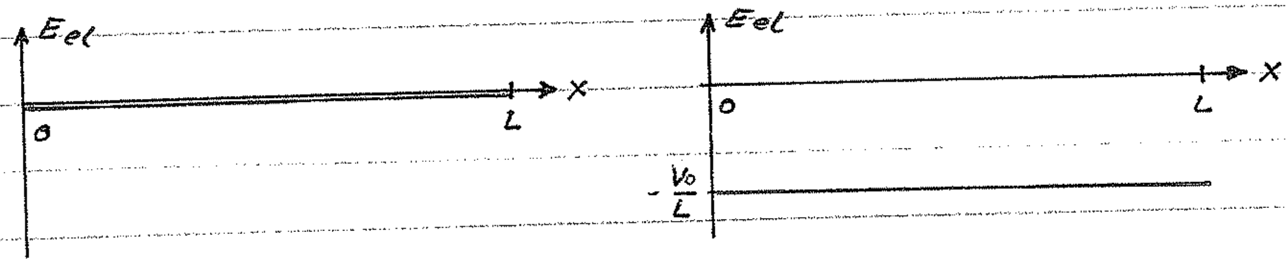
Equilibrium — Nonequilibrium



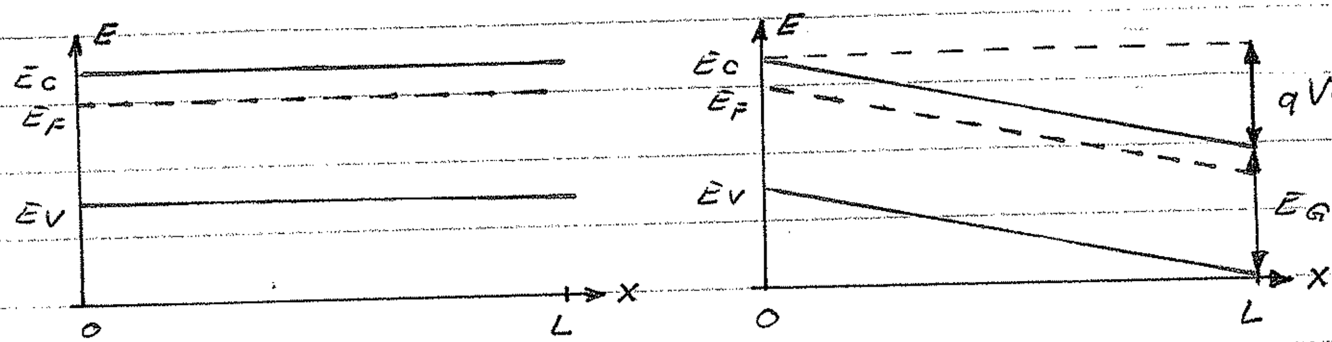
el. stat. Potential ψ [V]



el. Field $E_{el} = -d\psi/dx$



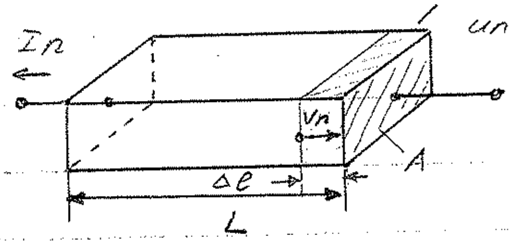
Energy $E = -q\psi$



The electron current density J_n flowing in the sample can be found by summing the product of the charge $(-q)$ on each el. times its velocity over all el. per unit vol.

$$\underline{\underline{\bar{J}_n}} = \frac{I_n}{A} = \sum_{i=1}^{n} (-q V_i) = -q n \bar{v}_n = \underline{\underline{q n \mu_n \bar{E}}}$$

unit vol. in which all el. cross unit aerea A in unit time $\Delta t = \Delta \ell / v_n$



A similar argument applies to holes. Since the charge of the hole is $+q$, we obtain

$$\underline{\underline{\bar{J}_p}} = \sum_{i=1}^{p} (q V_i) = q p \bar{v}_p = \underline{\underline{q p \mu_p \bar{E}}}$$

The total current flow in a semiconductor due to carrier drift is the sum of electron and hole current.

$$\boxed{\bar{J}_{Drift} = \bar{J}_n + \bar{J}_p = (q n \mu_n + q p \mu_p) \bar{E}}$$

The quantity in parentheses is known as conductivity σ. Thus

$$\boxed{\sigma = \frac{\bar{J}}{\bar{E}} = q n \mu_n + q p \mu_p} \qquad \text{conductivity}$$

or

$$\boxed{\rho = \frac{\bar{E}}{\bar{J}} = \frac{1}{q n \mu_n + q p \mu_p}} \qquad \text{resistivity}$$

For an n-type semiconductor ($n \gg p$) we can simplify the formula for the resistivity to

$$\boxed{\rho_n \cong \frac{1}{q n \mu_n}}$$ n-type semic. or conductor (Metal)

Similarly, we obtain for a p-doped semic.

$$\boxed{\rho_p \cong \frac{1}{q p \mu_p}}$$ p-type semic.

Recall: Ohm's law

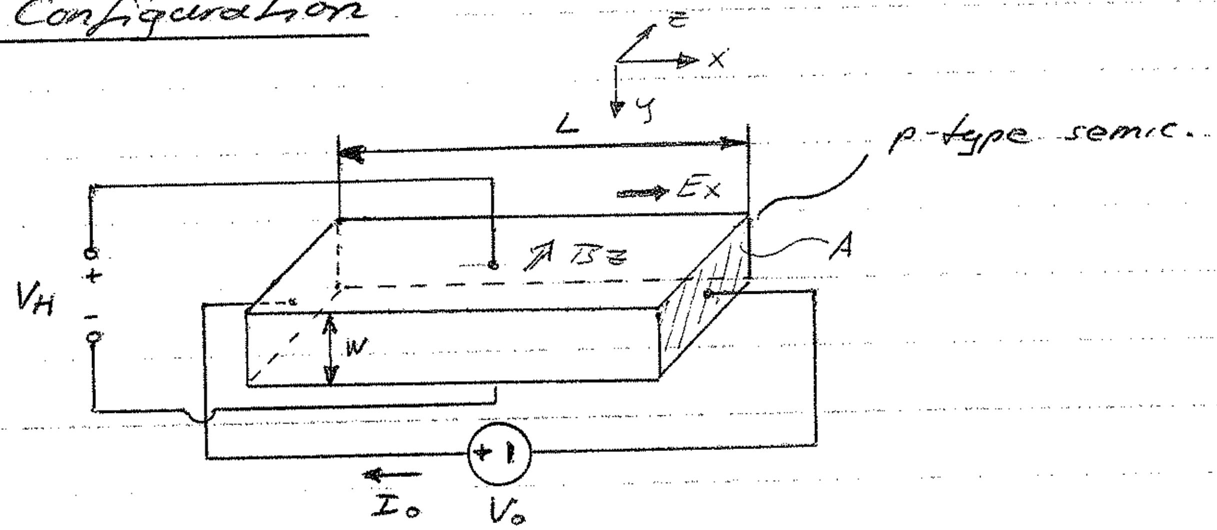
$$\bar{J} = \sigma \cdot \bar{E}$$

or

$$\bar{E} = \rho \cdot \bar{J} \qquad \rho = \frac{1}{\sigma}$$

Carrier Concentration Measurements
(The Hall Effect)

Basic Configuration



In the given configuration, a positive charge carrier inside the semiconductor experiences 2 forces:

1. Electrostatic force $\bar{F}_E = F_x = q \cdot E_x$ $\rightarrow v_x = \mu_p E_x$

2. Lorentz force $\bar{F}_L = q \cdot (\underbrace{\bar{V}}_{\text{velocity of particle}} \times \bar{B})$

$$\bar{F}_L = -F_y = q \; v_x \cdot B_z$$

The Lorentz force causes an accumulation of pos. charge carriers at the top of the sample that gives rise to an electric field in the y-direction. Since, in steady state, there is no net current flow along the y-axis, this el. field must balance the Lorentz force. Thus

$$q \, E_y = q \; v_x \cdot B_z$$

$$E_y = \frac{V_H}{W} = V_x \cdot B_z$$

With $v_x = \mu_p E_x = \mu_p \frac{V_0}{L}$ we obtain

$$\frac{V_H}{W} = \mu_p \frac{V_0}{L} B_z \Rightarrow \mu_p = \frac{V_H}{V_0} \frac{L}{W} \frac{1}{B_z}$$

Finally, in order to obtain a relationship which involves the carrier concentration p, we can replace the mobility by the equivalent expression

$$\mu_p = \frac{1}{qp}\,\sigma = \frac{1}{qp}\,\frac{J_x}{E_x} = \frac{1}{qp}\,\frac{I_0}{A \cdot E_x}$$

This yields the alternative solution

$$\frac{V_H}{W} = \frac{1}{qp}\,\frac{I_0}{A} \cdot B_z \Rightarrow \underline{\underline{p = \frac{I_0}{q \cdot V_H}\,\frac{W}{A}\,B_z}}$$

The quantity $\frac{1}{qp}$ is called Hall coefficient R_H.
The Hall voltage can then be expressed by

$$\underline{\underline{V_H = R_H\, I_0\, \frac{W}{A}\, B_z}}$$

Note: If V_H exhibits a negative sign, the semiconductor is n-doped. The Hall coefficient is then given by $\underline{R_H = -\frac{1}{qn}}$

Problem: Si is doped with 10^{22} P atoms/m^3. Find V_H in a sample with $W = 500\mu m$, $A = 2.5 \times 10^{-7} m^2$, $I_0 = 1mA$ and $B_z = 1\,\frac{Vs}{m^2}$

Solution: $\underline{\underline{V_H = -1.25\,mV}}$ (n-doped semicond.)

Problem Hall Effect

Given is a sample of Si with $L = 1cm$, $W = 500\,\mu m$ and $A = 2.5\times10^{-7}\,m^2$. Under a magnetic field $B_z = 1\,\frac{Vs}{m^2}$ the following values are measured:

$V_0 = 200\,mV$; $V_H = -1.25\,mV$

$I_0 = 1\,mA$

a) Calculate the type and concentration of the dopant.

b) Calculate the carrier mobility μ.

Solution

a) $V_H < 0 \Rightarrow$ n-type semiconductor

$$n = -\frac{I_0 \cdot W \cdot B_z}{q \cdot V_H \cdot A} = \frac{10^{-3}\times 5\times10^{-4}}{1.6\times10^{-19}\times1.25\times10^{-3}\times2.5\times10^{-7}}\,[m^{-3}]$$

$$\underline{\underline{n = 10^{22}\,m^{-3}}}$$

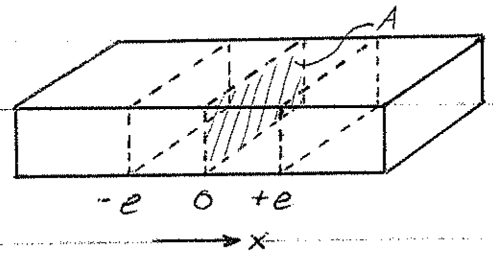
b) $$\mu_n = -\frac{V_H \cdot L}{V_0 \cdot W\, B_z} = \frac{1.25\times10^{-3}\times10^{-2}}{0.2\times5\times10^{-4}}\left[\frac{m^2}{Vs}\right]$$

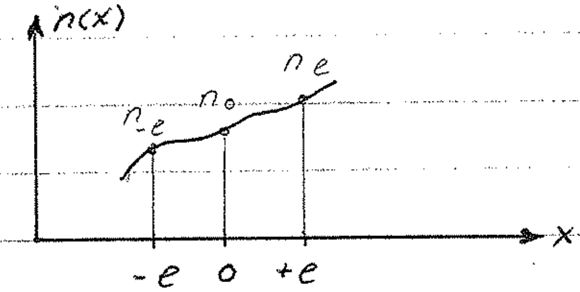
$$\underline{\underline{\mu_n = 1.25\times10^{-1}\,\frac{m^2}{Vs}}}$$

5.3 Carrier Diffusion

We assume a charge carrier density that varies in the x-direction

e.g.





We want to know how many charge carriers are crossing the plane at $x=0$ in unit time τ_c (mean free time)

ℓ is chosen such that $\underline{v_{th} \cdot \tau_c = \ell = L_c}$ (mean free path)

$\Rightarrow$ $\underline{\frac{\ell}{\tau_c} = v_{th}}$

On average,

50% of the charge carriers in volume between $x=-\ell$ and $x=0$ will cross the unit plane A at $x=0$ in time τ_c.

The average flow of carriers from the left to the right side is then

$$\underline{F_\ell} = \frac{1}{2} n(-\ell) \frac{A \cdot \ell}{A \tau_c} = \underline{\frac{1}{2} n(-\ell)\, v_{th}}$$

Similarly, the carrier flow from the right to the left side is

$$\underline{F_r} = \frac{1}{2} n(+\ell) \frac{A \ell}{A \tau_c} = \underline{\frac{1}{2} n(+\ell)\, v_{th}}$$

The net carrier flow from the left side to the right side is thus

$$\bar{F} = F_l - F_r = \frac{1}{2} V_{th} [n(-l) - n(+l)]$$

We now approximate the carrier densities at $x = \pm l$ by the first two terms of a Taylor series expansion, i.e.

$$n(\pm l) = n(0) \pm l \frac{dn(0)}{dx}$$

Thus

$$\bar{F} = \frac{1}{2} V_{th} [n(0) - l \frac{dn(0)}{dx} - n(0) - l \frac{dn(0)}{dx}]$$

$$= - V_{th} \cdot l \frac{dn(0)}{dx}$$

Def Diffusivity $D_n = V_{th} \cdot L_c = \frac{L_c}{\tau_c} L_c$ (electron)

Furthermore, since each charge carrier carries a charge of $-q$ (electron), the carrier flow gives rise to a current

$$\boxed{\bar{J}_n = -q\bar{F} = q D_n \frac{dn}{dx}}$$

Similarly, the current carried by holes is

$$\boxed{\bar{J}_p = +q\bar{F} = -q D_p \frac{dp}{dx}}$$

The total diffusion current in a semiconductor is then equal to

$$\boxed{\bar{J}_{Diff} = q D_n \frac{dn}{dx} - q D_p \frac{dp}{dx}}$$

Einstein Relation

The above eq. can be written in a more useful form using the theorem for the equipartition of the thermal energy. We can write

$$\frac{1}{2} m^* V_{th_x}^2 = \frac{1}{2} kT \quad \rightarrow V_{th_x} = \sqrt{\frac{kT}{m^*}}$$

Furthermore, the mobility has been defined as

$$\mu = \frac{q \tau_c}{m^*} \quad \rightarrow m^* = \frac{q \tau_c}{\mu}$$

The Diffusity D can now be written as

$$D = V_{th} \cdot L_c = \sqrt{\frac{kT}{m^*}} L_c = \sqrt{\frac{kT}{q \tau_c} \mu} L_c$$

$$= \sqrt{\frac{kT}{q} \mu \frac{L_c^2}{\tau_c}} = \sqrt{\frac{kT}{q} \mu \cdot D}$$

$$\Rightarrow \quad \boxed{D = \frac{kT}{q} \mu} \quad \text{Einstein Relation}$$

The total current flow in a semiconductor is then given by

$$\left\| \bar{J}_{Tot} = \bar{J}_{Drift} + \bar{J}_{Diff.} = q[n\mu_n + p\mu_p]\bar{E} + q[D_n \frac{dn}{dx} - D_p \frac{dp}{dx}] \right\|$$

Compare: Conductor (typ. Metal)

$$\left| \bar{J}_{Tot} = q \cdot n \cdot \mu_n \bar{E} \right|$$

Note: The concentration gradient in a semiconductor produces an additional internal field E which gives rise to a drift current.

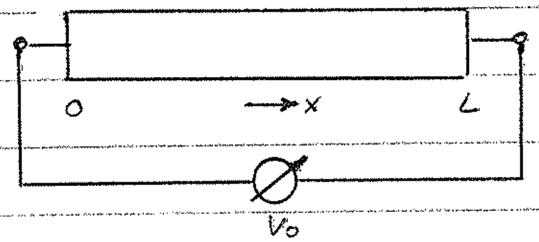
At equilibrium (no external forces applied), the total current flow must be equal to zero. Moreover, the currents of the electrons and holes must separately equal zero. Thus

$$\left| \begin{aligned} \bar{J}_{n_0} &= q\, n_0 \mu_n \bar{E}_{int.} + q\, D_n \frac{dn_0}{dx} = 0 \\ \bar{J}_{p_0} &= q\, p_0 \mu_p \bar{E}_{int} - q\, D_p \frac{dp_0}{dx} = 0 \end{aligned} \right|$$ Thermal equilibrium

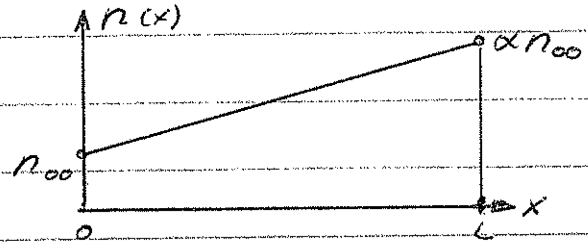
with $D = \frac{kT}{q}\mu$ we obtain

$$\left\| \begin{aligned} \bar{J}_{n_0} &= \mu_n [q\, n_0 \bar{E}_{int} + kT \frac{dn_0}{dx}] = 0 \\ \bar{J}_{p_0} &= \mu_p [q\, p_0 \bar{E}_{int} - kT \frac{dp_0}{dx}] = 0 \end{aligned} \right\|$$

Problem A bar of Si exhibits a constant concentration gradient along its x-axis (thermal equilibrium assumed, $T = 300°K$).



$n_{00} = 10^{17}\,m^{-3}$; $\alpha = 10^4$



$$\Rightarrow \quad \frac{dn}{dx} = \frac{(\alpha - 1)}{L}\, n_{00}$$

a) Calculate the voltage V_0 that you measure at thermal equilibrium between the two terminals of the Si bar.

b) Sketch the el. stat. potential ψ, the el. Field E_{el} and the Energy-Band Diagram of the bar at thermal equilibrium.

Solution: a)

Thermal equilibrium $\left| \begin{array}{l} \bar{J}_{n_0} = 0 \\ \bar{J}_{p_0} = 0 \end{array} \right|$

$$\bar{J}_{n_0} = \mu_n \left[q\, n_0\, \bar{E}_{el} + kT \frac{dn_0}{dx} \right] = 0$$

$$\Rightarrow \quad \bar{E}_{el} = -\frac{kT}{q} \frac{dn_0}{dx} \frac{1}{n_0}$$

$$n_0(x) = n_{00} + \frac{x}{L}(\alpha - 1)\, n_{00} = n_{00}\left[1 + \frac{x}{L}(\alpha - 1)\right]$$

$$E_{el} = -\frac{kT}{q}\,\frac{(\alpha-1)}{L}\,\frac{1}{1+\frac{x}{L}(\alpha-1)} = \underline{-\frac{kT}{q}\,\frac{(\alpha-1)}{L+x(\alpha-1)}}$$

Potential

$$\psi(x) = -\int_0^x E_{el}(\xi)\,d\xi = \frac{kT}{q}\int_0^x \frac{(\alpha-1)}{L+\xi(\alpha-1)}\,d\xi$$

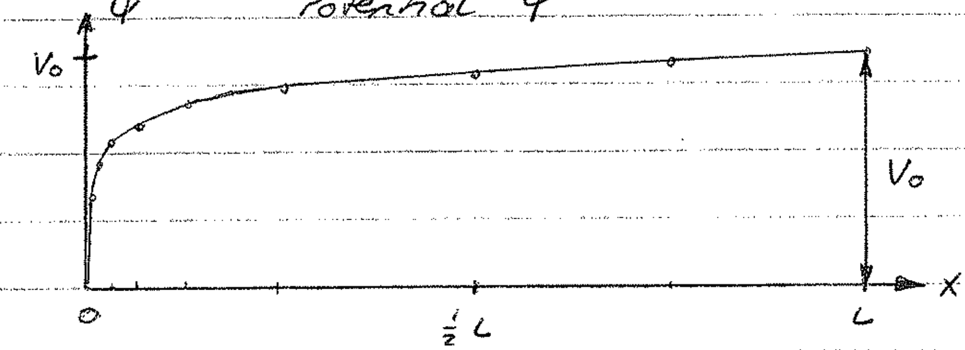
$$= \frac{kT}{q}\,\ln[L+\xi(\alpha-1)]\Big|_0^x$$

$$\boxed{\psi(x) = \frac{kT}{q}\,\ln\left[1+\frac{x}{L}(\alpha-1)\right]}$$

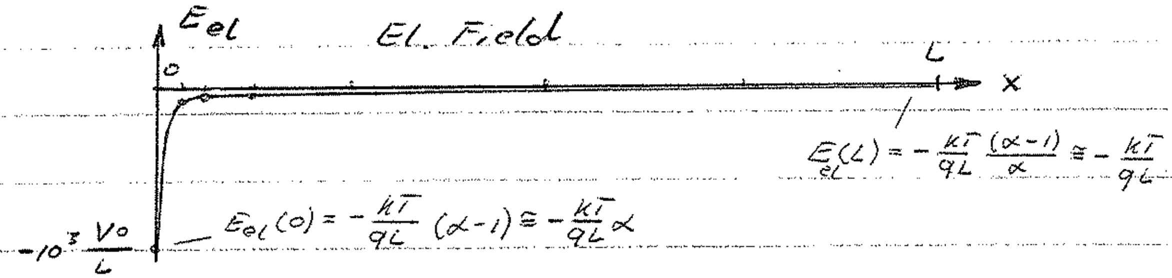
$$\boxed{V_0 = \psi(L) = \frac{kT}{q}\,\ln[\alpha] \cong 240\,mV}$$

b)

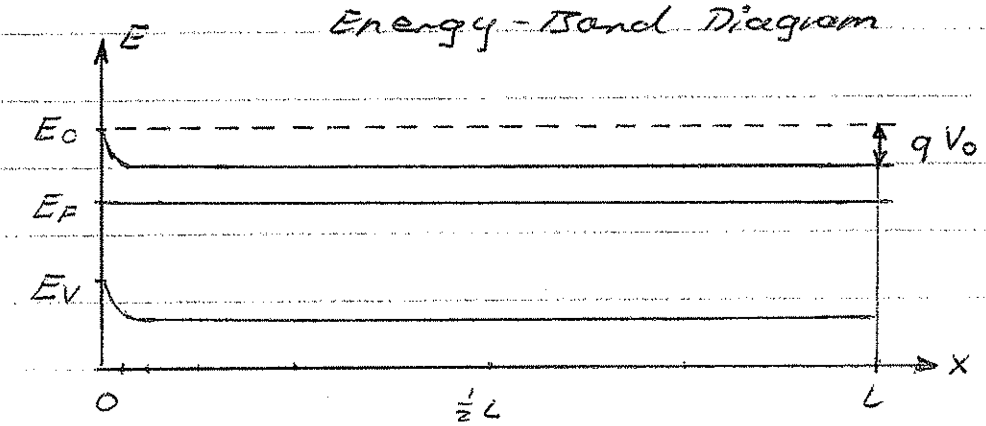
Potential ψ



El. Field



Energy-Band Diagram



5.4 Carrier Generation and Recombination Processes

Generation: Process whereby electrons and holes are created.

Recombination: Process whereby electrons and holes are annihilated or destroyed.

Generation-Recombination processes can be induced by

a) Light → Photogeneration (e.g. Photocell)

b) Thermal Energy (Heat) → thermal generation

c) an el. Field → Recomb. Energy is released via photons (e.g. LED, Laser diode)

Example Carrier injection via light

Since excess carriers are introduced to the semiconductor, $n \cdot p > n_{io}^2$ → nonequilibrium situation

We shine light on an n-type Si sample with $N_D = 10^{21} m^{-3} \Rightarrow p_o = \frac{n_{io}^2}{N_D} = 2.25 \times 10^{11} m^{-3}$

The light creates $\Delta n = \Delta p = 10^{18} m^{-3}$ excess charge carriers.

Thus

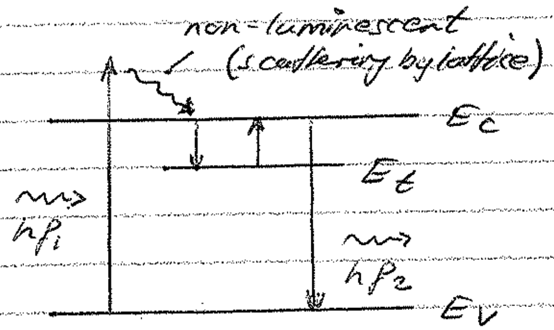
$$\left| \begin{aligned} n &= N_D + \Delta n = (10^{21} + 10^{18}) m^{-3} \cong 10^{21} m^{-3} \\ p &= p_o + \Delta p = (2.25 \times 10^{11} + 10^{18}) m^{-3} \cong 10^{18} m^{-3} \end{aligned} \right|$$

Carrier Generation- Recombination Processes

1. Luminescence

- Photon absorption → photoluminescence
- Electron absorption → cathodoluminescence (electron bombardment)
- Current induced → electroluminescence

Photoluminescence



Indirect R-G process
slow process
"Phosphorescence" Materials
e.g. ZnS
Color of light emitted depends on impurities.

Most common example: Fluorescent lamps

Electroluminescence

In LED's, an electric current causes the injection of minority carriers into regions of the crystal, where they can recombine with majority carriers → recombination radiation.

Impurities in Silicon

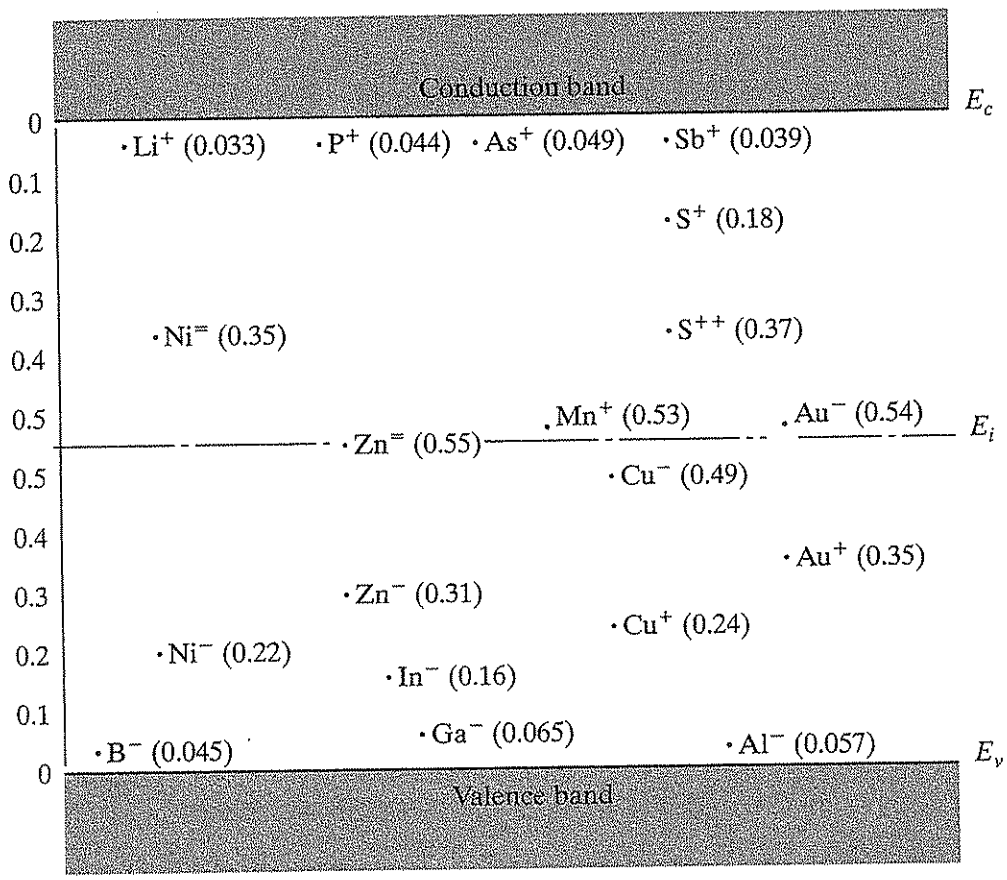


Figure 4–9 Energy levels of impurities in Si. The energies are measured from the nearest band edge (E_v or E_c); donor levels are designated by a plus sign and acceptors by a minus sign.

[4]References: S. M. Sze and J. C. Irvin, "Resistivity, Mobility, and Impurity Levels in GaAs, Ge and Si at 300 K," *Solid State Electronics*, vol. 11, pp. 599–602 (June 1968); E. Schibli and A. G. Milnes, "Deep Impurities in Silicon," *Materials Science and Engineering*, vol. 2, pp. 173–180 (1967).

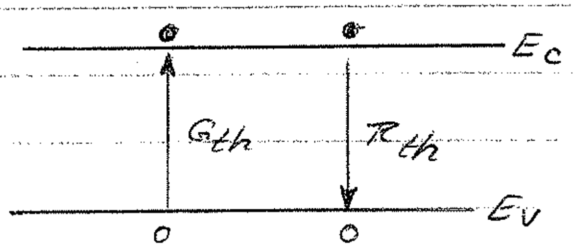
<u>Note</u>: In this example, the majority carrier concentration has hardly been affected, the minority carrier conc., however, has been encreased by several orders of magnitude. This is called <u>low-level injection</u>. Since $p \ll n \cong N_D$, the conductivity of the Si sample remains essentially unaffected.
If <u>$\Delta n = \Delta p$</u> would have been <u>comparable or larger</u> than the <u>majority carrier conc.</u> (e.g. N_D), we would call the process <u>high-level injection</u>.

$$\left| \begin{array}{l} \Delta n = \Delta p \ll N_D \rightarrow \text{low-level injection} \\ \Delta n = \Delta p \geq N_D \rightarrow \text{high-level injection} \end{array} \right|$$

Whenever thermal equilibrium is disturbed (i.e. $np \neq n_i^2$), processes exist to restore the system to equilibrium, where $np = n_i^2$. Thus, <u>generation</u> is <u>counter-balanced</u> by <u>recombination</u>.

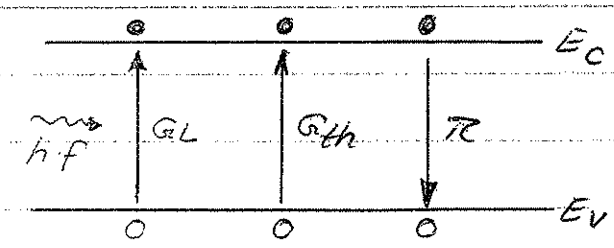
<u>Direct Recombination</u>

Thermal equilibrium



$$R_{th} = G_{th}$$

nonequilibrium state



$$R = G_L + G_{th}$$

(steady state)

for direct recomb. only

With $\mathcal{R} = \beta \cdot n \cdot p$, where $\beta = [\frac{m^3}{s}]$, we can write

<u>equilibrium</u> ~~(steady state)~~

$$\mathcal{R}_{th} = G_{th} = \beta n_{no} p_{no}$$

n-doped S.

<u>nonequilibrium</u> (steady state)

$$\mathcal{R} = G_L + G_{th} = \beta (n_{no} + \Delta n)(p_{no} + \Delta p)$$

n-doped S.

Furthermore, we know that $\Delta n = \Delta p$ (charge neutrality)
The recombination rate in the <u>nonequilibrium</u> case can now be written as

$$\mathcal{R} = G_L + G_{th} = \beta [n_{no} p_{no} + \Delta p (n_{no} + p_{no}) + \Delta p^2]$$

$$= G_{th} + \beta \Delta p [n_{no} + p_{no} + \Delta p]$$

The net recombination rate $\underline{\mathcal{U} \equiv \mathcal{R} - G_{th} \overset{\text{steady state}}{=} G_L}$ is then

$$\left| \mathcal{U} = \beta \Delta p [n_{no} + p_{no} + \Delta p] \right|$$

In case of <u>low-level injection</u>, we obtain

$$\left| \mathcal{U} \cong \beta \Delta p n_{no} = \frac{\Delta p}{\tau_p} = \frac{(p_n - p_{no})}{\tau_p} \right| \qquad (\Delta p \ll n_{no})$$

where

$$\underline{\underline{\tau_p = \frac{1}{\beta n_{no}}}} \quad \text{excess minority carrier lifetime}$$

Thus

$$\left| p_n = p_{no} + \Delta p = p_{no} + \underbrace{\tau_p G_L}_{\Delta p} \right| \qquad (G_L = \mathcal{U})$$

steady-state

The net rate of change of hole concentration is given by

$$\frac{dp_n}{dt} = G_L + G_{th} - R(t) = G_L - U(t)$$

with $U(t) = \frac{1}{\tau_p}[p_n(t) - p_{no}]$ we obtain

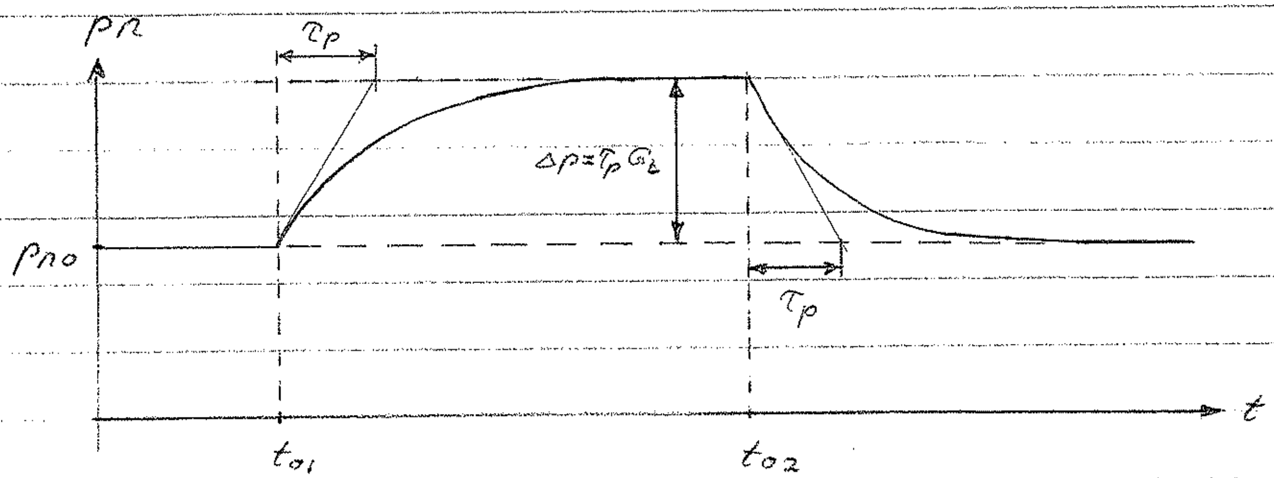
$$\boxed{\frac{dp_n}{dt} = G_L + \frac{p_{no}}{\tau_p} - \frac{p_n(t)}{\tau_p}}$$

Solution 1: light source is switched on at $t = t_0$

$$\boxed{p_n(t) = p_{no} + \tau_p G_L \left[1 - e^{-\frac{(t-t_0)}{\tau_p}}\right]} \quad t \geq t_0$$

Solution 2: light is turned off at $t = t_0$

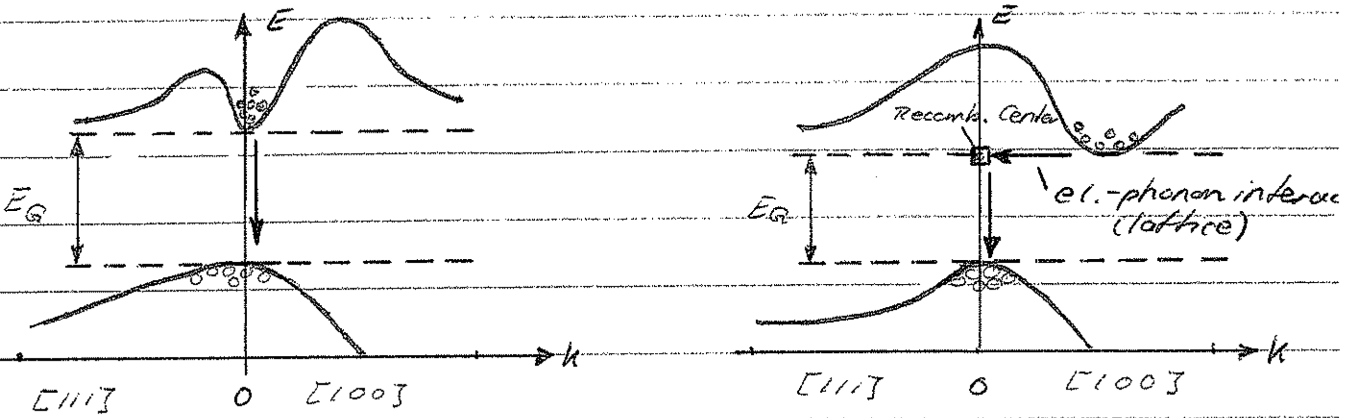
$$\boxed{p_n(t) = p_{no} + \tau_p G_L \, e^{-\frac{(t-t_0)}{\tau_p}}} \quad t \geq t_0$$



Indirect Recombination

For indirect-bandgap semiconductors (e.g. Si), a direct recombination process is very unlikely, because the electrons at the bottom of the conduction band have nonzero crystal momentum with respect to holes at the top of the valence band.

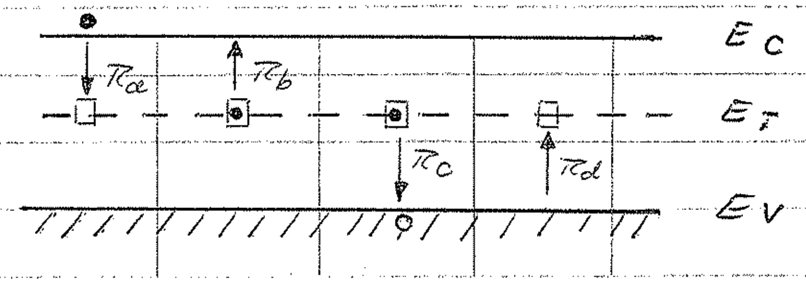
direct semicond. (e.g. GaAs) indirect semicond. (e.g. Si)



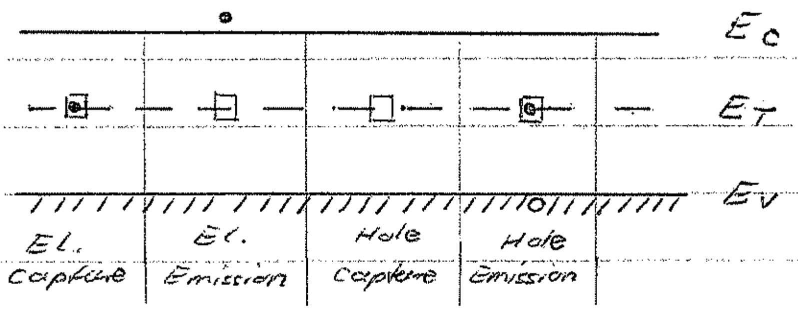
The dominant recomb. process in indirect-bandgap semicond. is, therefore, indirect transition via localized energy states in the forbidden energy gap. These states are realized by certain impurities (e.g. Au) or lattice defects.
Because the transition probability depends on the energy differences between the step and the cond. and valence band edges, these intermediate states substantially enhance the recombination process.

The 4 transitions that occur in the recombination process through intermediate-level states (also called recombination centers) are shown bellow.

before transition



after transition



El. Capture

$$\boxed{R_a = V_{th}\, \sigma_n\, n\, N_T (1 - F)}$$

Fermi distr. for Traps

Capture cross section Trap density

at room temp. $V_{th}\, \sigma_n \cong 10^5 \times 10^{-19} \ \frac{m^3}{s} = 10^{-14} \ \frac{m^3}{s}$

Volume swept out per unit time by electron

$$F = \frac{1}{1 + e^{(E_T - E_F)/kT}}$$

$$\Rightarrow \boxed{r_a = \frac{v_{th}\,\sigma_n\,n\,N_T}{1+\exp[(E_F-E_T)/kT]}}$$

El. Emission

$$\boxed{r_b = \underset{\text{Emission prob.}}{e_n}\, N_T\, F}$$

at thermal equilibrium $\Rightarrow r_{a_0} = r_{b_0}$

$$\Rightarrow e_{n_0} = v_{th}\,\sigma_n\,n_0\,\frac{(1-F)}{F} = v_{th}\,\sigma_n\,n_0\,\exp[(E_T-E_F)/kT]$$

$$\Rightarrow \boxed{r_{b_0} = v_{th}\,\sigma_n\,n_0\,N_T\,(1-F)}$$

Hole Capture

$$\boxed{r_c = v_{th}\,\sigma_p\,p\,N_T\,F}$$

Hole Emission

$$\boxed{r_d = e_p\,N_T\,(1-F)}$$

at thermal equilibrium $\Rightarrow r_{c_0} = r_{d_0}$

$$\Rightarrow e_{p_0} = v_{th}\,\sigma_p\,p_0\,\frac{F}{(1-F)} = v_{th}\,\sigma_p\,p_0\,\exp[-(E_T-E_F)/kT]$$

$$\Rightarrow \boxed{r_{d_0} = v_{th}\,\sigma_p\,p_0\,N_T\,F}$$

The net recombination rate U is given by

$$\boxed{\begin{aligned} U(t) &= R_a(t) - R_b(t) = R_c(t) - R_d(t) \\ &= \frac{v_{th}\,\sigma_n \sigma_p N_T\,(p_n n_n - n_{io}^2)}{\sigma_p\left[p_n + n_{io}\,e^{(E_{Fi}-E_T)/kT}\right] + \sigma_n\left[n_n + n_{io}\,e^{(E_T-E_{Fi})/kT}\right]} \end{aligned}}$$

Under low-level injection and $n_n \gg n_{io}\,e^{(E_T-E_{Fi})/kT}$ we can simplify the above formula to

$$\boxed{U(t) \cong v_{th}\,\sigma_p N_T\,(p_n - p_{no})}$$

The lifetime of a minority carrier, in our case a hole, can be defined as

$$\underline{\underline{\tau_p \equiv \frac{1}{v_{th}\,\sigma_p N_T}}}$$

<u>Recall</u>: Lifetime of minority carrier in direct-bandgap semiconductor

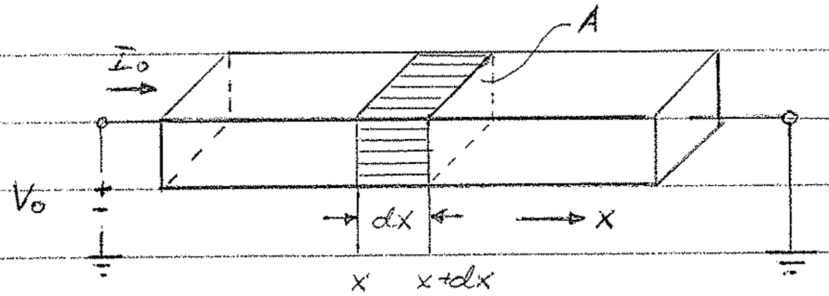
$$\underline{\tau_p \equiv \frac{1}{\beta n_n}} = \frac{1}{v_{th}\,\sigma_p n_n}$$

Note: In contrast to direct-bandgap semiconductors, the minority carrier lifetime in an indirect-bandgap semicond. is independent of the minority carrier concentration.

5.5 Continuity Equation

In this section we shall consider the overall effect when drift, diffusion, and recombination occur simultaneously in a semiconductor material. The governing eq. is called continuity equation.

In order to derive the one-dimensional continuity eq., we consider an infinitesimal slice of a semiconductor with thickness dx.



The overall change in the number of electrons per unit time in the slice of volume $A\,dx$ is

$$\frac{\partial n(x,t)}{\partial t} A\,dx = \left[\underbrace{\frac{J_n(x,t)A}{-q}}_{\text{carrier flow into } A\,dx} - \underbrace{\frac{J_n(x+dx,t)A}{-q}}_{\text{carrier flow out of } A\,dx} \right] + (\underbrace{G_n}_{\text{Generation}} - \overbrace{R_n}^{\text{Recombination}})A\,dx$$

with $J_n(x+dx,t) \cong J_n(x,t) + \frac{\partial J_n}{\partial x}dx$ we obtain

$$\boxed{\frac{\partial n(x,t)}{\partial t} = \frac{1}{q}\frac{\partial J_n(x,t)}{\partial x} + (G_n - R_n)}$$

Similarly, we can write for the change of holes per unit time in unit volume

$$\boxed{\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q}\frac{\partial J_p(x,t)}{\partial x} + (G_p - R_p)}$$

For the special case of low-level injection, the continuity eq. for minority carriers are ($R_n = \frac{\Delta n}{\tau_n}$; $R_p = \frac{\Delta p}{\tau_p}$)

$$\frac{\partial n_p}{\partial t} = \mu_n \frac{\partial}{\partial x}(n_p \cdot \bar{E}) + D_n \frac{\partial}{\partial x}\left(\frac{\partial n_p}{\partial x}\right) + G_n + \frac{n_{po}}{\tau_n} - \frac{n_p}{\tau_n}$$

$$\frac{\partial p_n}{\partial t} = -\mu_p \frac{\partial}{\partial x}(p_n \cdot \bar{E}) + D_p \frac{\partial}{\partial x}\left(\frac{\partial p_n}{\partial x}\right) + G_p + \frac{p_{no}}{\tau_p} - \frac{p_n}{\tau_p}$$

Finally, using Einstein's eq. $D = \frac{kT}{q}\mu$, we obtain

$$\frac{\partial n_p}{\partial t} = \mu_n\left(\frac{\partial n_p}{\partial x}E + n_p\frac{\partial E}{\partial x} + \frac{kT}{q}\frac{\partial^2 n_p}{\partial x^2}\right) + G_n + \frac{n_{po}}{\tau_n} - \frac{n_p}{\tau_n}$$

$$\frac{\partial p_n}{\partial t} = -\mu_p\left(\frac{\partial p_n}{\partial x}E + p_n\frac{\partial E}{\partial x} + \frac{kT}{q}\frac{\partial^2 p_n}{\partial x^2}\right) + G_p + \frac{p_{no}}{\tau_p} - \frac{p_n}{\tau_p}$$

Continuity eq. for minority carriers under low-level injection

Note: In addition to the continuity eq., Poisson's eq. must also be satisfied, i.e. in 1-dim.

$$\frac{\partial E}{\partial x} = \frac{1}{\epsilon}\rho \quad \text{where} \quad \rho = q(p^+ - n^- + N_D^+ - N_A^-)$$

dielectric permittivity of semiconductor

Minority Carrier Diffusion equations

They are derived from the continuity eqs. by invoking the following set of simplifying assumptions:

1. The analysis is limited to minority carriers
2. $\mathcal{E}_{G} \cong 0$ in the semic. region to be analyzed
3. Minority carrier conc. are independent of position e.g. $n_0 \neq n_0(x)$
4. Low-level injection conditions prevail
5. No other processes, except possibly photogeneration, are taking place

Rewriting the continuity eqs. under these conditions yields:

$$\frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} + G_n - R_n \qquad \left(G_n = G_L \text{ and } R_n = \frac{\Delta n}{\tau_n}\right)$$

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} + G_p - R_p \qquad \left(G_p = G_L \text{ and } R_p = \frac{\Delta p}{\tau_p}\right)$$

Since $\frac{\partial n}{\partial t} = \frac{\partial}{\partial t}[n_0] + \frac{\partial}{\partial t}[\Delta n] = \frac{\partial \Delta n}{\partial t}$ we finally obtain:

$$\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} + G_L - \frac{\Delta n}{\tau_n}$$

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{\partial^2 \Delta p}{\partial x^2} + G_L - \frac{\Delta p}{\tau_p}$$

minority carrier diffusion eqs.

Summary Chapters IV and V

Carrier Concentration

$$n_o(T) = N_c(T)\exp\left[-\frac{E_C - E_F}{kT}\right] \qquad (E_c - E_F) > 3kT$$

$$p_o(T) = N_V(T)\exp\left[-\frac{E_F - E_V}{kT}\right] \qquad (E_F - E_V) > 3kT$$

$$N(T) = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$n_o(T)\,p_o(T) = n_{io}^2(T) \qquad \text{(Law of mass action)}$$

Carrier Transport

$$\bar{J}_n = q\mu_n n\bar{E} + qD_n\frac{\partial n}{\partial x}$$

$$\bar{J}_p = q\mu_p p\bar{E} - qD_p\frac{\partial p}{\partial x}$$

$$\bar{J}_{Tot} = \bar{J}_n + \bar{J}_p$$

$$D = \frac{kT}{q}\mu \qquad \text{(Einstein Rel.)}$$

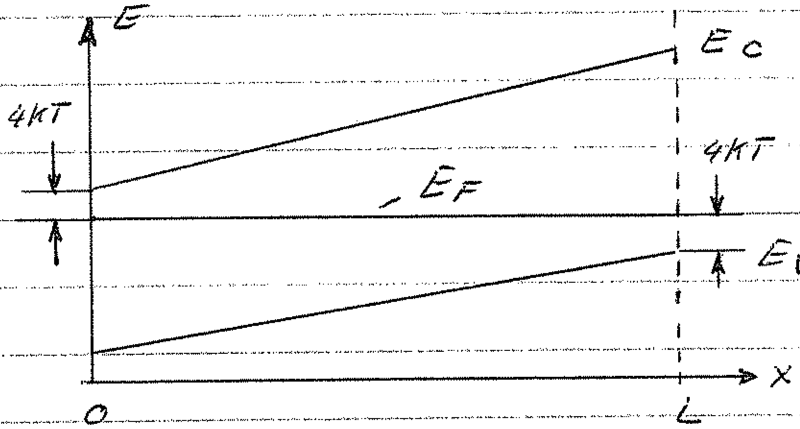
Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - R_p$$

$$\frac{\partial E}{\partial x} = \frac{1}{\epsilon}\rho \qquad \text{(Poisson eq.)} \qquad \rho = q[p^+ - n^- + N_D^+ - N_A^-]$$

Problem #1 A bar of Silicon ($T = 290°K$) exhibits the following energy-band diagram: (thermal equilibrium)



a) Sketch $n(x)$ and $p(x)$

b) Determine the el. Field inside the semicond. if $L = 10^{-2}\,m$

Solution

a) $n(x) = N_c \exp\left[-\frac{E_c - E_F}{kT}\right]$

$p(x) = N_V \exp\left[-\frac{E_F - E_V}{kT}\right]$

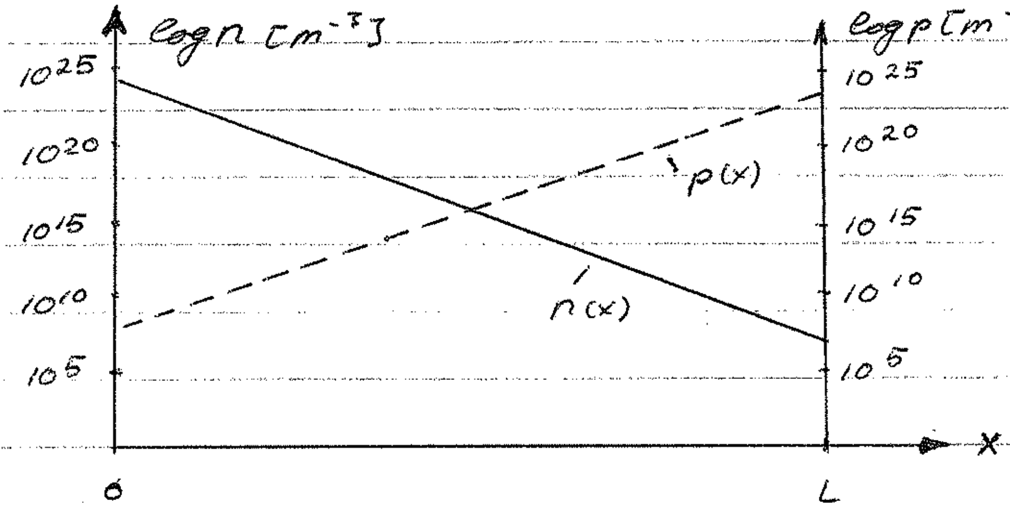
$(E_c - E_F) = 4kT + (E_g - 8kT)\frac{x}{L}$

$(E_F - E_V) = E_g - 4kT - (E_g - 8kT)\frac{x}{L}$

$$\Rightarrow \quad \left\| \begin{array}{l} n(x) = N_c\, e^{-4}\, e^{-\left(\frac{E_g}{kT} - 8\right)\frac{x}{L}} \\ p(x) = N_V\, e^{-\left(\frac{E_g}{kT} - 4\right)}\, e^{+\left(\frac{E_g}{kT} - 8\right)\frac{x}{L}} \end{array} \right\|$$

$kT = 0.025\,eV \quad \Rightarrow \quad \frac{E_g}{kT} = 44.8$

$$\Rightarrow \boxed{\begin{aligned} n(x) &= N_c\, e^{-4}\, e^{-36.8\frac{x}{L}} = 5.1\times 10^{23} e^{-36.8\frac{x}{L}}\ m^{-3} \\ p(x) &= N_V\, e^{-40.8}\, e^{+36.8\frac{x}{L}} = 3.4\times 10^{7} e^{+36.8\frac{x}{L}}\ m^{-3} \end{aligned}}$$



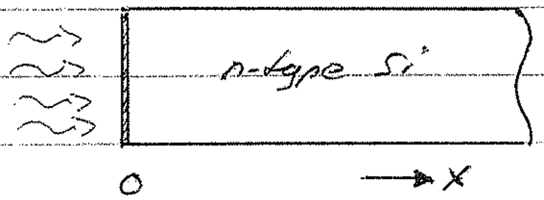
b) $E_{el}(x) = -\frac{d\psi(x)}{dx}$

$\psi(x) = -\frac{1}{q} E_c(x) = -\frac{1}{q}\left[E_{c0} + (E_g - 8kT)\frac{x}{L}\right]$

$\Rightarrow E_{el}(x) = \frac{1}{q}(E_g - 8kT)\frac{1}{L} = \underline{const}$

$\underline{\underline{E_{el} = 92\ \frac{V}{m}}}$

Problem #2 A uniformly doped semi-infinite bar of silicon with $N_D = 10^{21}\ m^{-3}$ is illuminated from the left hand side so as to create $\Delta p_{n0} = 10^{16}\ m^{-3}$ excess holes at $x = 0$. The wavelength of the illumination is such that no light penetrates into the interior ($x \geq 0$) of the bar.
Determine $\Delta p_n(x)$.



__Solution__

Problem discusses a steady-state condition.
Boundary conditions for Δp_n
1) $\Delta p_n(0) = \Delta p_{n0} = 10^{16}\ m^{-3}$
2) $\Delta p_n(\infty) = \Delta p_{n\infty} = 0$ (perturbation due to the light source at $x=0$ cannot extend to $x=\infty$)

Minority-carrier diffusion eq. for __steady-state__ cond.

$$\left| \frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} + G_L - \frac{\Delta p_n}{\tau_p} = 0 \right|$$

$$\Rightarrow \Delta p_n(x) = G_L \tau_p + D_p \tau_p \frac{\partial^2 \Delta p_n(x)}{\partial x^2}$$

for $x \geq 0$ we have $\underline{G_L = 0}$

Thus

$$\left| \Delta p_n(x) = D_p \tau_p \frac{\partial^2 \Delta p_n(x)}{\partial x^2} \right| \qquad x \geq 0$$

Solution of above diff. eq.

$$\left\| \Delta p_n(x) = C_1 e^{kx} + C_2 e^{-kx} \right\|$$

$$\frac{\partial^2 \Delta p_n}{\partial x^2} = k^2 \Delta p_n \quad \Rightarrow \quad \underline{\underline{k^2 = \frac{1}{D_p \tau_p}}}$$

Boundary cond.

$$\left| \begin{array}{ll} \Delta p_n(\infty) = 0 & \Rightarrow C_1 = 0 \\ \Delta p_n(0) = \Delta p_{no} & \Rightarrow C_2 = \Delta p_{no} \end{array} \right|$$

$$\Rightarrow \left\| \Delta p_n(x) = \Delta p_{no} \exp\left[- \frac{x}{\sqrt{D_p \tau_p}} \right] \right\|$$

Note: $D_p = V_{th} \cdot L_p = \frac{L_p^2}{\tau_p} \quad \Rightarrow \quad \underline{\sqrt{D_p \tau_p} = L_p}$ (Diff. Length)