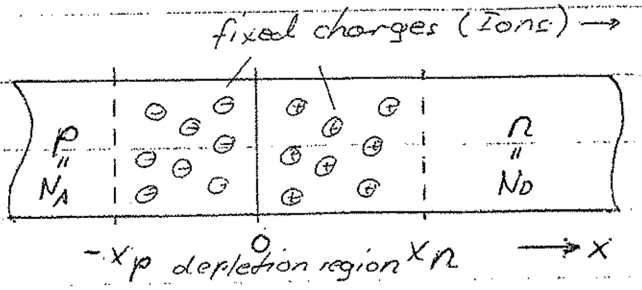
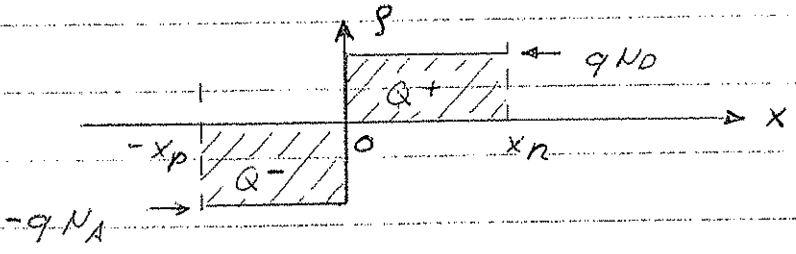
VI Semiconductor Junctions

6.1 The p-n Junction in equilibrium

abrupt junction Diode (alloying semic with impurity, e.g. Al)

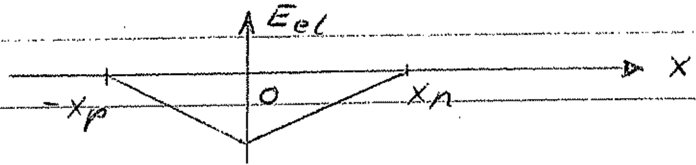


charge Density



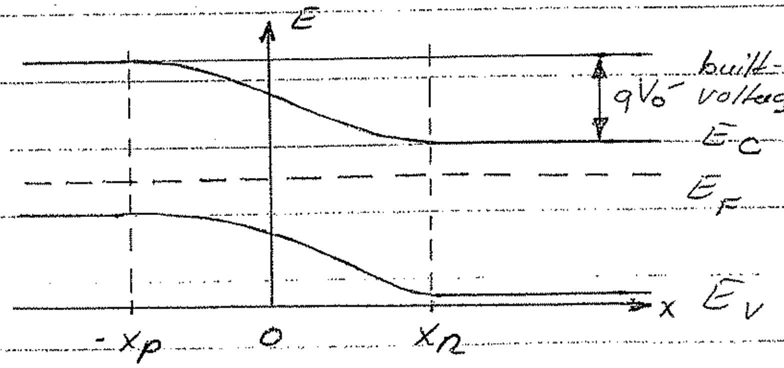
electrical Field

$$E_{el} = \frac{1}{\varepsilon}\int \rho(x)\,dx$$



Energy-Band

$$E_c(x) = q\int E_{el}(x)\,dx = \frac{q}{\varepsilon}\iint \rho(x)\,dx^2$$



Quantitative Relationships for ideal junction

charge density

$$\rho(x) = \begin{cases} -q N_A & -x_p < x \le 0 \\ q N_D & 0 < x \le x_n \\ 0 & \text{else} \end{cases}$$

Note: $x_p N_A \overset{\text{charge neutrality}}{=} x_n N_D$

el. Field

$$E_{el}(x) = \frac{1}{\varepsilon}\int \rho(x)\,dx = \begin{cases} -\frac{q}{\varepsilon} N_A (x + x_p) & -x_p < x \le 0 \\ -\frac{q}{\varepsilon} \underbrace{N_A \left(x_p - \frac{N_D}{N_A} x\right)}_{\overset{\shortparallel}{N_D (x_n - x)}} & 0 < x \le x_n \\ 0 & \text{else} \end{cases}$$

el. stat. Potential

$$\psi(x) = -\int E_{el}(x)\,dx = \begin{cases} 0 & x < -x_p \\ \frac{q}{\varepsilon} N_A \left(\frac{1}{2} x_p^2 + x x_p + \frac{1}{2} x^2\right) & -x_p < x \le 0 \\ \frac{q}{\varepsilon} N_A \left(\frac{1}{2} x_p^2 + x x_p - \frac{1}{2}\frac{N_D}{N_A} x^2\right) & 0 < x \le x_n \\ \frac{q}{\varepsilon} N_D x_n^2 \frac{1}{2}\left(1 + \frac{N_D}{N_A}\right) & x > x_n \end{cases}$$

Energy

$$E(x) = -q\,\psi(x) = \begin{cases} 0 & x < -x_p \\ -\frac{q^2}{\varepsilon} N_A \left(\frac{1}{2} x_p^2 + x x_p + \frac{1}{2} x^2\right) & -x_p < x \le 0 \\ -\frac{q^2}{\varepsilon} N_A \left(\frac{1}{2} x_p^2 + x x_p - \frac{1}{2}\frac{N_D}{N_A} x^2\right) & 0 < x \le x_n \\ -\frac{q^2}{\varepsilon} N_D x_n^2 \frac{1}{2}\left(1 + \frac{N_D}{N_A}\right) & x > x_n \end{cases}$$

The ideal p-n Junction (Thermal eq.)

Charge Density

charge Neutrality

$q N_A x_p = q N_D x_n$

$$\rho(x) = \begin{cases} -q N_A & -x_p \leq x < 0 \\ q N_D & 0 < x \leq x_n \\ 0 & \text{else} \end{cases}$$

Electric Field

$$E_{el}(x) = \frac{1}{\varepsilon} \int \rho(x)\, dx = \begin{cases} -\frac{q}{\varepsilon} N_A (x + x_p) & -x_p \leq x < 0 \\ -\frac{q}{\varepsilon} N_D (x_n - x) & 0 < x \leq x_n \\ 0 & \text{else} \end{cases}$$

El. stat. Potential

$$\psi(x) = -\int E_{el}(x)\, dx = \begin{cases} 0 & x < -x_p \\ \frac{q}{2\varepsilon} N_A (x_p^2 + 2 x x_p + x^2) & -x_p \leq x < 0 \\ \frac{q}{2\varepsilon} N_D \left(\frac{N_D}{N_A} x_n^2 + 2 x x_n - x^2\right) & 0 < x \leq x_n \\ \frac{q}{2\varepsilon} N_D x_n^2 \left(1 + \frac{N_D}{N_A}\right) & x > x_n \end{cases}$$

Energy

$$E(x) = -q\, \psi(x)$$

Built-in voltage across depletion region V_0

At thermal equilibrium, the current must be zero everywhere.

and

$$J_n = q\left[n_0 \mu_n E + D_n \frac{dn_0}{dx}\right] = 0$$

$$J_p = q\left[p_0 \mu_p E - D_p \frac{dp_0}{dx}\right] = 0$$

Solving for E

$$E = -\frac{1}{n_0 \mu_n} D_n \frac{dn_0}{dx} = -\frac{kT}{q}\frac{1}{n_0}\frac{dn_0}{dx}$$

or

$$E = \frac{1}{p_0 \mu_p} D_p \frac{dp_0}{dx} = \frac{kT}{q}\frac{1}{p_0}\frac{dp_0}{dx}$$

Finally: $\psi(x) = -\int E(x)\,dx$ el. stat. potential

$V_0 = \psi(x_n) - \psi(-x_p)$

$$V_0 = \frac{kT}{q}\int_{-x_p}^{x_n}\frac{dn_0}{n_0} = \frac{kT}{q}\ln\left[\frac{n_0(x_n)}{n_0(-x_p)}\right]$$

or

$$V_0 = -\frac{kT}{q}\int_{-x_p}^{x_n}\frac{dp_0}{p_0} = -\frac{kT}{q}\ln\left[\frac{p_0(x_n)}{p_0(-x_p)}\right]$$

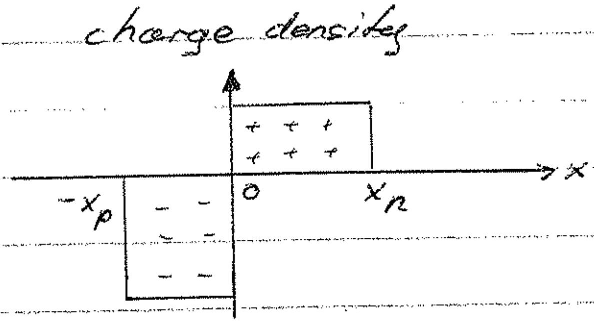
Note: $n_0(x_n) = N_D$ $n_0(-x_p) = \frac{n_{i0}^2}{N_A}$ or $p_0(x_n) = \frac{n_{i0}^2}{N_D}$ $p_0(-x_p) = N_A$

$$\Rightarrow \quad \boxed{V_0 = \frac{kT}{q}\ln\left[\frac{N_A N_D}{n_{i0}^2}\right]}$$

e.g. Si at 300°K, $N_A = N_D = 10^{21}\,m^{-3}$ $n_{i0} = 1.5 \times 10^{16}\,m^{-3}$

$\Rightarrow$ $V_0 \cong 0.578\,V$

Width of depletion region w_d



$$\boxed{w_d = x_p + x_n}$$

cross section of junction

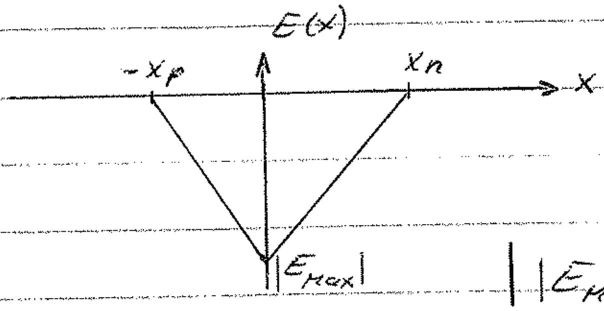
Charge neutrality: $N_A x_p A = N_D x_n A$

$$\boxed{x_p = x_n N_D / N_A}$$

Poisson's eq. $\operatorname{div} \bar{E} = \frac{1}{\varepsilon}\rho$ (e.g. Si: $\varepsilon = 11.7 \times \varepsilon_0$)

Solving for $E(x)$ yields

$$\boxed{E(x) = \begin{cases} -\frac{q}{\varepsilon} N_A (x + x_p) & -x_p \leq x \leq 0 \\ -\frac{q}{\varepsilon} N_A \underbrace{\left(x_p - \frac{N_D}{N_A} x\right)}_{= N_D (x_n - x)} & 0 < x \leq x_n \\ 0 & \text{else} \end{cases}}$$



$$\boxed{|E_{Max}| = |E(0)| = \frac{q}{\varepsilon} N_A x_p = \frac{q}{\varepsilon} N_D x_n}$$

At the junction ($x=0$) the el. field is maximum in magnitude.

el. stat. potential $\psi(x) = -\int E(x)\,dx$

Solution

$$\psi(x) = \begin{cases} 0 & x < -x_p \\ \frac{q}{\epsilon} N_A \left(\frac{1}{2} x_p^2 + x x_p + \frac{1}{2} x^2\right) & -x_p < x \leq 0 \\ \frac{q}{\epsilon} N_A \left(\frac{1}{2} x_p^2 + x x_p - \frac{1}{2} \frac{N_D}{N_A} x^2\right) & 0 < x \leq x_n \\ \frac{q}{\epsilon} N_D x_n^2 \frac{1}{2} \left(1 + \frac{N_D}{N_A}\right) & x > x_n \end{cases}$$

Voltage across junction: (thermal equilibrium)

$$V_0 = \psi(x_n) - \psi(-x_p) = \frac{q}{\epsilon} N_D x_n^2 \frac{1}{2}\left(1 + \frac{N_D}{N_A}\right)$$

$$= \frac{kT}{q} \ln\left[\frac{N_A N_D}{n_{i0}^2}\right]$$

$J_n = J_p = 0$ ← cond. from thermal equil.

$$\Rightarrow \quad \frac{q}{\epsilon} N_D x_n^2 \frac{1}{2}\left(1 + \frac{N_D}{N_A}\right) = \frac{kT}{q} \ln\left[\frac{N_A N_D}{n_{i0}^2}\right]$$

$$x_n^2 = 2 \frac{\epsilon}{q} \frac{N_A \cdot V_0}{N_D (N_A + N_D)} = 2 \frac{\epsilon}{q^2} \frac{kT N_A \ln\left[\frac{N_A N_D}{n_{i0}^2}\right]}{N_D (N_A + N_D)}$$

$$x_p^2 = 2 \frac{\epsilon}{q} \frac{N_D \cdot V_0}{N_A (N_A + N_D)} = 2 \frac{\epsilon}{q^2} \frac{kT N_D \ln\left[\frac{N_A N_D}{n_{i0}^2}\right]}{N_A (N_A + N_D)}$$

$$\boxed{w_d = x_p + x_n = \sqrt{2 \frac{\epsilon V_0 (N_A^2 + N_D^2)}{q N_A N_D (N_A + N_D)}} = \sqrt{2 \frac{\epsilon kT (N_A^2 + N_D^2) \ln\left[\frac{N_A N_D}{n_{i0}^2}\right]}{q^2 N_A N_D (N_A + N_D)}}}$$

e.g. Si: $N_A = N_D = 10^{21} m^{-3}$; $n_{i0} = 1.5 \times 10^{16} m^{-3}$

$x_n = x_p = 4.3 \times 10^{-7} m$

$\underline{\underline{w_d = 8.6 \times 10^{-7} m}}$

Problem: Given is a Si p-n junction ($T = 300°K$) with

$N_A = 5 \times 10^{20}\, m^{-3}$

$N_D = 10^{21}\, m^{-3}$; $n_{io} = 1.5 \times 10^{16} m^{-3}$; $\varepsilon_r = 11.7$

Determine

a) V_0

b) x_n and x_p

c) $E_{el}(x)$, $\psi(x)$ and Energy-Band Diagram

Solution

a) $$\underline{V_0} = \frac{kT}{q} \ln\left[\frac{N_A N_D}{n_{io}^2}\right] = \underline{\underline{0.56\, V}}$$

b) $$\underline{x_n} = \sqrt{\frac{2\varepsilon_r \varepsilon_0 N_A V_0}{q N_D (N_A + N_D)}} = \underline{\underline{4.9 \times 10^{-7} m}}$$

$$\underline{x_p} = \sqrt{\frac{2\varepsilon_r \varepsilon_0 N_D V_0}{q N_A (N_A + N_D)}} = \underline{\underline{9.8 \times 10^{-7} m}}$$

c) El. Field

$$E_{el}(x) = \frac{1}{\varepsilon_r \varepsilon_0} \int \rho(x)\, dx = \begin{cases} -\frac{q}{\varepsilon_r \varepsilon_0} N_A (x + x_p) & -x_p \le x < 0 \\ -\frac{q}{\varepsilon_r \varepsilon_0} N_D (x_n - x) & 0 < x \le x_n \\ 0 & \text{else} \end{cases}$$

$$\underline{\underline{|E_{el_{max}}|}} = -E_{el}(0) = \frac{q}{\varepsilon_r \varepsilon_0} N_A x_p = \frac{q}{\varepsilon_r \varepsilon_0} N_D x_n$$

$$= \sqrt{\frac{2q N_A N_D V_0}{\varepsilon_r \varepsilon_0 (N_A + N_D)}} = \underline{\underline{7.6 \times 10^{5} \frac{V}{m}}}$$

El. stat. Potential

$$\psi(x) = -\int E_{el}(x)\,dx = \begin{cases} 0 & x < -x_p \\ \frac{q}{2\varepsilon_r\varepsilon_0} N_A (x_p^2 + 2x x_p + x^2) & -x_p \le x < 0 \\ \frac{q}{2\varepsilon_r\varepsilon_0} N_D (\frac{N_D}{N_A} x_n^2 + 2x x_n - x^2) & 0 \le x \le x_n \\ \frac{q}{2\varepsilon_r\varepsilon_0} N_D x_n^2 (1 + \frac{N_D}{N_A}) & x > x_n \end{cases}$$

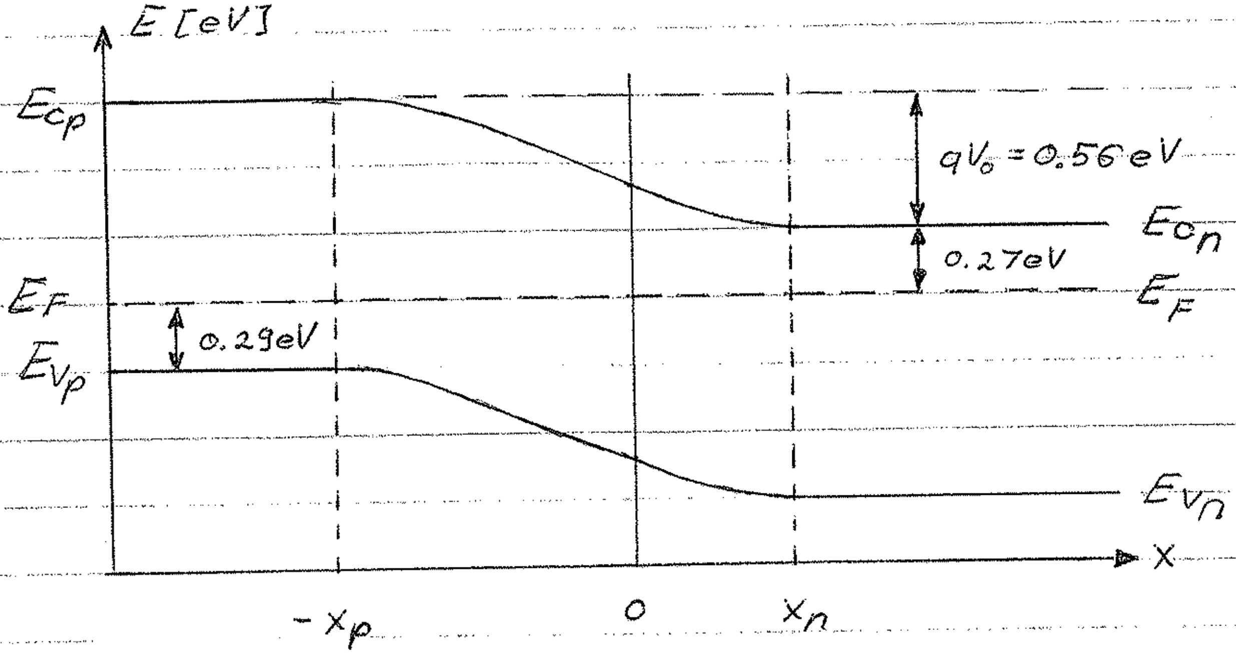
$\psi(-x_p) = 0$

$\psi(0) = 0.37\,V$

$\psi(x_n) = 0.56\,V$

Energy

$$E(x) = -q\,\psi(x)$$



Note: We have derived the formula for the depletion width under the assumption of thermal equilibrium. However, this formula is also valid if the p-n junction is reverse biased, i.e. the p-side is negative with respect to the n-side.

Thus, under reverse-bias V_a (V_a is negative)

$$\boxed{W_d = \sqrt{\frac{2\varepsilon (V_0 - V_a)(N_A^2 + N_D^2)}{q N_A N_D (N_A + N_D)}}} \qquad V_a < 0$$

Note: This eq. is also valid for small forward-bias voltages in the range of a few tenth of V_0

Depletion-Region Capacitance

The capacitance per unit area across the depletion region is defined as

$$C_d' = \left| \frac{dQ'}{dV_a} \right|$$

where Q' denotes the charge per unit area on one side of the depletion region

e.g. n-side: $Q_n' = q N_D x_n = \sqrt{\frac{2\varepsilon N_A N_D q}{(N_A + N_D)} (V_0 - V_a)}$

Hence

$$\boxed{\begin{aligned} C_d' &= \sqrt{\frac{\varepsilon N_A N_D q}{2(N_A+N_D)}}\,\frac{1}{\sqrt{V_0-V_a}} = C_{do}'\sqrt{\frac{V_0}{(V_0-V_a)}} \\ C_{do}' &\cong \sqrt{\frac{\varepsilon N_A N_D q}{2(N_A+N_D)\,V_0}} \end{aligned}}$$

C_d is a voltage controlled capacitor. It can be used as a tuning device in oscillators.

Example: Si p-n junction at 300°K

$N_A = N_D = 10^{21}\,m^{-3}$; $n_{io} = 1.5 \times 10^{16}\,m^{-3}$

$\varepsilon = 11.7 \times \varepsilon_0$

a) Built-in voltage V_0

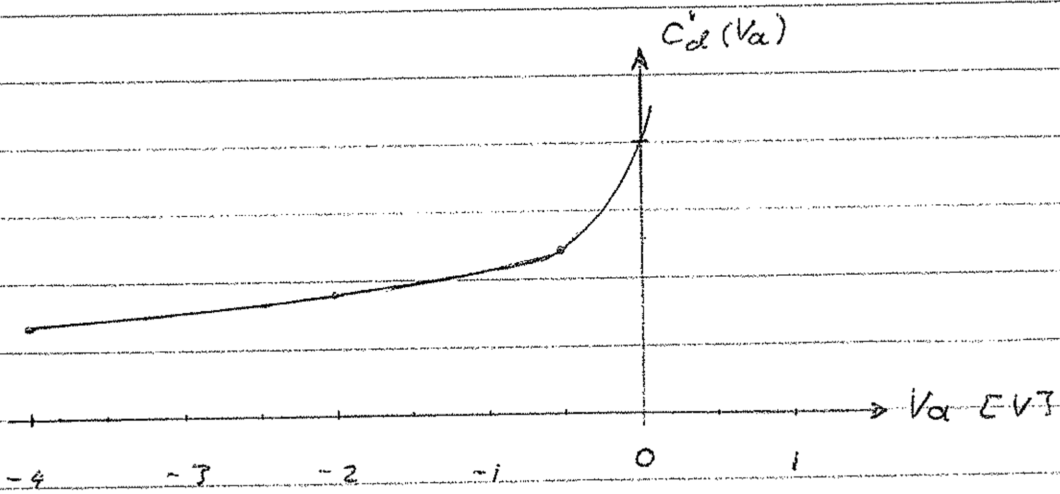
$$\underline{V_0} = \frac{kT}{q}\ln\left[\frac{N_A N_D}{n_{io}^2}\right] = \underline{0.578\,V}$$

b) Depletion Capacitance C_d'

$$\underline{C_{do}'} = \sqrt{\frac{\varepsilon N_A N_D q}{2(N_A+N_D)V_0}} = \underline{8.46 \times 10^{-5}\,\frac{As}{Vm^2}}$$

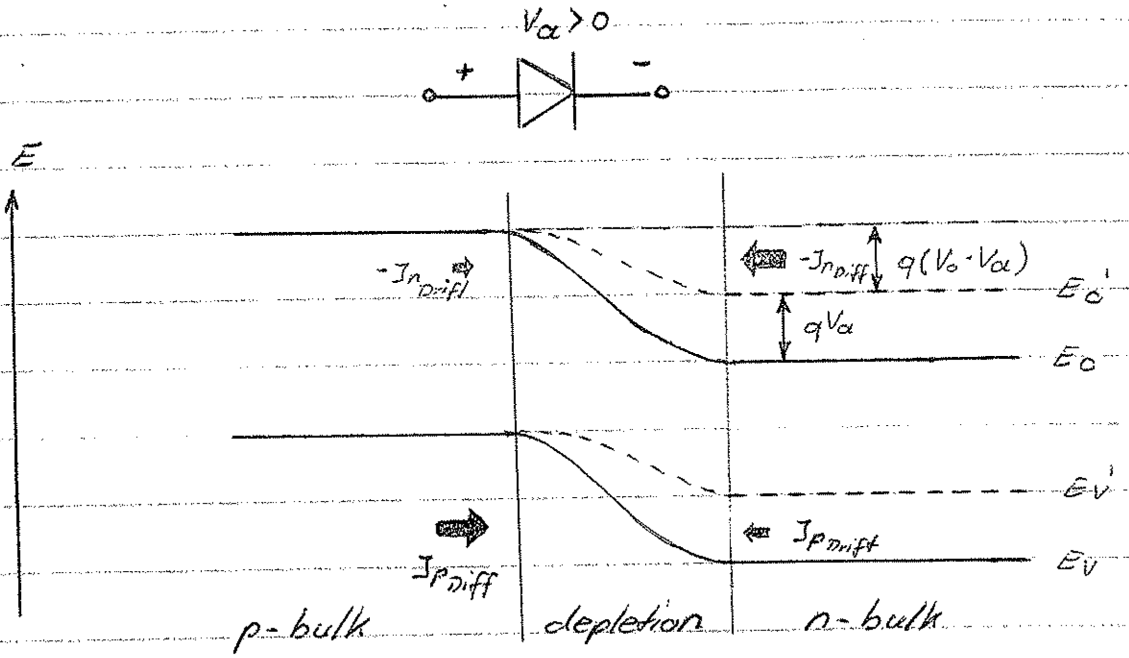
e.g. $A = 10^{-6}\,m^2$

$\Rightarrow \underline{C_{do} = 84.6\,pF}$



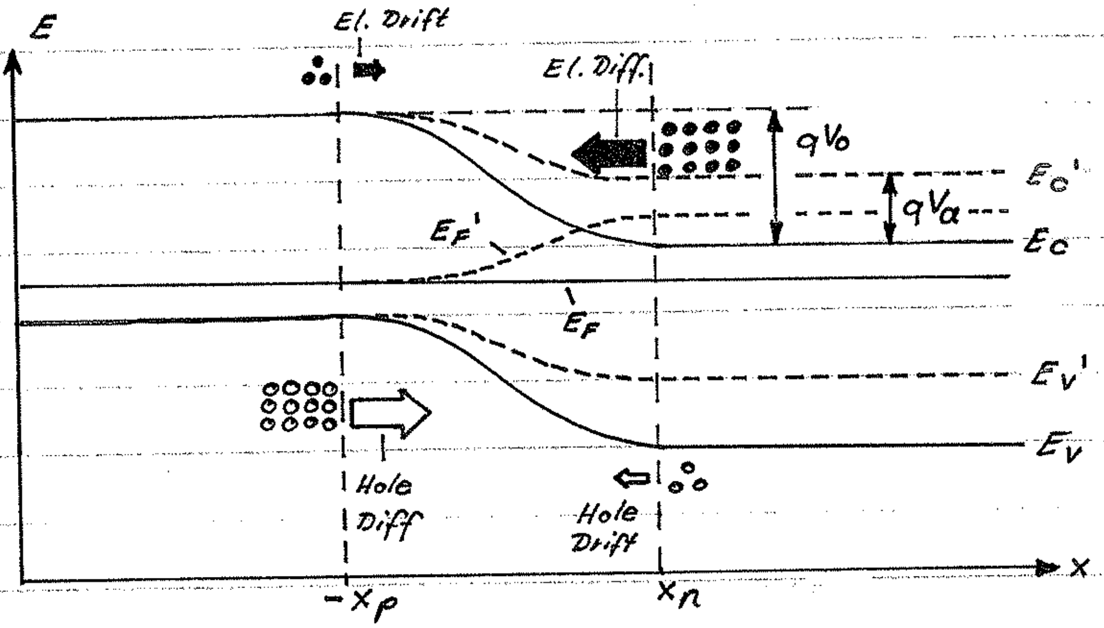
6.2 The p-n junction under nonequilibrium conditions

6.2.1 The forward-biased junction



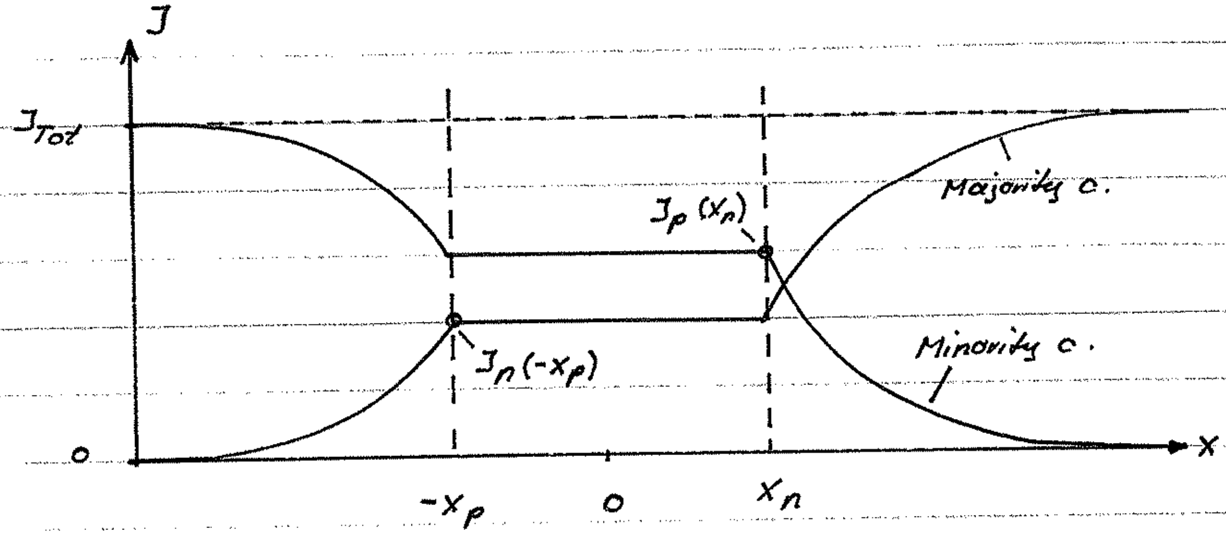
The reduced barrier yields an increase in both the electron and the hole diffusion current components over the thermal equilibrium values. A large number of holes p_p (or electrons n_n) have energies greater than the barrier height $q(V_0 - V_a)$ and therefore, more holes can diffuse into the n-material (or electrons into the p-material). The drift current components, however, remain essentially the same as their thermal equilibrium values, since a change in barrier height has almost no effect on the number of minority carriers (p_n or n_p) and their ability to drift down the potential hill.

The forward-biased p-n junction



Current densities

Simplification: Carrier generation or recombination in depletion region is neglected



Current - Voltage Relationship

From previous results, we know that the voltage drop across the depletion region is

Thermal equilibrium: $V_0 = \frac{kT}{q} \ln\left[\frac{n_0(x_n)}{n_0(-x_p)}\right]$

Forward-biased ($V_a > 0$): $V_0 - V_a = \frac{kT}{q} \ln\left[\frac{n(x_n)}{n(-x_p)}\right]$

Solving for V_a yields

$$\boxed{V_a = \frac{kT}{q} \ln\left[\frac{n_0(x_n)}{n_0(-x_p)} \frac{n(-x_p)}{n(x_n)}\right]}$$

Under low-level injection cond. $n(x_n) \cong n_0(x_n)$

$$\Rightarrow \boxed{V_a = \frac{kT}{q} \ln\left[\underbrace{\frac{n(-x_p)}{n_0(-x_p)}}_{\left(1 + \frac{\Delta n(-x_p)}{n_0(-x_p)}\right)}\right]} \Rightarrow \Delta n(-x_p) = n_0(-x_p)\left[e^{\frac{V_a q}{kT}} - 1\right]$$

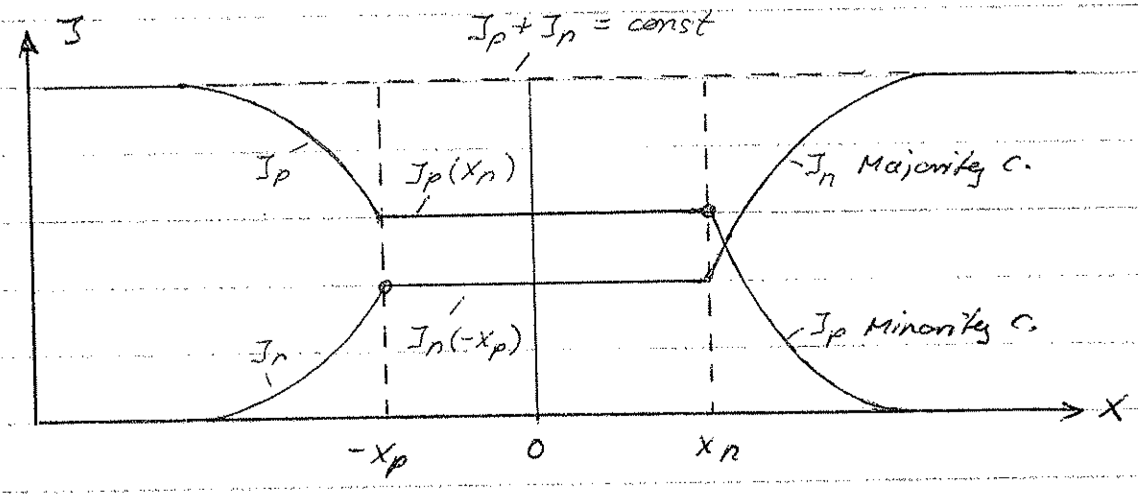
Similarly, we obtain

$$\boxed{V_a = \frac{kT}{q} \ln\left[1 + \frac{\Delta p(x_n)}{p_0(x_n)}\right]} \Rightarrow \Delta p(x_n) = p_0(x_n)\left[e^{\frac{V_a q}{kT}} - 1\right]$$

We now assume that the minority carrier current outside the depletion region is entirely due to diffusion (i.e. we neglect the drift component). Furthermore, we assume the depletion region to be narrow enough to preclude recombination between electrons and holes.

Thus $$\left| \begin{array}{l} J_p(-x_p) = J_p(x_n) \\ J_n(-x_p) = J_n(x_n) \end{array} \right|$$

The current density across the p-n junction is qualitatively shown below.



The total current across the p-n junction is therefore equal to

$$\left\| J_{Tot} = J_n(-x_p) + J_p(x_n) \right\|$$

where $$\left| \begin{array}{l} J_n(-x_p) = +q D_n \left. \frac{dn_p(x)}{dx} \right|_{x=-x_p} \\ J_p(x_n) = -q D_p \left. \frac{dp_n(x)}{dx} \right|_{x=x_n} \end{array} \right|$$

$n_p(x)$ for $x \leq -x_p$ and $p_n(x)$ for $x \geq x_n$ are obtained from the minority carrier diffusion equations under steady state conditions. (Note that $\frac{dn}{dx} = \frac{d\Delta n}{dx}$ and $\frac{dp}{dx} = \frac{d\Delta p}{dx}$)

The two minority carrier diff. eq. under steady state cond. are:

$$D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} = 0$$

$$D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} = 0$$

τ: Minority carrier lifetime

The solutions are: $L = \sqrt{D\tau}$: Diffusion length

$$\Delta n_p(x) = n_0(-x_p)\overbrace{\left[e^{\frac{qV_a}{kT}} - 1\right]}^{\Delta n(-x_p)} e^{\frac{x+x_p}{L_n}} \qquad x \leq -x_p$$

$$\Delta p_n(x) = p_0(x_n)\underbrace{\left[e^{\frac{qV_a}{kT}} - 1\right]}_{\Delta p(x_n)} e^{-\frac{x-x_n}{L_p}} \qquad x \geq x_n$$

The electron and hole diffusion currents in the depletion region are, therefore

$$J_n(-x_p) = q\frac{D_n}{L_n} n_0(-x_p)\left[e^{\frac{qV_a}{kT}} - 1\right]$$

$$J_p(x_n) = q\frac{D_p}{L_p} p_0(x_n)\left[e^{\frac{qV_a}{kT}} - 1\right]$$

Finally, we can replace $n_0(-x_p)$ by $\frac{n_{io}^2}{N_A}$ and $p_0(x_n)$ by $\frac{n_{io}^2}{N_D}$

The total current density through the junction is then

$$J_{Tot} = q n_{io}^2 \left[\frac{D_n}{L_n}\frac{1}{N_A} + \frac{D_p}{L_p}\frac{1}{N_D}\right]\left(e^{\frac{qV_a}{kT}} - 1\right)$$

$$= kT n_{io}^2 \left(\frac{\mu_n}{L_n}\frac{1}{N_A} + \frac{\mu_p}{L_p}\frac{1}{N_D}\right]\left(e^{\frac{qV_a}{kT}} - 1\right)$$

The voltage - current relationship is typically given in the following form:

$$\left\| \; I_d = I_S \left[e^{\frac{V_a}{V_T}} - 1 \right] \; \right\|$$

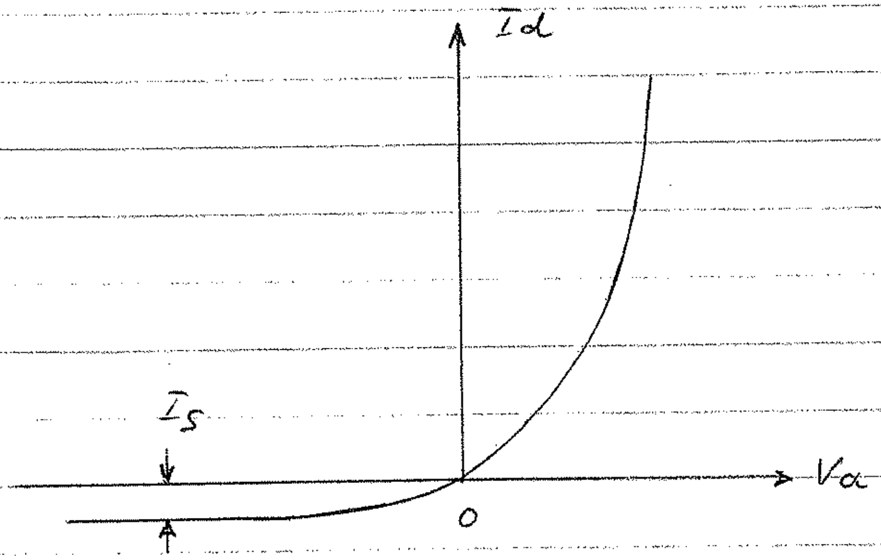
$$\left| \; I_S = A q n_{io}^2 \left[\frac{D_n}{L_n} \frac{1}{N_A} + \frac{D_p}{L_p} \frac{1}{N_D} \right] \; \right|$$ reverse saturation current

$$\left| \; V_T = \frac{kT}{q} \; \right|$$ thermal voltage

A : cross-sectional area of junction

<u>Note</u>: Most p-n junctions that occur in devices are of the p^+-n or p-n^+ type. Hence, the formula for I_S can be simplified correspondingly (e.g. for p^+-n junction $\rightarrow I_S \cong A q n_{io}^2 \frac{D_p}{L_p} \frac{1}{N_D}$)

V-I characteristics



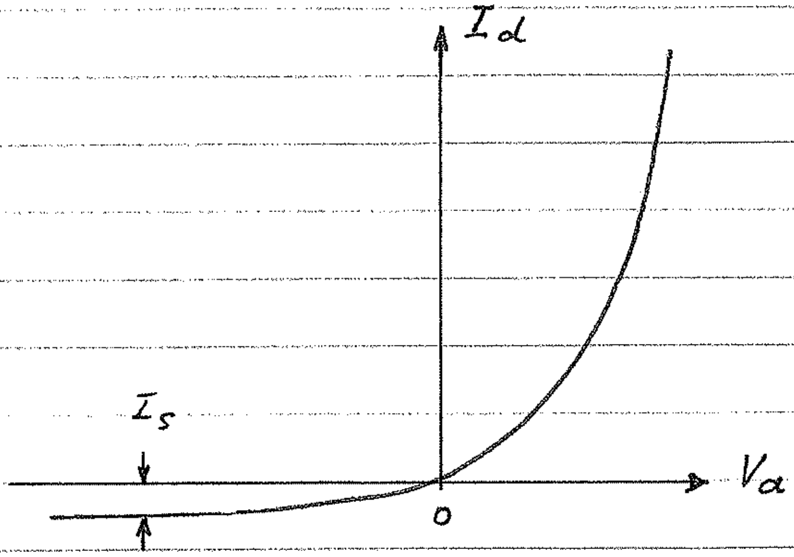
Including carrier recombination:

Recombination component

$$\left\| \; I_S = A q n_{io} \left(\frac{D_n}{L_n} \frac{n_{io}}{N_A} + \frac{D_p}{L_p} \frac{n_{io}}{N_D} + \frac{W_d}{\tau_n + \tau_p} \right) \; \right\|$$

The forward-biased p-n junction

V/I characteristics



$$\boxed{I_d = I_s \left[e^{\frac{V_a}{V_T}} - 1 \right]}$$

$V_T = \frac{kT}{q}$: Thermal voltage

$I_s = A q n_{io} \left[\frac{D_n}{L_n} \frac{n_{io}}{N_A} + \frac{D_p}{L_p} \frac{n_{io}}{N_D} \right]$: reverse sat. curr.

A: cross-sectional area of junction

Deviations from ideal Diode Characteristics

In deriving the pn junction current under forward bias, we have applied the following 2 assumptions:

1. No electron-hole recombination occurs in the depletion region.
2. The electric field in the bulk (neutral) regions on either side of the junction is negligibly small.

Neither assumption is fully applicable in a practical junction.

Electron-hole recombinations do occur and thus increase the total current. Analysis has shown that the excess current due to recombinations in the depletion region is proportional to $n_i \cdot e^{\frac{1}{2}\frac{V_a}{V_T}}$ while the excess current due to recombinations in the neutral regions is proportional to p_n and n_p and therefore n_i^2/N_D and n_i^2/N_A and increases according to $e^{\frac{V_a}{V_T}}$.

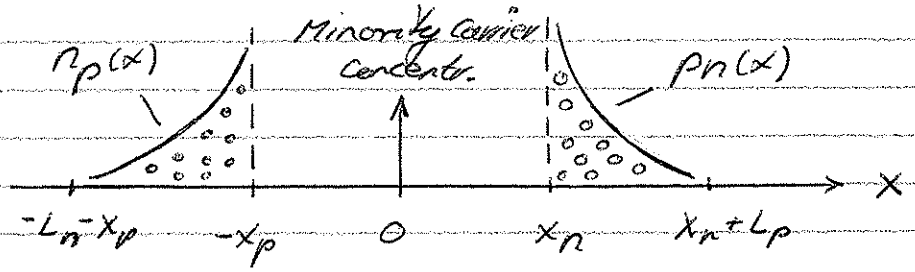
The carrier recombination effect can be included by rewriting the equation for the junction current as

$$\boxed{I_d = I_s'\left[e^{\frac{V_a}{nV_T}} - 1\right]} \qquad 1 < n < 2$$

The correction factor n in the exponent also helps to account for the non-zero el. field in the bulk regions, which leads to a higher junction terminal voltage V_a, i.e. $V_a = V_p + V_{depl} + V_n$

Charge Storage under forward Bias

In contrast to a reversly biased junction, where the change in charge is due to a change in the depletion width, the forward biased junction accumulates charge in form of excess minority carriers at the edges of the depletion layer.



The stored charge Q_S is directly proportional to the current flowing through the junction.

$$\left| Q_S = \tau_T \cdot I_d \right|$$

τ_T: Transit Time

An increase in the junction forward biased voltage thus causes a corresponding increase in minority carrier charge at the edges of the depletion layer. We can therefore define the stored-charge capacitance C_S, also referred to as diffusion capacitance as follows

$$\left| C_S = \frac{dQ_S}{dV_d} = \tau_T \frac{dI_d}{dV_d} \right|$$

or

$$\left| C_S \equiv \tau_T \cdot \frac{I_d}{V_T} \right| \qquad V_T = \frac{kT}{q}$$

To gain insight into the dependence of the transit time τ_T on technological and physical parameters, we can use the familiar junction (diode) current-voltage relationship to develop a similar equation for Q_S and express τ_T as

$$\boxed{\tau_T = \frac{Q_S}{I_d}}$$

If we assume an n^+/p junction, we can express the current as

$$\boxed{I_d \cong A \cdot q \frac{D_n}{L_n} \Delta n_p(-x_p)}$$

Approximating the minority carrier concentration on the p-side of the junction by a triangular shape allows us to write the excess charge Q_S stored as

$$\boxed{Q_S \cong q A \cdot L_n \frac{1}{2} \Delta n_p(-x_p)}$$

The resulting expression for the transit time is,

$$\boxed{\tau_T \cong \frac{1}{2} \frac{L_n^2}{D_n}}$$

We have previously shown that

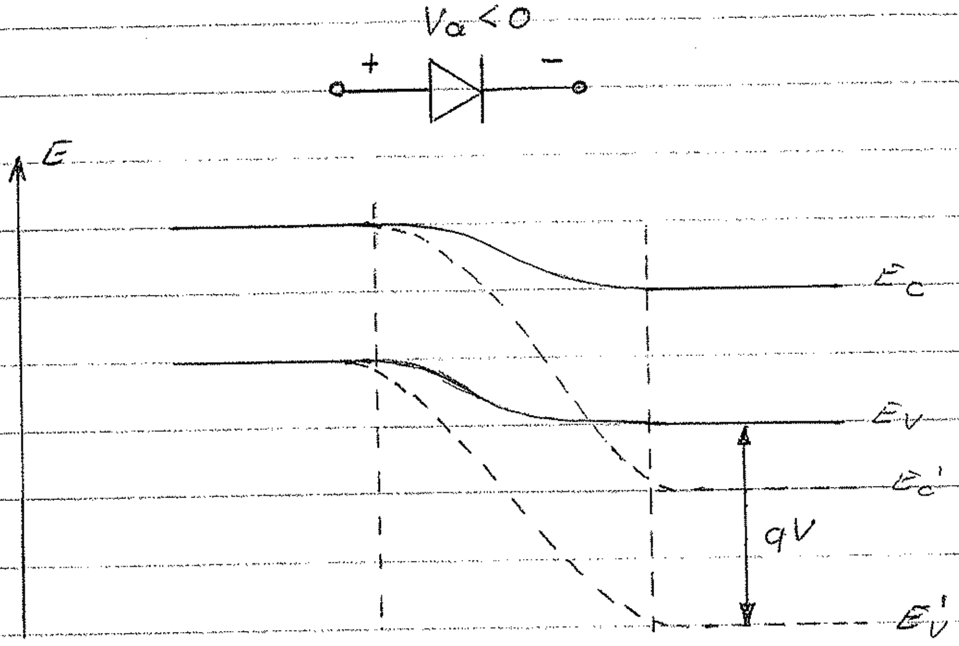
$$L_n^2 = D_n \cdot \tau_n \qquad \tau_n : \text{Minority Carrier Lifetime}$$

Thus

$$\boxed{\tau_T \cong \frac{1}{2} \tau_n} \quad n^+/p \text{ junction}$$

For junctions with more balanced doping concentrations, τ_T is a combination of τ_n and τ_p.

6.2.2 The reverse-biased p-n junction



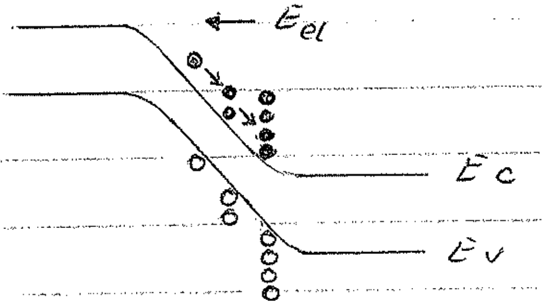
The increase in barrier height reduces the diffusion current components since less carriers have sufficient energy to get over the potential wall. However, the drift current components (holes from the n-region to the p-region or electrons from the p-region to the n-region) remain at their thermal equilibrium values. Hence, the net current through the reverse-biased junction is negative and very small. Because the drift current is essentially independent of barrier height, the current becomes a constant after several tenth of volts of reverse bias.
A large reverse voltages (e.g. 5-100 V) the reverse current can suddenly increase → junction Breakdown.

Reverse Breakdown

Breakdown is due to one of two phenomena
- ↗ Avalanche B.
- ↘ Zener B. (Tunneling)

Avalanche Breakdown

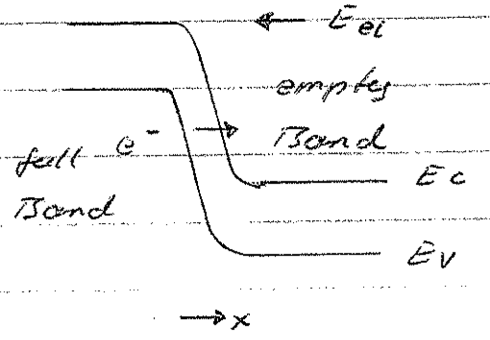
The carriers in the depletion region have sufficient energy to create additional electron-hole pairs via collisions with bound electrons

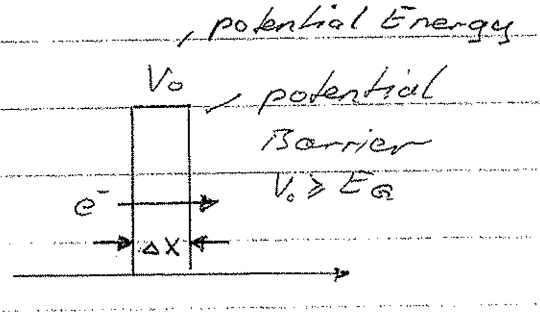


quantitative relationship for the reverse current

$$\boxed{I_d = -I_S \cdot M} \qquad M = \frac{1}{1 - \left|\frac{V_a}{V_{BR}}\right|^m} \qquad 3 \le m \le 6$$

Zener Breakdown (Tunneling)





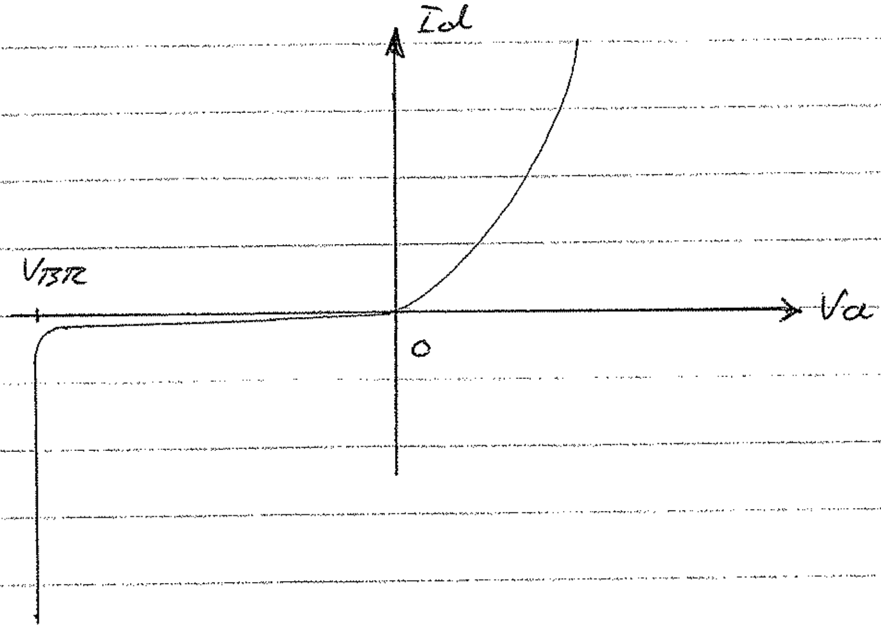
Q. M. tunneling

requirements for tunneling: thin barrier $\Delta x \leq 10^{-8} m$
high el. Field $\mathcal{E}_{el} \geq 10^{8} \frac{V}{m}$

Note A thin barrier is achieved if both sides of the junction are highly doped. Hence, zener breakdown is limited to $p^{+}-n^{+}$ junctions. $W_d = \sqrt{\frac{2 \varepsilon V_0 (N_A + N_D)}{q N_A N_D}} \Rightarrow N_A \equiv N_D \cong 10^{25} m^{-3}$

Since the breakdown voltage of a zener diode V_Z can be controlled precisely by the fabrication process, this element is frequently used in voltage regulation applications.

A more realistic I-V characteristic of the p-n junction is, therefore



6.3 Metal-Semiconductor Contacts

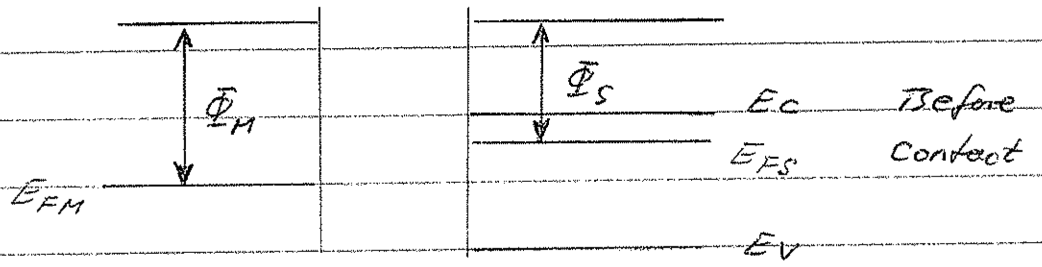
__Def.__ Work function Φ

Φ is defined as the energy required to remove an electron from a material assuming the electron was located at the Fermi energy.

__Note:__ This def. holds for metals and semiconductors, although, in the latter case, there are probably no electrons located at the Fermi level.

6.3.1 Metal n-type semiconductor contact

A) $\Phi_M > \Phi_S$



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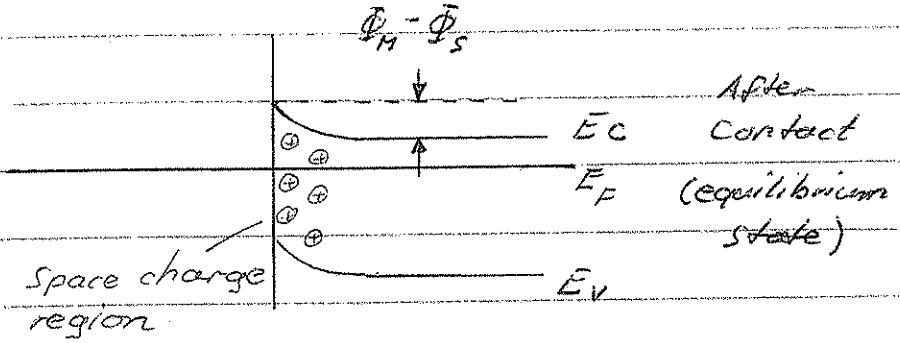


since $(E_C - E_F)$ has been increased at the junction by the amount $(\Phi_M - \Phi_S)$, the semiconductor in the vicinity of the junction becomes less n-type, intrinsic or even p-type. This creates a space charge region which influences the electron transfer from the metal to the semiconductor and vice versa.

- If we apply a pos. external voltage V_a from the metal to the semiconductor, the barrier is lowered and the current will increase.
- If we apply a neg. external voltage V_a, the potential barrier is increased and the current will be reduced.

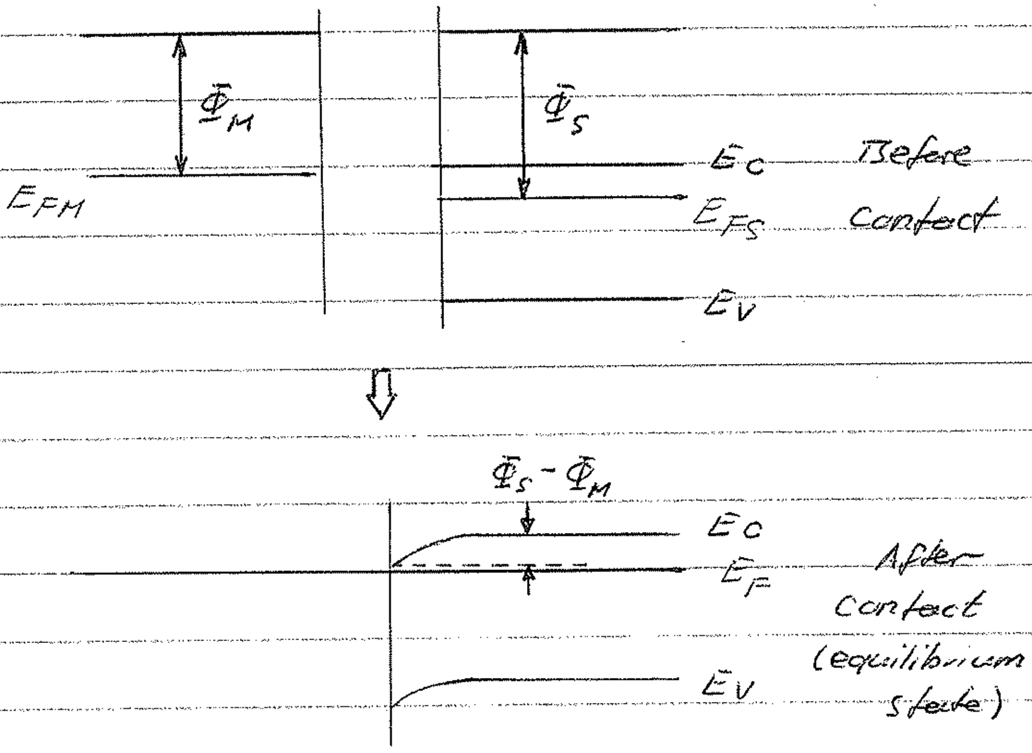
Thus, the metal-semiconductor junction behaves similar to a p-n junction

This type of rectifying metal-semiconductor contact is called a "Schottky-Barrier Diode".

Note: Schottky-Barrier Diodes display larger saturation currents I_S than p-n junction Diodes. (The actual difference depends on the barrier height). Consequently, the voltage drop for the same forward-current will also be smaller.

Finally, since the applied voltage of a Schottky-Barrier Diode controls the flow of majority carriers, no minority carriers are stored during forward-bias. This results in a shorter switching time from forward to reverse-biased mode.

B) $\Phi_M < \Phi_S$



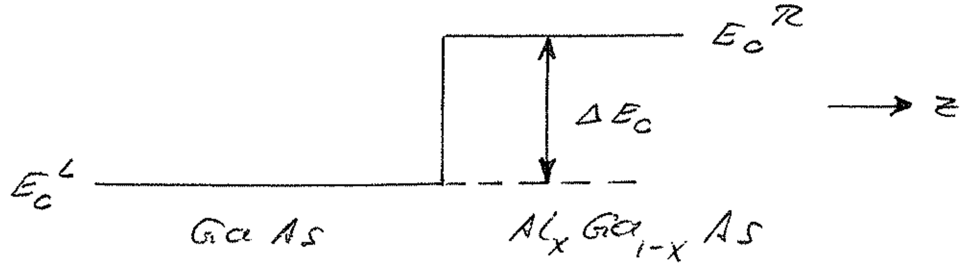
In this case, the number of free electrons at the junction is always increased and the contact becomes completely ohmic.

6.4 Semiconductor Heterojunctions

1. The heterojunction Barrier

Heterojunctions are formed by interfacing two different semiconductor materials.

Let us consider the junction between GaAs and $Al_xGa_{1-x}As$ which exhibits a certain conduction band discontinuity ΔE_C. We assume that the bands are in a fixed position, as shown below, and we attempt to calculate the current from GaAs to AlGaAs and vice versa.



To simplify the calculations, we assume (Bethe) that the distribution function is equal to the Fermi-Dirac function (or approx. Maxwell-Boltzmann) having a quasi-Fermi level E_{QF} which is different in the two materials but constant on each side. This means that a strong electron-electron interaction must be present, which causes the distribution function to look Fermi-like independent of the fact that electrons can be lost at a high rate to the neighboring material.

Junction current

Classically, all el. with a velocity component perpendicular to the interface (z-direction) and large enough to overcome the band-edge discontinuity ΔE_C contribute to the junction current.
The current density from the left (GaAs) to the right ($Al_x Ga_{1-x} As$) is then

$$\left| j_{LR} = \frac{q}{4\pi^3} \int_{k_x k_y} dk_x \, dk_y \int_{k_z > k_{z0}} v_z \, dk_z \, f_0(k) \right|$$

where k_{z0} is the minimum k-vector component required to overcome the barrier. Classically, we can calculate k_{z0} from

$$\left| \Delta E_C = \frac{1}{2} m^* v_{z0}^2 \right| \quad \text{and} \quad \left| m^* v_z = \hbar k_z \right|$$

$$dv_z = \frac{\hbar}{m^*} dk_z$$

Note: There exist several quantum corrections for the above case (quantum transmiss. coeff., deviations from eff. mass theorem and tunneling), which will be discussed later.

If we replace $f_0(k)$ by a Maxwellian distribution at a quasi-Fermi level E_{Fa}^L

$$\boxed{f_0(k) = e^{-\frac{(E - E_{Fa}^L)}{k_B T}} = e^{\frac{E_{Fa}^L}{k_B T}} \, e^{-\frac{\hbar^2 k^2}{2 m^* k_B T}}}$$

E_{Fa}^L is measured from the left side cond. band edge E_c^L

and k by $\frac{m^*}{\hbar} v$, we may rewrite the current dens. as

$$\boxed{j_{LR} = \frac{q}{4\pi^3} \left(\frac{m^*}{\hbar}\right)^3 e^{\frac{E_F^L}{k_B T}} \int_{-\infty}^{\infty} dv_x \, dv_y \int_{v_{z0}}^{\infty} v_z \, dv_z \, e^{-\frac{m^*}{2 k_B T}(v_x^2 + v_y^2 + v_z^2)}}$$

solving the x-y integration in cylindrical coordinates where $v^2 = v_x^2 + v_y^2$ and $dv_x \, dv_y = v \, dv \, d\varphi$ yields

$$\int_{-\infty}^{\infty} e^{-\frac{m^*}{2 k_B T}(v_x^2 + v_y^2)} dv_x \, dv_y = \int_0^{2\pi} \int_0^{\infty} e^{-\frac{m^*}{2 k_B T} v^2} v \, dv \, d\varphi$$

$$= 2\pi \int_0^{\infty} e^{-\frac{m^*}{2 k_B T} v^2} v \, dv = 2\pi \frac{k_B T}{m^*} \left[- e^{-\frac{m^* v^2}{2 k_B T}} \right]_0^{\infty} = \underline{2\pi \frac{k_B T}{m^*}}$$

The expression for the current density simplifies to

$$j_{LR} = \frac{q}{2\pi^2} k_B T \frac{m^{*2}}{\hbar^3} e^{\frac{E_F^L}{k_B T}} \int_{v_{z0}}^{\infty} v_z \, e^{-\frac{m^* v_z^2}{2 k_B T}} dv_z$$

$$= \frac{q}{2\pi^2} (k_B T)^2 \frac{m^*}{\hbar^3} e^{\frac{E_F^L}{k_B T}} e^{-\frac{m^* v_{z0}^2}{2 k_B T}}$$

$$\boxed{j_{LR} = \underbrace{\frac{q}{2\pi^2} k_B^2 \frac{m^*}{\hbar^3}}_{\text{Richardson constant } A^* \left[\frac{A}{m^2 \, {}^\circ K^2}\right]} T^2 \, e^{\frac{(E_F^L - \Delta E_c)}{k_B T}}}$$

<u>Note</u>: E_F^L is referenced to E_c^L

i.e. $E_F^L \mathrel{\hat{=}} (E_F^L - E_c^L)$

e.g. free el. at $T_0 = 300°K$

$A^* T_0^2 \cong 10^3 \left[\frac{A}{m^2}\right]$ $\qquad$ $A^* = 1.2 \times 10^{-2} \left[\frac{A}{m^2 \, {}^\circ K^2}\right]$

If we replace the normalized quasi-Fermi level E_F^L by the absolute energy difference $(E_F^L - E_c^L)$, we can rewrite the current as

$$\left| j_{LR} = A^* T^2 e^{\frac{E_F^L - [E_c^L + \Delta E_c]}{k_B T}} \right| \qquad E_c^L + \Delta E_c = E_c^R$$

Similarly, we obtain for the current from the $Al_x Ga_{1-x} As$ towards the GaAs

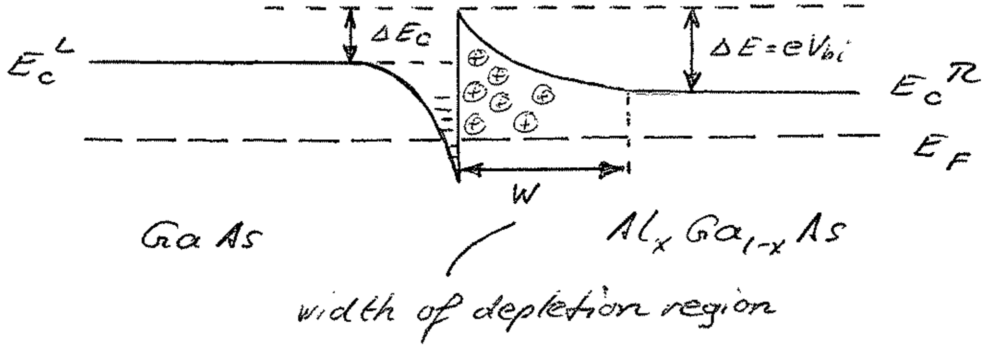
$$\left| j_{RL} = A^* T^2 e^{\frac{E_F^R - E_c^R}{k_B T}} \right|$$

Hence, the total current flow is given by

$$\left\| j_{Tot} = j_{LR} - j_{RL} = A^* T^2 e^{-\frac{E_c^R}{k_B T}} \left[e^{\frac{E_F^L}{k_B T}} - e^{\frac{E_F^R}{k_B T}} \right] \right\|$$

Note: In thermal equilibrium, the net current flow is zero. Consequently, the two quasi-Fermi levels E_F^L and E_F^R must be equal. We, therefore, have to conclude that the original step-like band diagram changes shape as equilibrium is approached. This change of the band structure is achieved by redistributing the free charge carriers at the interface of the 2 materials. The time constant involved in this charge redistribution process

is typically in the picosecond range.
If we assume that the $Al_xGa_{1-x}As$ is doped with a density N_D of donors and the GaAs is undoped, the band structure at the interface looks approximately as shown below.

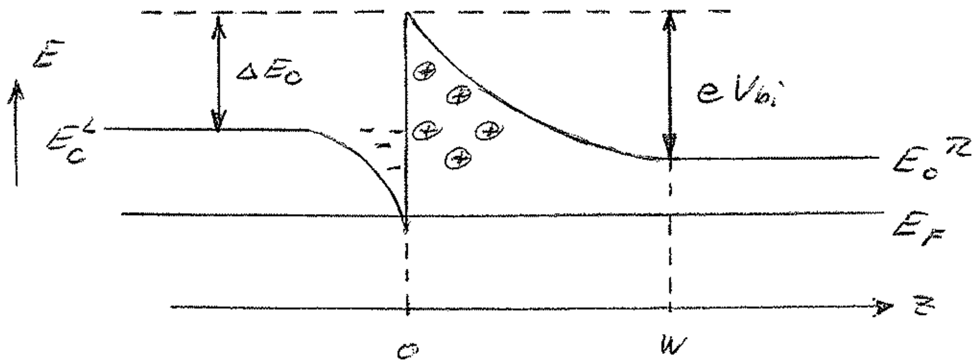


The diffusion of electrons from the $Al_xGa_{1-x}As$ creates a space charge region (positive donor ions) which gives rise to a potential barrier. As this barrier is growing, it slows down the electron transfer to the GaAs and finally prevents electrons from leaving the $Al_xGa_{1-x}As$ at a higher rate than electrons are returning from the GaAs (due to the built in field). Then, equilibrium has been reached.

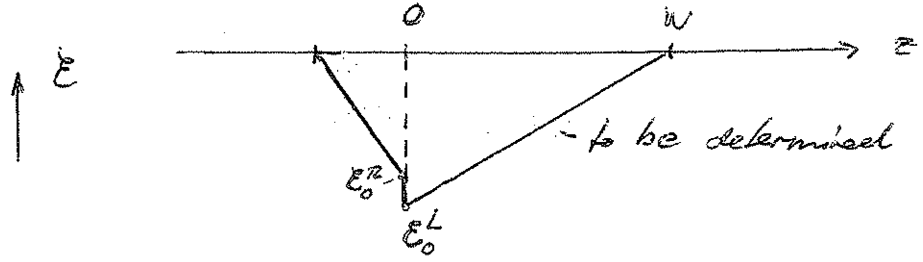
Note: Equilibrium does not necessarily mean there is no carrier transport going on; it merely states that the net carrier transport is zero (e.g. $j_{LR} = j_{RL}$). In case of the heterojunction barrier, we observe a delicate balance between diffusion and drift current.

Depletion Width and Built-in Voltage

Energy-Bands (thermal equilibrium)



Electric Field



The Poisson eq. in the depletion region $(0 < z < W)$ can be written as

$$\boxed{\frac{\partial Ɛ}{\partial z} = \frac{q}{\varepsilon}\left(p - n + N_D^+ - N_A^-\right) \cong \frac{q}{\varepsilon} N_D^+} \qquad Ɛ = -\frac{\partial \overset{\text{el. stat. pot.}}{\Phi}}{\partial z}$$

and the built-in voltage V_{bi} is obtained from

$$\boxed{V_{bi} = -\int_0^W \underset{\downarrow}{Ɛ(z)}\, \underset{\uparrow}{dz}}$$

We can solve this equation by partial integration

Thus

$$\boxed{V_{bi} = - z \cdot \mathcal{E}(z)\Big|_0^W + \int_0^W z \frac{\partial \mathcal{E}}{\partial z}\, dz}$$

Replacing $\frac{d\mathcal{E}}{dz}$ by $+\frac{q}{\varepsilon} N_D^+$ and assuming $\mathcal{E}(W) \cong 0$ yields

$$\boxed{V_{bi} \cong + \frac{1}{2} W^2 \frac{q}{\varepsilon} N_D^+}$$

Since $\underline{|q V_{bi}| = \Delta E_c + E_c^L - E_c^R}$ we can write

$$\boxed{\frac{1}{2} W^2 \frac{q^2}{\varepsilon} N_D^+ \cong \Delta E_c + E_c^L - E_c^R}$$

or

$$\boxed{W \cong \sqrt{\frac{2\varepsilon}{q^2 N_D^+} \left[\Delta E_c + E_c^L - E_c^R \right]}}$$

So far, we have discussed only the right hand side of the junction where we have essentially no free charge carriers (depletion region). The field on the left side can also be derived from Poisson's eq., but this time under the presence of free charge carriers. The carrier distribution for the left side is obtained from the drift-diffusion eq. for zero current, i.e.

$$q n \mu \mathcal{E} + \mu k T_c \nabla_r n = 0$$

or

$$\boxed{n + \frac{k T_c}{q \mathcal{E}} \nabla_r n = 0}$$

For a single gradient in the z-direction we have

$$n(z) + \frac{kT_c}{q\,\mathcal{E}(z)} \frac{\partial n}{\partial z} = 0 \qquad \Rightarrow n(z) = n_0 \exp\left[-\frac{q}{kT_c} \int \mathcal{E}(z)\,dz\right]$$

This yields the solution

$$\boxed{n(z) = n_0 \exp\left[\frac{q\,\phi(z)}{kT_c}\right]} \qquad \text{where} \quad \phi(z) = -\int \mathcal{E}(z)\,dz$$

carrier conc. at z=-∞ el. stat. potential [V]

The connection rules for the el. stat. potential ϕ at the interface are:

$$\begin{array}{rl} 1) & q\,(\phi_0^R - \phi_0^L) = \Delta E_{C0} \\ 2) & \varepsilon_L \dfrac{\partial \phi_0^L}{\partial z} = \varepsilon_R \dfrac{\partial \phi_0^R}{\partial z} \end{array}$$

ϕ_0: potential at interface

$$\varepsilon_L\, \mathcal{E}_0^L = \varepsilon_R\, \mathcal{E}_0^R$$

Poisson's eq. applied to the left side of the junction reads now: (holes or acceptors neglected)

$$\frac{\partial^2 \phi}{\partial z^2} = -\frac{q}{\varepsilon}\left(N_D^+ - n_0^- \exp\left[\frac{q\,\phi(z)}{kT_c}\right]\right)$$

Note: If the left side is homogeneously doped with N_D donors, $n_0 = N_D$

The Poisson eq. can now be written as:

$$\left| \frac{\partial^2 \phi}{\partial z^2} = -\frac{q N_D}{\varepsilon}\left(1 - \exp\left[\frac{q}{kT_c}\phi(z)\right]\right) \right| \quad \left| \int_{-\infty}^{z} -\frac{\partial \phi}{\partial \xi}\, d\xi \right.$$

Now we multiply both sides of this eq. with $\mathcal{E}(z) = -\frac{\partial \phi}{\partial z}$ and integrate over z. This yields

for the left side

$$-\int_{-\infty}^{z} \underset{\uparrow}{\frac{\partial^2 \phi}{\partial \xi^2}}\, \underset{\downarrow}{\frac{\partial \phi}{\partial \xi}}\, d\xi = -\frac{1}{2}\left[\frac{\partial \phi}{\partial z}\right]^2 = -\frac{1}{2}\,\mathcal{E}(z)^2$$

Boundary condition $\phi(-\infty) = 0$
$\mathcal{E}(-\infty) = 0$

for the right side

$$+\frac{q N_D}{\varepsilon}\int_{-\infty}^{z}\left[\frac{\partial \phi}{\partial \xi} - \frac{\partial \phi}{\partial \xi}\exp\left[\frac{q}{kT_c}\phi\right]\right]d\xi$$

$$= +\frac{q N_D}{\varepsilon}\left[\phi(z) - \frac{kT_c}{q}\left(e^{\frac{q}{kT_c}\phi(z)} - 1\right)\right]$$

Thus

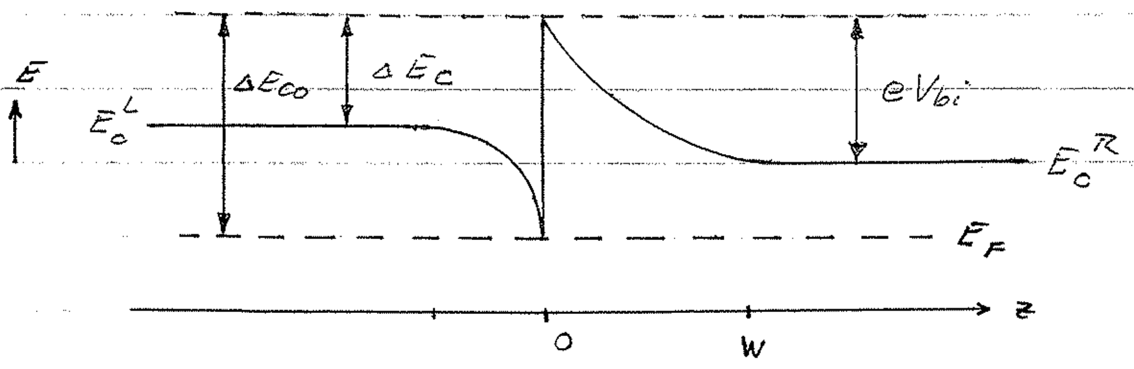
$$\left\| \mathcal{E}(z)^2 = \frac{2 q N_D}{\varepsilon}\left[\frac{kT_c}{q}\left(e^{\frac{q}{kT_c}\phi(z)} - 1\right) - \phi(z)\right] \right\| \qquad z \leq 0$$

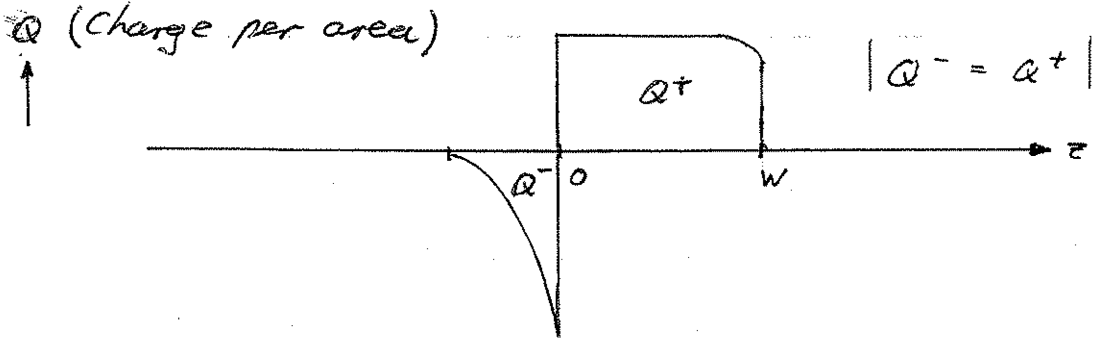
with $\underline{\frac{kT_c}{q} = V_T}$ we can write $\left| \mathcal{E}(z)^2 = \frac{2qN_D}{\varepsilon} V_T\left[e^{\frac{\phi(z)}{V_T}} - 1 - \frac{\phi(z)}{V_T}\right] \right|$

The total excess charge (per unit area) on the left side of the junction is obtained by integrating the left hand side of Poisson's equation between $z = -\infty$ and 0 and multiplying the result with ε.

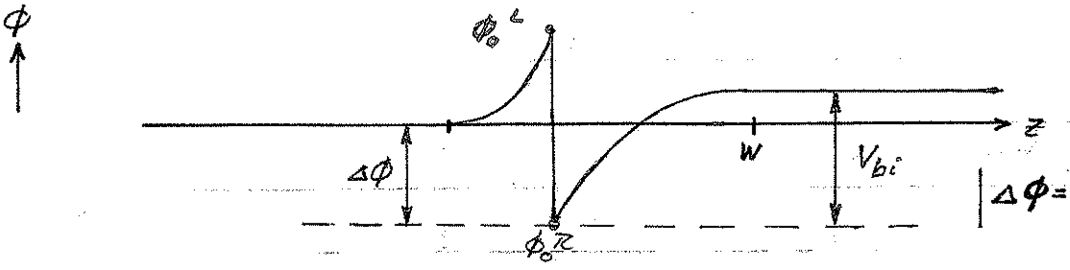
$$\left\| Q_{tot}^{-} = \varepsilon^{L}\frac{\partial \phi_0^{L}}{\partial z} = -\varepsilon^{L}\underset{\substack{| \\ \text{El. Field at } z=0}}{\mathcal{E}_0^{L}} \right\| \qquad Q_{tot}^{-} = -\sqrt{2q\,\varepsilon^{L} N_D\left[V_T\left(e^{\frac{\phi_0}{V_T}} - 1\right) - \phi_0\right]}$$

Heterojunction - Classical Solution

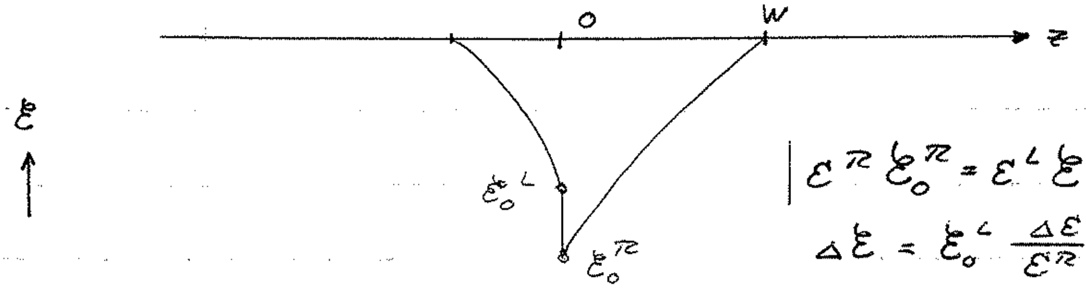




$$\left| Q^- = Q^+ \right|$$



$$\left| \Delta\phi = \frac{\Delta E_c}{q} \right|$$



$$\left| \varepsilon^R \mathcal{E}_0^R = \varepsilon^L \mathcal{E}_0^L \right|$$

$$\Delta\mathcal{E} = \mathcal{E}_0^L \frac{\Delta\varepsilon}{\varepsilon^R}$$

Note: The discontinuity Δφ involves the crystal atoms and cannot be derived from Poisson's equation

If the GaAs on the left side were doped with N_A acceptors, electrons initially accumulating in the GaAs would recombine with holes, and for small values of the interface potential a depletion layer would form before electrons are further accumulated. At higher values of the potential ϕ electrons start to accumulate in the p-type material and an inversion layer forms. If the electron concentration in this layer exceeds the original hole concentration N_A, we talk about strong inversion. The minimum potential required for this condition is

$$\left| \phi_S = \frac{2kT_c}{q} \ln\left(\frac{N_A}{n_i}\right) = 2V_T \ln\left(\frac{N_A}{n_i}\right) \right| \quad \text{onset of strong inversion}$$

Note: If GaAs is intrinsic $|\phi_S \equiv 0|$

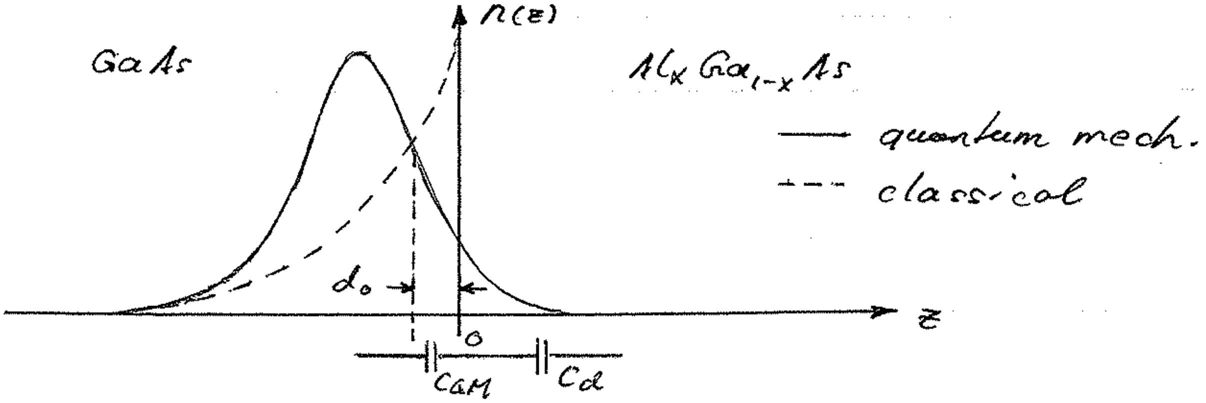
Effects of Size Quantization (Quantum Mechanical)

The inversion layer that forms on the GaAs side of the junction is typically very narrow. For an electron density around $10^{24} m^{-3}$ the width of the layer is in the order of 10-100Å only. Therefore, quantum effects can be very important.

Since an exact treatment of this quantum mechanical approach is very involved, we will highlight the most important results only.

Charge distribution in inversion layer (QM)

Since the energy profile of the inversion layer (perpendicular to the interface) looks very much like a 1-dim. potentiall well, it becomes obvious that the highest electron density does not occur at the interface but at some distance off the boundary between the two materials (see Fig. below)



As is evident from the above figure, the quantum mech. solution gives rise to an additional almost insulating layer of width d_0 at the interface (d_0 is typically in the range of 10 Å). This yields an extra quantum capacitance

$$\boxed{C_{QM} \approx \frac{\bar{\varepsilon}}{d_0}} \qquad \text{where } \bar{\varepsilon} = \varepsilon_0 \tfrac{1}{2}(\varepsilon^L + \varepsilon^R)$$

This series capacitance C_{QM} is only significant for very thin thicknesses of $Al_xGa_{1-x}As$ (or SiO_2 in case of MOS devices) and has little influence in state-of-the-art devices. $C_{Tot} = C_d \frac{1}{(1 + C_d/C_{QM})}$ $(C_{QM} \gg C_d)$

Enhanced Electron Mobility (QM)

We have seen that the quantum corrections to the total charge density at interfaces is usually small. However, the effects of size quantization on the electronic properties can be significant.

The enhanced electron mobility at interfaces is mostly due to a separation of scattering centers (impurities) and electrons (dopants remain on the $Al_xGa_{1-x}As$ side while the electrons mostly reside in the GaAs). It is important to note that not only impurities can be remote, but also phonon modes characteristic for only one medium can be separated from the electrons.

With a simplified 2-dim. model one can show that the matrix element for impurity scattering close to the interface is equal to

$$\left\| \, |M_{kk'}|^2 \cong \frac{q^4}{4A^2\varepsilon^2\left(\tilde{q}_\parallel^2 + \tilde{q}_{s\parallel}^2/\varepsilon_r\right)^2} \exp[-2\tilde{q}_\parallel |z_0|] \, \right\|$$

Transition Matrix Element

where A: Interface area

z_0: distance of impurity from electron

$\tilde{q}_\parallel = k_\parallel - k_\parallel'$ wave vector parallel to interface

Note that the matrix element and so the impurity scattering rate decreases exponentially with the distance z_0 of the impurity from the electron. → <u>modulation doping</u>

Typically the mobility may change by a factor of 2, but under special conditions the mobility can be enhanced by more than a factor of 10.

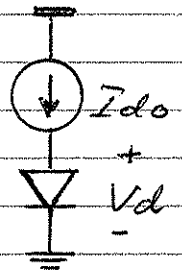
Problem We would like to use a pn junction as a thermal sensor.

To do so, we keep the junction current constant and measure the junction voltage as a function of temperature.

Use the diode equation

$$\boxed{I_d \cong I_s\, e^{\frac{V_d}{V_T}}} \quad \text{where } I_s = A\cdot q\, n_{i0}^2 \frac{D_x}{L_x}\frac{1}{N_x}$$

to find an expression for the expected change of the diode voltage Vd



Solution

1. We assume that $\frac{D_x}{L_x}$ approximately remains constant over the temperature range of interest.

$$\therefore \boxed{\frac{dI_s}{dT} \cong \frac{dn_{i0}^2}{dT}}$$

2. From $n_{i0} = N_c(T)\, e^{-\frac{E_c - E_{Fi}}{kT}} = N_v(T)\, e^{-\frac{E_{Fi} - E_v}{kT}}$

we obtain

$$n_{i0}^2(T) = N_c(T)\cdot N_v(T)\, e^{-\frac{E_c - E_v}{kT}}$$

where

$$N_c(T)\cdot N_v(T) = 4\left(\frac{2\pi kT}{h^2}\right)^3 \left(m_n^a m_p^a\right)^{3/2}$$

We can thus write

$$I_S(T) = I_{S0}\left(\frac{T}{T_0}\right)^3 e^{\frac{E_G}{kT_0}\left(1-\frac{T_0}{T}\right)}$$

The derivative of I_S w.r.t. T is

$$\frac{dI_S(T)}{dT} = \underbrace{I_{S0}\left(\frac{T}{T_0}\right)^3 e^{\frac{E_G}{kT_0}\left(1-\frac{T_0}{T}\right)}}_{I_S(T)}\left[\frac{3}{T}+\frac{E_G}{kT^2}\right]$$

$$\therefore \left\| \frac{dI_S(T)}{dT} = \frac{I_S(T)}{T}\left[3+\frac{E_G}{kT}\right] \right\| \qquad (1)$$

Note: $\frac{E_G}{kT} = \frac{V_G}{V_T}$ where $V_G = \frac{E_G}{q}$ Bandgap voltage

3. Solving the original diode equation for the diode voltage yields

$$\left| V_d \cong V_T \ln\left(\frac{I_d}{I_S}\right) \right| \text{ where } I_d = I_{d0}$$

$$\left\| \frac{dV_d(T)}{dT} = \frac{dV_T}{dT}\ln\left(\frac{I_{d0}}{I_S}\right) - V_T\frac{1}{I_S}\frac{dI_S}{dT} \right\| \qquad (2)$$

4. Finally, inserting (1) into (2) yields

$$\frac{dV_d(T)}{dT} = \frac{V_T}{T}\ln\left(\frac{I_{d0}}{I_S}\right) - \frac{V_T}{T}\left[3+\frac{V_G}{V_T}\right]$$

or

$$\left\| \frac{dV_d(T)}{dT} = \frac{1}{T}\left[V_d - 3V_T - V_G\right] \right\| \qquad (3)$$

Numerical Solutions

Find the values for $\frac{dV_d(T)}{dT}$ between 300k and 400k if we know that V_d @ 300k is 0.7V

T [k]	dVd/dT [mV/k]	Vd [mV]
300	-1.659	700.0
320	-1.675	666.8
340	-1.690	633.3
360	-1.704	599.5
380	-1.718	565.4
400	-1.731	531.0

$V_{G_{Si}} = 1.12V$

Notes: The numerical values depend crucially on the initial diode voltage

second order approximation for diode voltage: (300k - 400k)

$$\| V_d(T) \cong V_{d0} + \Delta T k_1 + \Delta T^2 k_2 \|$$

where

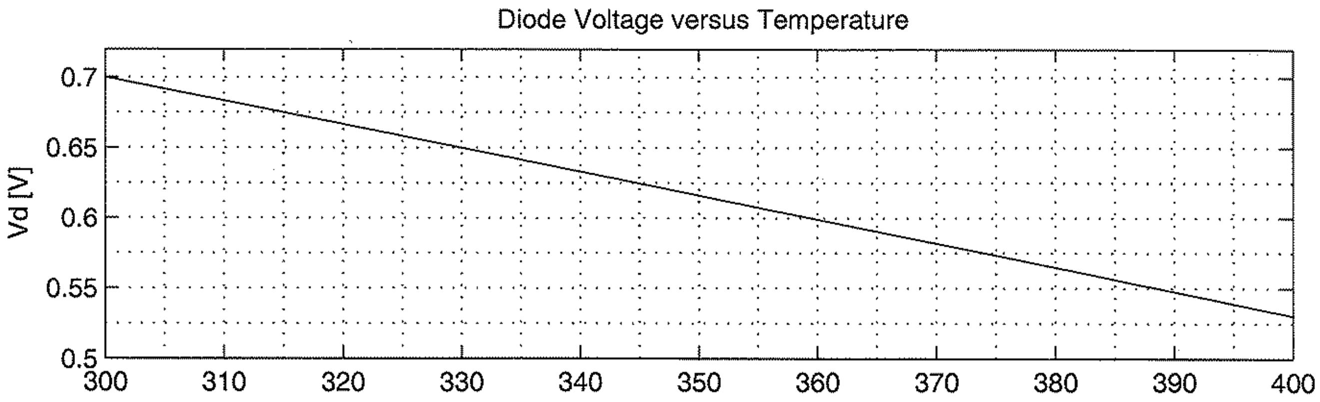
$k_1 \cong -1.697\ mV/k$

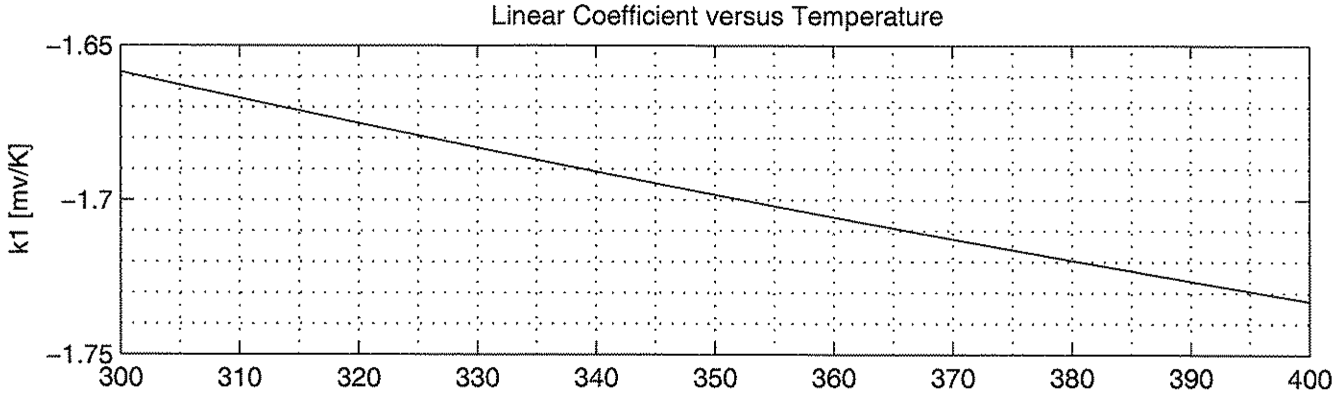
$k_2 \cong -0.37\ \mu V/K^2$

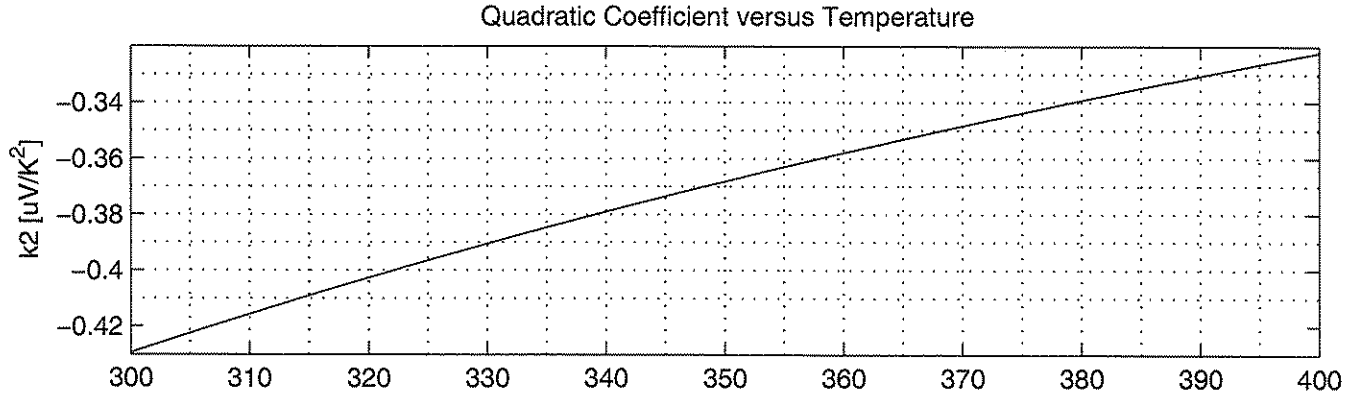
Values are referenced to 350K

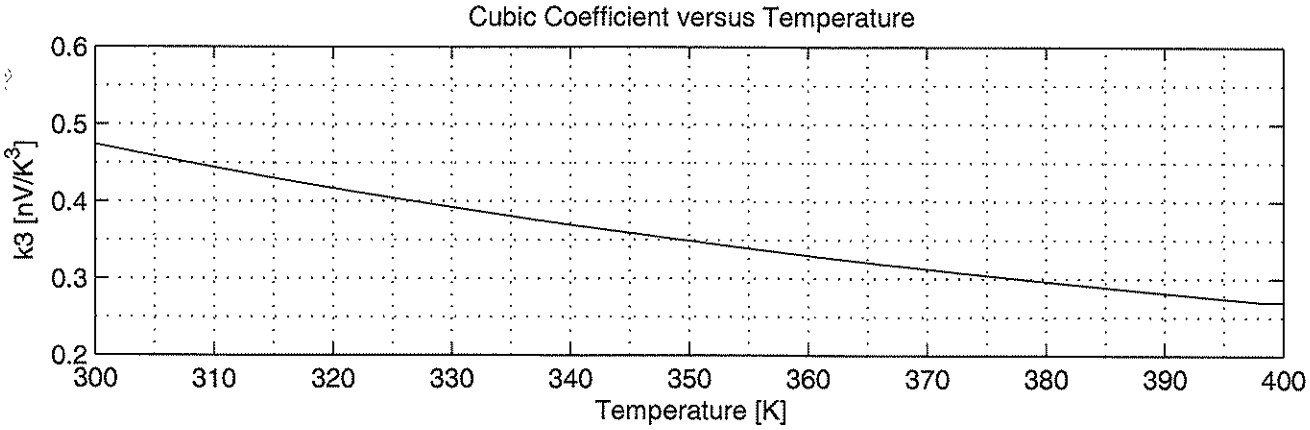
Notes: If one were to compensate for the linear deviation of the diode voltage (e.g. in a voltage reference circuit), one would end up with a quadratic (parabolic) error, i.e., $\Delta V_d = k_2 \cdot \Delta T^2$

VI-P4


Diode Voltage versus Temperature


Linear Coefficient versus Temperature


Quadratic Coefficient versus Temperature


Cubic Coefficient versus Temperature