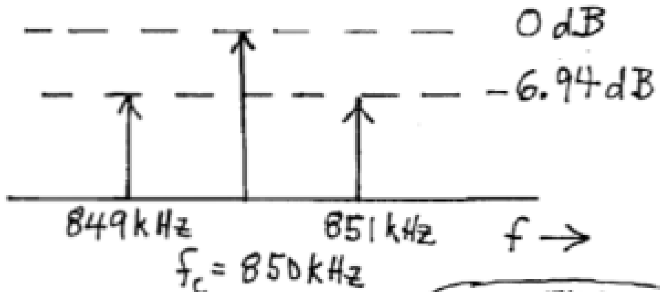
5-1. (a.) $dBk = 10\log_{10}\left(\frac{5000}{1000}\right) = \underline{\underline{6.99\ dBk}}$

(b.) $P = \frac{A_c^2}{2R} \Rightarrow A_c = \sqrt{2PR} = \sqrt{2(5000)(50)} = \underline{\underline{707\ \text{volts}}}$

$$\underline{\underline{s(t) = 707\left[1 + 0.9\cos(2000\pi t)\right]\cos(2\pi\, 850\,000\, t)}}$$

(c.) $$s(t) = 707\cos\omega_c t + \frac{0.9(707)}{2}\cos[(\omega_c - \omega_m)t] + \frac{0.9(707)}{2}\cos[(\omega_c + \omega_m)t]$$



$20\log_{10}\left(\frac{318}{707}\right) = -6.94$

(d.) $P_{AVE} = \frac{(707)^2}{2(50)} + \frac{(318)^2}{2(50)} + \frac{(318)^2}{2(50)} = \underline{\underline{7.021\ kW}}$

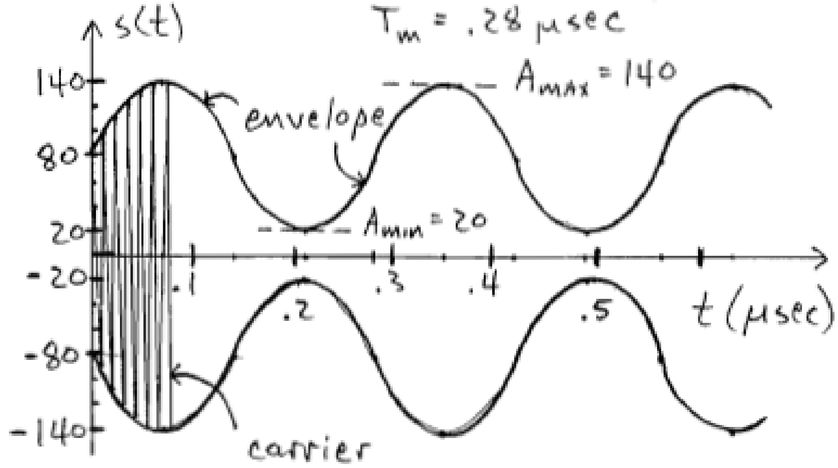
(e.) $PEP = \frac{[(707)(1.9)]^2}{2(50)} = \underline{\underline{18.045\ kW}}$

5-4. (a.) $m(t) = -0.2 + 0.6 \sin \omega_1 t$

$f_m = f_1 = 3.57$ MHz ; $A_c = \underline{\underline{100}}$

$$s(t) = 100\left(0.8 + 0.6 \sin \omega_1 t\right) \cos \omega_c t$$

$T_m = .28\ \mu\text{sec}$



(b.) $\%\ \text{pos. mod.} = \dfrac{A_{max} - A_c}{A_c}(100) = \dfrac{140-100}{100}(100)$

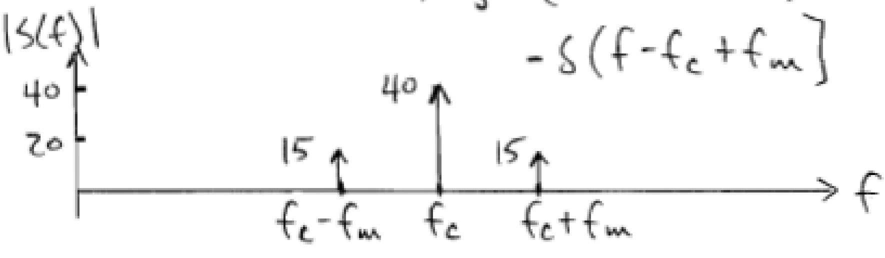
$= \underline{\underline{40\%}}$

$\%\ \text{neg. mod.} = \dfrac{A_c - A_{MIN}}{A_c}(100) = \dfrac{100-20}{100}(100)$

$= \underline{\underline{80\%}}$

(c.) $\underline{f>0}$

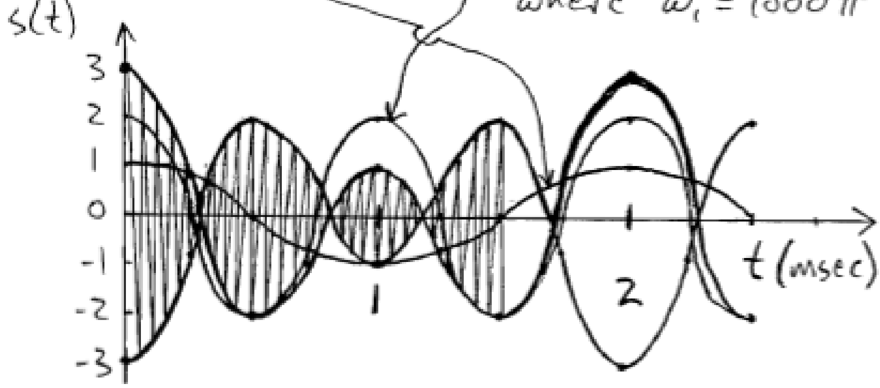
$$S(f) = 40\,\delta(f-f_c) - j15\left[\delta(f-f_c-f_m) - \delta(f-f_c+f_m)\right]$$



5-7. (a.) DSB-SC $m(t) = \cos\omega_1 t + 2\cos 2\omega_1 t$

$$\underline{\underline{s(t) = \left[\cos\omega_1 t + 2\cos 2\omega_1 t\right]\cos\omega_c t}}$$

where $\omega_1 = 1000\pi$

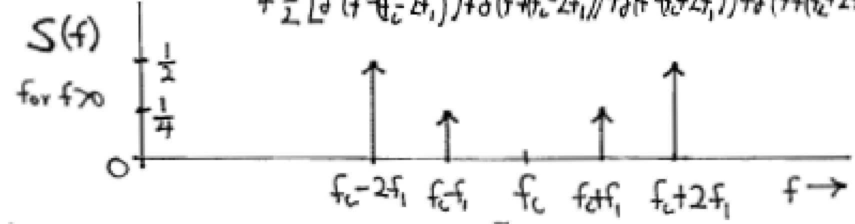


(b.) $s(t) = \frac{1}{2}\left[\cos(\omega_c - \omega_1)t + \cos(\omega_c + \omega_1)t\right]$

$$+ \cos(\omega_c - 2\omega_1)t + \cos(\omega_c + 2\omega_1)t$$

5-7 (b) Cont'd $S(-f) = S(f)$ even

$$S(f) = \mathcal{F}[s(t)] = \frac{1}{4}\left[\delta(f-(f_c-f_1)) + \delta(f+(f_c-f_1)) + \delta(f-(f_c+f_1)) + \delta(f+(f_c+f_1))\right]$$
$$+ \frac{1}{2}\left[\delta(f-(f_c-2f_1)) + \delta(f+(f_c-2f_1)) + \delta(f-(f_c+2f_1)) + \delta(f+(f_c+2f_1))\right]$$



(c.) $P_{AV_{norm}} = \frac{1}{2}\left[(\tfrac{1}{2})^2 + (\tfrac{1}{2})^2 + (1)^2 + (1)^2\right] = \underline{\underline{1.25\ W}}$

(d.) $A_{max} = 3 \Rightarrow PEP_{norm} = \frac{(3)^2}{2} = \underline{\underline{4.5\ W}}$

5-11.

$$h(t)=\int_{-\infty}^{\infty}H(f)\,e^{j2\pi ft}\,df$$

$$=\lim_{\substack{\alpha\to 0\\ \alpha>0}}\left[\int_{-\infty}^{0} j\,e^{\alpha f}e^{j2\pi ft}\,df+\int_{0}^{\infty}-j\,e^{-\alpha f}e^{j2\pi ft}\,df\right]$$

$$h(t)=\lim_{\alpha\to 0}\left[\frac{j\,e^{(\alpha+j2\pi t)f}}{(\alpha+j2\pi t)}\Bigg|_{-\infty}^{0}+\frac{j\,e^{-(\alpha-j2\pi t)f}}{(\alpha-j2\pi t)}\Bigg|_{0}^{\infty}\right]$$

$$=\lim_{\alpha\to 0}\left[\frac{j}{\alpha+j2\pi t}-\frac{j}{\alpha-j2\pi t}\right]=\frac{1}{\pi t}$$

$$\Rightarrow h(t)=\frac{1}{\pi t}$$

5-16. Note: T has units of Hz.

(a)

$$m(t)=\frac{\sin(\pi T t)}{\pi T t}\longleftrightarrow M(f)=\frac{1}{T}\Pi\left(\frac{f}{T}\right)=\frac{1}{T}\left[\Pi\left(\frac{f-\frac{T}{4}}{T/2}\right)+\Pi\left(\frac{f+\frac{T}{4}}{T/2}\right)\right]$$

$$\Rightarrow \mathcal{F}[\hat{m}(t)]=M(f)\begin{Bmatrix}-j, & f>0\\ j, & f<0\end{Bmatrix}=\frac{\frac{1}{2}}{\frac{T}{2}}\left[-j\Pi\left(\frac{f-T/4}{T/2}\right)+j\Pi\left(\frac{f+T/4}{T/2}\right)\right]$$

$$\Rightarrow \hat{m}(t)=-j\frac{1}{2}\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}e^{j2\pi\frac{T}{4}t}+j\frac{1}{2}\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}e^{-j2\pi\frac{T}{4}t}$$

$$=\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}\,\frac{e^{j\pi\frac{T}{2}t}-e^{-j\pi\frac{T}{2}t}}{2j}=\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}\sin\left(\pi\frac{T}{2}t\right)=\frac{\sin^2(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}$$

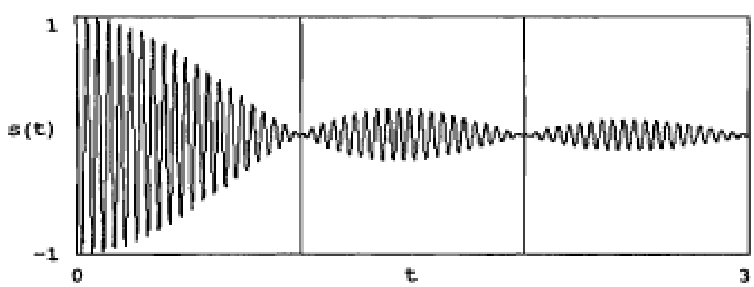
(b)

$t := 10^{-7}, 0.002 .. 3 \qquad T := 2$

$f_c := 20 \qquad \omega_c := 2\cdot\pi\cdot f_c$

$$m(t) := \frac{\sin(\pi\cdot T\cdot t)}{\pi\cdot T\, t} \qquad m_h(t) := \frac{\left[\sin\left[\pi\cdot\frac{T}{2}\cdot t\right]\right]^2}{\pi\cdot\frac{T}{2}\cdot t}$$

$$s(t) := m(t)\cdot\cos\left[\omega_c\cdot t\right]-m_h(t)\cdot\sin\left[\omega_c\cdot t\right]$$



5-20. (a.) $\underline{0\%}$ AM

(b.) $P_{norm} = A_c^2/2 = 10^2/2 = \underline{50 W}$

(c.) $\Delta\theta_{max} = \underline{10 \text{ radians}}$

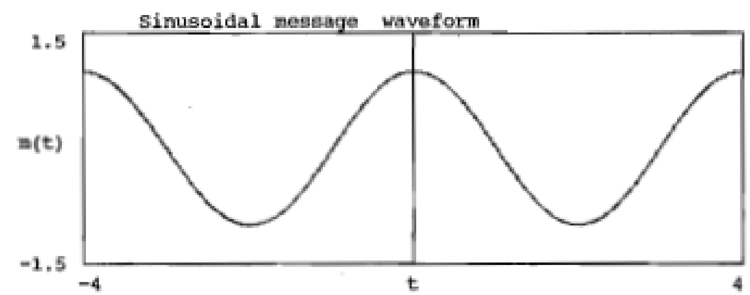
(d.) $\omega_d(t) = \frac{d\theta(t)}{dt} = -10(2000\pi)\sin(2000\pi t)$

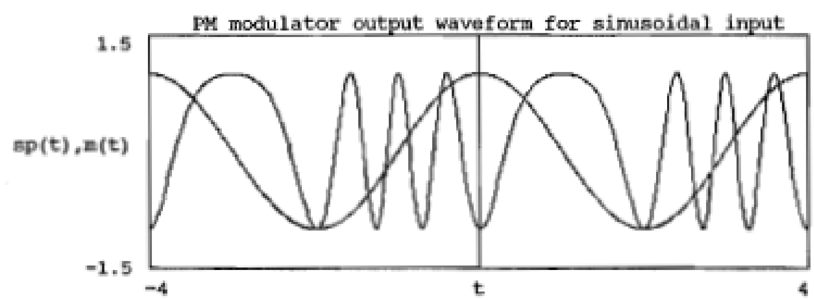
$$\Delta F \simeq \frac{\Delta\omega_d}{2\pi} = \frac{10(2000\pi)}{2\pi} = 10^4 = \underline{10 \text{ kHz}}$$

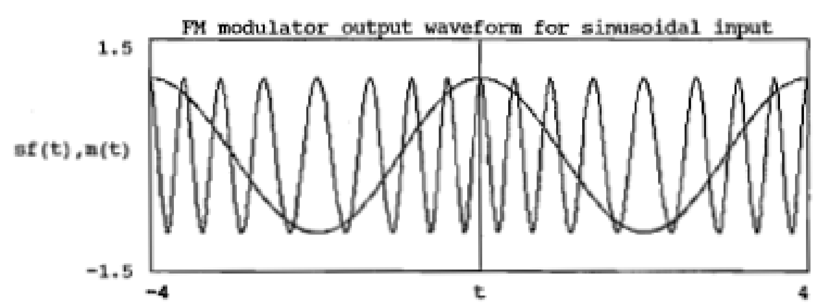
5-21.

```
t := -5,-4.99 ..5    fc := 1   fm := 0.25 fc    Dp := π   Df := π
m(t) := cos(2·π fm·t)
sp(t) := cos(2 π·fc·t + Dp·m(t))
sf(t) := cos(2·π·2·fc·t + θ(t))
```

$$\theta(t) := \left[\frac{Df}{2\cdot\pi\cdot fm}\right]\cdot\sin(2\ \pi\ fm\cdot t)$$







5-23. (a.) $B_T = 2(\beta+1)B = 2(\beta+1)f_m = 2(2+1)(15) = \underline{\underline{90 \text{ kHz}}}$

(b.) With infinite BW: $P_T = \frac{1}{2}A_c^2$ Note: See Prob. 5-30 solution for (1).

Finite BW: $P_T = \frac{1}{2}A_c^2 \sum_{n=-N}^{N} J_n^2(\beta) = \frac{1}{2}A_c^2 \sum_{n=-N}^{N} J_n^2(2)$ (1)

For $B_T = 90\text{kHz} \Rightarrow 2N(15) = 90 \Rightarrow N = 3$

$$P_T = \frac{1}{2}A_c^2 \left\{ J_0^2(2) + 2\left[J_1^2(2) + J_2^2(2) + J_3^2(2)\right]\right\}$$

Using Table 5-2 or Using MathCAD or MATLAB,

$$P_T = \frac{1}{2}A_c^2 \left\{ [0.2239]^2 + 2\left[(0.5767)^2 + (0.3528)^2 + (0.1289)^2\right]\right\}$$

$$\Rightarrow P_T = \frac{1}{2}A_c^2 [0.9985] \quad \Rightarrow \% P_{\text{contained}} = \frac{\frac{1}{2}A_c^2[0.9985]}{\frac{1}{2}A_c^2} \times 100 = \underline{\underline{99.8\%}}$$

5-28. $s(t) = 100\cos(\varphi(t))$

$$f_i(t) = f_c + \frac{D_f}{2\pi} m(t)$$

$$\Delta F = \frac{D_f V_p}{2\pi} = \frac{D_f \sqrt{2}}{2\pi} = 30 \text{ KHz}$$

$$\Rightarrow D_f = \frac{30(10^3)2\pi}{\sqrt{2}} = \underline{1.333 \times 10^5 \text{ rad}/\text{V}\cdot\text{s}}$$

5-28. Cont'd (a.) $m(t) = 2.5\cos(3\pi \times 10^4 t)$

$$\beta_f = \frac{\Delta F}{f_m} = \frac{1.333(10^5)\,2.5}{2\pi(1.5)10^4} = \underline{3.536} \doteq \underline{3.5}$$

$$G(f) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\,\delta(f - nf_m) \quad ; \; f_m = 15\text{KHz}$$

$$S(f_c \pm nf_m) = \frac{A_c}{2} J_n(\beta) \qquad ; \; A_c = 100$$

n	$J_n(3.5)$	$\lvert S(f_c \pm nf_m)\rvert$
0	-0.380	19.0
1	0.137	6.85
2	0.459	22.95
3	0.399 *	19.95
4	0.220	11.0
5	0.090	4.5
6	0.029	1.45
7	0.008	0.4

* for $3 \le n \le 7$, $\beta = 3.6$

(b.) $m(t) = 1\cos(6\pi \times 10^4 t)$; $A_c = 1$; $f_m = 30\text{KHz}$

$$\beta_f = \frac{1.333(10^5)}{2\pi(3)10^4} = 0.7072 \doteq 0.7$$

n	$J_n(0.7)$	$\lvert S(f_c \pm nf_m)\rvert$
0	0.881	44.05
1	0.329	16.45
2	0.059	2.95
3	0.007 *	0.35

* by interpolation (0.6 – 0.8)

$\boxed{5\text{-}29.}$

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j[\beta\sin\alpha - n\alpha]}\,d\alpha \Rightarrow J_{-n}(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j[\beta\sin\alpha + n\alpha]}\,d\alpha$$

Let $\alpha = \pi - \alpha' \Rightarrow \alpha' = \pi - \alpha$ & $d\alpha = -d\alpha'$

Thus,

$$J_{-n}(\beta) = \frac{1}{2\pi}\int_{2\pi}^{0} e^{j[\beta\sin(\pi-\alpha') + n(\pi-\alpha')]}(-d\alpha')$$

$$= \frac{e^{jn\pi}}{2\pi}\int_{0}^{2\pi} e^{j[\beta\sin\alpha' - n\alpha']}\,d\alpha'$$

$\sin(\pi - x) = \sin(x)$

$$\Rightarrow J_{-n}(\beta) = (-1)^n \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j[\beta\sin\alpha' - n\alpha']}\,d\alpha' = (-1)^n J_n(\beta) \quad (5\text{-}58)$$