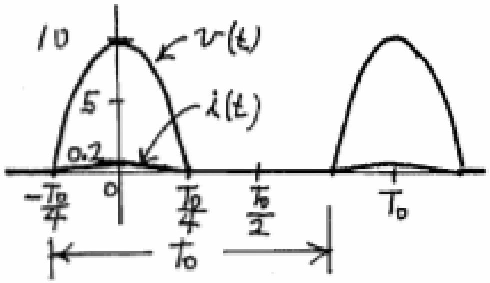
$\boxed{2\text{-}4.}$

(a.) $i(t) = \frac{v(t)}{R} = \frac{v(t)}{50}$

$$i(t) = \begin{cases} 0.2\cos(\omega_0 t), & |t - nT_0| < \frac{T_0}{2} \\ 0, & \text{elsewhere} \end{cases}$$




(b.) $V_{DC} = \langle v(t) \rangle = \frac{V_p}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \cos(\omega_0 t)\,dt = \frac{2V_p}{T_0}\frac{\sin(\omega_0 \frac{T_0}{4})}{\omega_0}$

$$= \frac{2V_p}{\cancel{T_0}}\frac{\sin\left(\frac{2\pi}{\cancel{T_0}}\frac{\cancel{T_0}}{4}\right)}{\frac{2\pi}{\cancel{T_0}}} = \frac{\cancel{2}}{\cancel{2}\pi}V_p \sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow V_{DC} = \frac{V_p}{\pi} \underset{\uparrow}{=} \frac{10}{\pi} = \underline{\underline{3.183 \text{ volts}}}$$

$V_p = 10$

$$\Rightarrow I_{DC} = \frac{I_p}{\pi} \underset{\uparrow}{=} \frac{0.2}{\pi} = \underline{\underline{0.064 \text{ Amps}}}$$

$I_p = 0.2$

2-4. Cont'd (c.) $V_{rms}^2 = \langle v^2(t) \rangle = \frac{1}{T_0}\int_0^{T_0/2} v^2(t)\,dt = \frac{V_p^2}{T_0}\int_{-T_0/4}^{T_0/4} \cos^2 \omega_0 t\,dt$

$$\Rightarrow V_{rms}^2 = \frac{V_p^2}{T_0}\int_{-T_0/4}^{T_0/4} \frac{1}{2}\left[1 + \cos(2\omega_0 t)\right]dt = \frac{V_p^2}{2T_0}\left[2\frac{T_0}{4} + \frac{\sin(2\omega_0 t)}{2\omega_0}\Big|_{-T_0/4}^{T_0/4}\right]$$

$$= \frac{V_p^2}{2\cancel{T_0}}\frac{2\cancel{T_0}}{4} = \frac{V_p^2}{4} = V_{rms}^2$$

$$\Rightarrow V_{rms} = \frac{V_p}{2} = \frac{10}{2} = \underline{\underline{5 \text{ volts rms}}}$$

$$I_{rms} = \frac{I_p}{2} = \frac{0.2}{2} = \underline{0.1 \text{ amps}}$$

(d) $P = \langle p(t) \rangle = V_{rms} I_{rms} = (5)(0.1) = \underline{\underline{0.5 \text{ watts}}}$

**2-16.**

$$S(f) = \int_{-\infty}^{\infty} s(t)\, e^{-j\omega t}\, dt = \int_{0}^{T_0} A t\, e^{-j\omega t}\, dt$$

$$= A\left[e^{-j\omega t}\left(\frac{t}{-j\omega} + \frac{1}{\omega^2}\right)\right]\Bigg|_0^{T_0}$$

$$\int x e^{ax}\, dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right]$$

$$= A\left\{e^{-j\omega T_0}\left(\frac{T_0}{-j\omega} + \frac{1}{\omega^2}\right) - \frac{1}{\omega^2}\right\}$$

$$= \frac{A}{(2\pi f)^2}\left\{e^{-j2\pi f T_0} - 1\right\} + \frac{A T_0\, e^{-j2\pi f T_0}}{-j2\pi f}$$

$$\Rightarrow S(f) = \frac{-A}{(2\pi f)^2} + A e^{-j2\pi f T_0}\left(\frac{1}{(2\pi f)^2} + j\frac{T_0}{2\pi f}\right)$$

**2-26.**

$$w(t) = \int_{-\infty}^{\infty} W(f)\, e^{j2\pi f t}\, df$$

$$\Rightarrow w(0) = \int_{-\infty}^{\infty} W(f)\, \cancelto{1}{e^{j0}}\, df = \int_{-\infty}^{\infty} W(f)\, df$$

**2-35.**

$$w(t) = w_1(t)\, w_2(t)$$

$$W(f) = \int_{-\infty}^{\infty} w(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} w_1(t)\, w_2(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} W_1(\lambda)\, e^{j2\pi\lambda t}\, d\lambda\right] w_2(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} W_1(\lambda) \underbrace{\int_{-\infty}^{\infty} w_2(t)\, e^{-j2\pi(f-\lambda)t}\, dt}_{W_2(f-\lambda)}\, d\lambda = \int_{-\infty}^{\infty} W_1(\lambda)\, W_2(f-\lambda)\, d\lambda = W(f)$$

$\boxed{2\text{-}44.}$ (a) Before the general result is worked out, work out the result for the special case where:

$$\omega_1 = \omega_2 \text{ and } \theta_1 = \theta_2$$

$$\Rightarrow w(t) = (A_1 + A_2)\cos(\omega_1 t + \theta_1)$$

$$R_w(\tau) = \langle w(t)\, w(t+\tau)\rangle = (A_1+A_2)^2 \langle \cos(\omega_1 t + \theta_1)\cos(\omega_1(t+\tau)+\theta_1)\rangle$$

$$= (A_1+A_2)^2 \left\langle \tfrac{1}{2}\cos\omega_1\tau + \tfrac{1}{2}\cos(2\omega_1 t + \omega_1\tau + 2\theta_1)\right\rangle$$

$$= \frac{(A_1+A_2)^2}{2}\left\{ \langle \cos(\omega_1\tau)\rangle \xrightarrow{} \cos(\omega_1\tau) + \langle \cos(2\omega_1 t + \omega_1\tau + 2\theta_1)\rangle \xrightarrow{} 0 \right.$$

$$\Rightarrow R_w(\tau) = \frac{(A_1+A_2)^2}{2}\cos(\omega_1\tau) \quad \text{when } \omega_1 = \omega_2 \text{ and } \theta_1 = \theta_2.$$

2-44.(a.) Cont'd

Now work out the general result.

$$w(t) = A_1\cos(\omega_1 t+\theta_1) + A_2\cos(\omega_2 t+\theta_2)$$

$$R_w(\tau) = \langle [A_1\cos(\omega_1 t+\theta_1) + A_2\cos(\omega_2 t+\theta_2)][A_1\cos(\omega_1(t+\tau)+\theta_1) + A_2\cos(\omega_2(t+\tau)+\theta_2)] \rangle$$

$$\begin{aligned} = \; & A_1^2 \langle \cos(\omega_1 t+\theta_1)\cos(\omega_1(t+\tau)+\theta_1) \rangle \\ & + A_1A_2 \langle \cos(\omega_1 t+\theta_1)\cos(\omega_2(t+\tau)+\theta_2) \rangle \\ & + A_1A_2 \langle \cos(\omega_2 t+\theta_2)\cos(\omega_1(t+\tau)+\theta_1) \rangle \\ & + A_2^2 \langle \cos(\omega_2 t+\theta_2)\cos(\omega_2(t+\tau)+\theta_2) \rangle \end{aligned}$$

Thus,

$$R_w(\tau) = \frac{A_1^2}{2}\cos(\omega_1\tau) + \frac{A_1A_2}{2}\overset{\begin{Bmatrix} \cos(\omega_2\tau+\theta_2-\theta_1), & \omega_1=\omega_2 \\ 0, & \omega_1\neq\omega_2 \end{Bmatrix}}{\cancel{\langle \cos[(\omega_2-\omega_1)t} + \omega_2\tau + \theta_2 - \theta_1] \rangle}$$

$$+ \frac{A_1A_2}{2}\overset{\begin{Bmatrix} \cos(\omega_1\tau+\theta_1-\theta_2), & \omega_1=\omega_2 \\ 0, & \omega_1\neq\omega_2 \end{Bmatrix}}{\cancel{\langle \cos[(\omega_1-\omega_2)t} + \omega_1\tau + \theta_1 - \theta_2] \rangle} + \frac{A_2^2}{2}\cos(\omega_2\tau)$$

$$= \frac{A_1^2}{2}\cos(\omega_1\tau) + \frac{A_2^2}{2}\cos(\omega_2\tau) + \frac{A_1A_2}{2}\begin{Bmatrix} [\cos(\omega_1\tau+\theta_2-\theta_1) + \cos(\omega_1\tau+\theta_1-\theta_2)], & \omega_1=\omega_2 \\ 0 & , \omega_1\neq\omega_2 \end{Bmatrix}$$

$$\Rightarrow R_w(\tau) = \frac{A_1^2}{2}\cos(\omega_1\tau) + \begin{Bmatrix} A_1A_2\cos(\theta_2-\theta_1)\cos(\omega_1\tau), & \omega_1=\omega_2 \\ 0 & , \omega_1\neq\omega_2 \end{Bmatrix} + \frac{A_2^2}{2}\cos(\omega_2\tau)$$

Thus, the general result is:

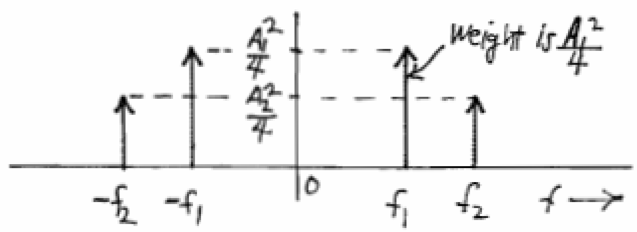
$$R_w(\tau) = \begin{Bmatrix} \frac{1}{2}[A_1^2 + 2A_1A_2\cos(\theta_2-\theta_1) + A_2^2]\cos(\omega_1\tau), & \omega_1=\omega_2 \\ \frac{A_1^2}{2}\cos(\omega_1\tau) + \frac{A_2^2}{2}\cos(\omega_2\tau) & , \omega_1\neq\omega_2 \end{Bmatrix}$$

2-44(a.) Cont'd. Note that for $\omega_1=\omega_2$ and $\theta_1=\theta_2$, this general result reduces to the result that was first obtained for the special case.

(b.) Evaluating $\mathcal{F}[R_w(\tau)] = \mathcal{P}_w(f)$, we get

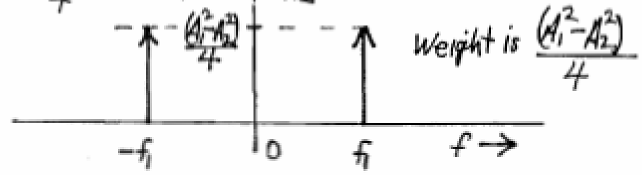
$$\mathcal{P}_w(f) = \begin{cases} \frac{1}{4}\left[A_1^2 + 2A_1A_2\cos(\theta_2-\theta_1) + A_2^2\right]\left[\delta(f-f_1)+\delta(f+f_1)\right], & f_1=f_2 \\ \frac{A_1^2}{4}\left[\delta(f-f_1)+\delta(f+f_1)\right] + \frac{A_2^2}{4}\left[\delta(f-f_2)+\delta(f+f_2)\right], & f_1\neq f_2 \end{cases}$$

(c.)



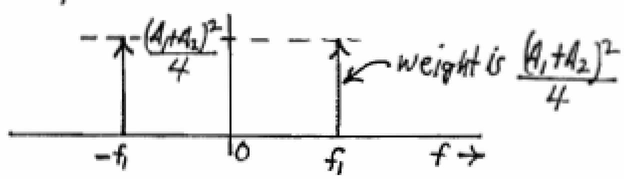

(d.) $\mathcal{P}_w(f) = \frac{1}{4}\left[A_1^2 + 2A_1A_2\overset{0}{\cos(\theta_2-\theta_1-90^\circ)} + A_2^2\right]\left[\delta(f-f_1)+\delta(f+f_1)\right]$

$= \frac{(A_1^2-A_2^2)}{4}\left[\delta(f-f_1)+\delta(f+f_1)\right]$




(e.) $\mathcal{P}_w(f) = \frac{1}{4}\left[A_1^2 + 2A_1A_2\overset{1}{\cos(0)} + A_2^2\right]\left[\delta(f-f_1)+\delta(f+f_1)\right]$

$= \frac{(A_1+A_2)^2}{4}\left[\delta(f-f_1)+\delta(f+f_1)\right]$



2-48.

$$w(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_0^{T_0} \sum_{k=-\infty}^{\infty} \delta(t-kT_0)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\sum_{k=-\infty}^{\infty} e^{-jn\frac{2\pi}{T_0}kT_0} = \frac{1}{T_0}\sum_{k=-\infty}^{\infty} e^{-jnk2\pi}$$

$$w(t) = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} \frac{1}{T_0} \exp\left[jn(\omega_0 t - k2\pi)\right]$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{T_0} e^{jn\omega_0 t}$$ ⟸ periodic with period $2\pi$

$$\mathcal{F}\{w(t)\} = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{T_0}\, e^{jn\omega_0 t} e^{-j\omega t}\, dt$$

$$= \frac{1}{T_0}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-j2\pi(f-nf_0)t}\, dt = \underline{\underline{f_0 \sum_{n=-\infty}^{\infty} \delta(f-nf_0)}}$$

**2-54.** $$c_n = \frac{1}{T}\int_{\tau_0}^{\tau_0+b} A\, e^{-jn\omega t}\, dt$$

$$= \frac{-A}{T}\frac{1}{jn\omega} e^{-jn\omega t}\Big|_{\tau_0}^{\tau_0+b}$$

$$= \frac{-A}{jn\omega T}\left(e^{-jn\omega(\tau_0+b)} - e^{-jn\omega\tau_0}\right)$$

$$= \frac{-A}{jn\omega T} e^{-jn\omega(\tau_0+\frac{b}{2})}\left(e^{-jn\frac{\omega b}{2}} - e^{jn\frac{\omega b}{2}}\right)$$

$$= \frac{2A}{n\omega T} e^{-jn\omega(\tau_0+\frac{b}{2})}\left(\frac{e^{jn\frac{\omega b}{2}} - e^{-jn\frac{\omega b}{2}}}{j2}\right)$$

$\omega = \frac{2\pi}{T}$

$$= \frac{A}{n\pi} e^{-jn\omega(\tau_0+\frac{b}{2})} \sin\left(\frac{n\pi b}{T}\right)$$

$$c_n = \frac{Ab}{T} e^{-jn\omega(\tau_0+\frac{b}{2})} \frac{\sin\left(\frac{n\pi b}{T}\right)}{n\pi b/T}$$

**2-56.** Use (2-110) and (2-112)

(a.) $$c_n = f_0\, P(f)\Big|_{f=nf_0}$$

where $P(f) = \mathcal{F}[p(t)] = \int_{-\infty}^{\infty} p(t)\, e^{-j\omega t}\, dt$

For $f=0$
$$P(0) = \int_0^T At\, dt = \frac{At^2}{2}\Big|_0^T = \frac{AT^2}{2}, \quad f=0$$

For $f \neq 0$
$$P(f) = \int_0^T At\, e^{-j\omega t}\, dt$$

2-56. (a.) Cont'd

Let $u = At \quad dv = e^{-j\omega t}$

$du = A\,dt \quad v = e^{-j\omega t}/-j\omega$

$$P(f) = \frac{At\,e^{-j\omega t}}{-j\omega}\Big|_0^T + \frac{A}{j\omega}\int_0^T e^{-j\omega t}\,dt$$

$$= \frac{jATe^{-j\omega T}}{\omega} + \frac{A}{\omega^2}\left(e^{-j\omega T} - 1\right)$$

$$P(f) = \frac{A\left[e^{-j\omega T} + j\omega T e^{-j\omega T} - 1\right]}{\omega^2}, \quad f \neq 0$$

$$c_n = \frac{1}{T_0}P\left(\omega = \frac{n2\pi}{T_0}\right) = f_0\,P\left(\omega = n2\pi f_0\right)$$

$$c_n = \left\{\begin{array}{ll} \dfrac{AT^2}{2T_0}, & n = 0 \\ \dfrac{A\left[e^{-j2\pi n f_0 T}(1 + jn2\pi f_0 T) - 1\right]}{T_0\,\omega^2}, & n \neq 0 \end{array}\right\}$$

(b.) $x_n = \mathrm{Re}\{c_n\}$ ; $y_n = \mathrm{Im}\{c_n\}$

$$c_n = A\,\frac{\left\{\left[\cos(n2\pi f_0 T) - j\sin(n2\pi f_0 T)\right] \cdot \left[1 + jn2\pi f_0 T\right] - 1\right\}}{T_0\,\omega^2}$$

2-56. (b.) Cont'd

$$x_n = \begin{cases} \dfrac{A T^2}{2T_0}, & n=0 \\ A\left\{\dfrac{\cos(n2\pi f_0 T) + n2\pi f_0 T \sin(n2\pi f_0 T) - 1}{T_0 \omega^2}\right\}, & n \neq 0 \end{cases}$$

$$y_n = \begin{cases} 0, & n=0 \\ A\left\{\dfrac{n2\pi f_0 T \cos(n2\pi f_0 T) - \sin(n2\pi f_0 T)}{T_0 \omega^2}\right\}, & n \neq 0 \end{cases}$$

(c.)

$$D_n = \begin{cases} c_0, & n=0 \\ 2\sqrt{x_n^2 + y_n^2}, & n \geq 1 \end{cases}, \quad \phi_n = \begin{cases} 0, & n=0 \\ \tan^{-1}\left(\dfrac{y_n}{x_n}\right), & n \geq 1 \end{cases}$$