Lab-IV

Analysis of the FM Spectrum ,FM Generation and FM Demodulation

Aim:

To get a basic understanding of the nature of phase modulated (PM) and frequency modulated (FM) signals. These notes define the angle modulated signal, of which PM and FM are special cases. FM generation using a VCO. Confirmation of selected aspects of the FM spectrum. Demodulation using a zero crossing counter demodulator.

Definition of Modulation:

Consider the signal:

$$y(t) = E.\cos(\omega t + \phi) \quad .... \; 1$$

This signal possesses, by definition:

- an amplitude E
- a total phase $(\omega t + \phi)$
- an instantaneous frequency defined as the time rate of change of total phase.

Any one of these three parameters may be modulated by a message $A.\cos\mu t$. Which ever parameter is chosen, then, by definition:

- the *rate* of variation of the chosen parameter should be directly proportional to the rate of variation of the message alone (μ)

- the *amount* of variation of the chosen parameter should be directly proportional to the amplitude of the message alone (A)

Other parameters may vary at the same time, as will be seen in what follows, but these variations will not be strictly in accordance with the above definitions.

Phase modulation (PM) – Definition :

According to the above requirements a signal will be phase modulated, by the message $A.\cos\mu t$, if:

$$\text{total phase} = \omega t + k_1.\cos\mu t \qquad ....2$$

and provided k_1 is linearly proportional to A, the message amplitude.

Hence:

$$PM = E \cos(\omega t + k_1.\cos\mu t) \qquad ....3$$

is a phase modulated signal.

Note that, for PM:

$$\text{instantaneous frequency} = \omega - k_1.\ \mu.\sin\mu t \qquad .... 4$$

Although the frequency is also varying with the message, the variation is not directly proportional to the message amplitude alone. Hence, by definition, this is not frequency modulation.

Frequency modulation (FM) – Definition :

According to the above requirements a signal will be frequency modulated, by the message A.cosμt, if:

$$\text{instantaneous frequency} = \omega + k_2\cos\mu t \qquad .... 5$$

and provided k_2 is linearly proportional to A, the message amplitude.

The total phase is obtained by integration of the instantaneous frequency, and thus the signal itself must be:

$$FM = E.\cos(\omega t + (k_2/\mu).\sin\mu t\ ) \qquad .... 6$$

Although the phase is also varying with the message, the variation is not directly proportional to the message amplitude alone. Hence, by definition, this is not phase modulation.

Angle modulation (FM) – Definition :

The defining equation, for both PM and FM, can be written in the form:

$$Y(t) = E.\cos(\omega t + \beta.\sin\mu t\ ) \qquad .... 7$$

One can choose β to represent either PM or FM as the case may be, and according to the definitions above. Thus:

for PM $\beta = \Delta\phi$, the peak phase deviation 8

and for FM $\beta = \Delta\phi / \mu$ 9

The parameter β is often called the *modulation index*.
Both PM and FM fall into a class known as *angle modulated* signals.

Receivers:

There are demodulators for these signals.

The demodulator in a PM receiver responds in a linear manner to the variations in phase of the PM signal, and the receiver output is a copy of the original message. Likewise the demodulator in an FM receiver responds in a linear manner to the instantaneous frequency variations of the FM signal.

There is an output from a PM receiver if the input is an FM signal, and from an FM receiver if the input is a PM signal. But these outputs will not be related to the message in a linear manner.

a general expression is defined for a modulated signal, is defined as:

$$y(t) = a(t).\cos[\omega t + \varphi(t)] \qquad ....10$$

where a(t) and φ (t) were defined as involving components at or near the message frequency only.
The envelope was defined as being |a(t)|. Phase variations are described by φ (t).

> *By definition, the output from both a PM and an FM receiver will be φ(t), provided a(t) is a constant.*

Spectrum Analysis:

The spectrum of the angle modulated signal y(t) of eqn.(7) above can be obtained by trigonometrical expansion.

Firstly you will need to know that:

$$\cos(\beta.\sin\varphi) = J_0(\beta) + 2\,[\,J_2(\beta).\cos 2\varphi + J_4(\beta).\cos 4\varphi + ...] \qquad .... 11$$

$$\sin(\beta.\sin\varphi) = 2\,[\,J_1(\beta).\sin\varphi + J_3(\beta).\sin 3\varphi + ...] \qquad .... 12$$

$$\cos(\beta.\cos\varphi) = J_0(\beta) - 2\,[\,J_2(\beta).\cos 2\varphi - J_4(\beta).\cos 4\varphi + ...] \qquad .... 13$$

$\sin(\beta.\cos\varphi) = 2\ [\ J_1(\beta).\cos\varphi - J_3(\beta).\cos3\varphi + J_5(\beta).\cos5\varphi - ....]$ 14

Here $J_n(\beta)$ is a Bessel function of the first kind, argument β, and order n.
You will also need to know that:

$$J_{-n}(\beta) = (-1)^n\ J_n(\beta) \qquad \text{........ 15}$$

Using the above formulae, y(t) of eqn.(7) can be expanded into two infinite series.

These can be combined, and then condensed, into the compact form:

$$y(t) = E.\sum_{n=-\infty}^{n=\infty} J_n(\beta).\cos(\omega_0 + n\mu)t \qquad \text{........ 16}$$

Spectral Properties:

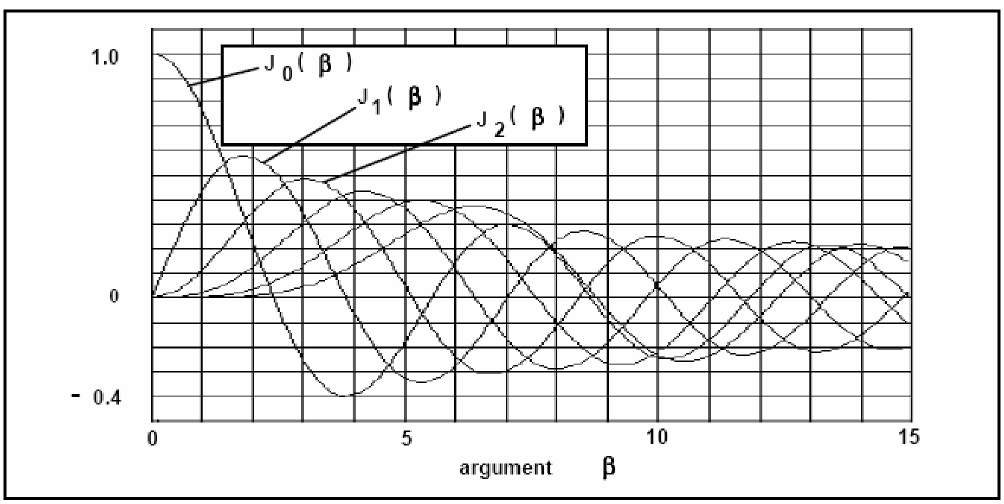

Figure 1: Plots of Bessel Functions

Notice that, as a function of β, all the curves are damped oscillatory. Except for $J_0(\beta)$, they start from zero, rise to a positive maximum, then oscillate about zero with ever decreasing amplitudes (damped). The exception, $J_0(\beta)$, starts from an amplitude of unity. Their zero crossings are *not* uniform, so the functions are not periodic. By a careful examination of eqn.(16), and the plots of the Bessel functions in Figure 1, the following properties can be deduced.

- spectral components are numbered left and right (±) counting from the central component at ω (number n = 0).
- the spectral lines are spaced μ/(2π) Hz apart
- amplitude of the ±n_{th} component from the centre is E.J_n(β). Because of eqn.(15) these two components are of equal amplitudes. Thus the spectrum is symmetrical about the central component at ω.
- the bandwidth is mathematically infinite, but in engineering terms the signal is considered confined within limits which contain all 'significant components'.
- as β increases, the bandwidth, however defined, increases
- as β increases, individual spectral lines do *not* increase in amplitude monotonically. Their amplitudes are determined by J_n(β), plots of which appear in Figure 1.
- for particular values of β the amplitude of particular sidefrequency pairs becomes zero (these are the 'Bessel zeros').
- the total power in the spectrum is constant, and independent of β.
- the largest ever component is the one at ω rad/s (often called the 'carrier'), for the special case when β = 0

Amplitude Limiting:

Amplitude limiters are used extensively in angle modulated systems.

It is an easy matter to describe the function of an amplitude limiter:

> *an amplitude limiter removes variations in the envelope of a signal.*

The input in the present context is a narrow band modulated signal defined by

$$y(t) = a(t).\cos[\omega t + \varphi(t)] \qquad \text{........ 17}$$

where both a(t) and φ (t) contain components at or near the message frequency only.

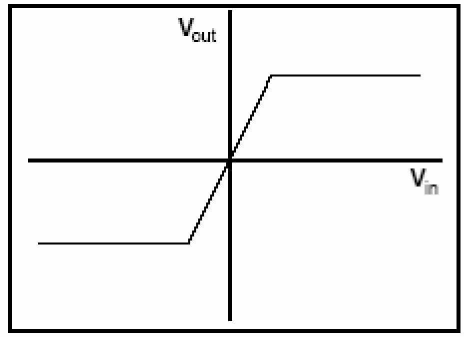

Figure 2: Limiter characteristic

An amplitude limiter can be imagined as an amplifier with a characteristic such as that of Figure 2.

Being a non-linear device it is not an easy matter to analyse its operation on a general signal. It will be sufficient for our purposes to declare that:

> *the amplitude limiter will convert a(t) of eqn.(10) to a constant.*

there will be groups of components (bands) located around the harmonics of ω. These bands have useful properties. Thus the amplitude limiter is followed by a bandpass filter to select one of these wanted bands.

Generation of FM using a VCO:

The *controllable* part of the VCO is its frequency, which may be varied about a mean by an external control voltage. The variation of frequency is remarkably linear, with respect to the control voltage,
over a large percentage range of the mean frequency. This then suggests that it would be ideal as an FM generator for communications purposes.

Unfortunately such is not the case.

The relative instability of the centre frequency of these VCOs renders them unacceptable for modern day communication purposes. The uncertainty of the centre frequency does not give rise to problems at the receiver, which may be taught to track the drifting carrier. The problem is that spectrum regulatory authorities insist, and with good reason, that communication transmitters maintain their (mean) carrier frequencies within close limits.

It is possible to stabilise the frequency of an oscillator, relative to some fixed reference, with automatic frequency control circuitry. But in the case of a VCO which is being frequency modulated there is a conflict, with the result that the control circuitry is complex, and consequently expensive. For applications where close frequency control is not mandatory, the VCO is used to good effect

Procedure:

A suitable set-up for measuring some properties of a VCO is illustrated in Figure 3.

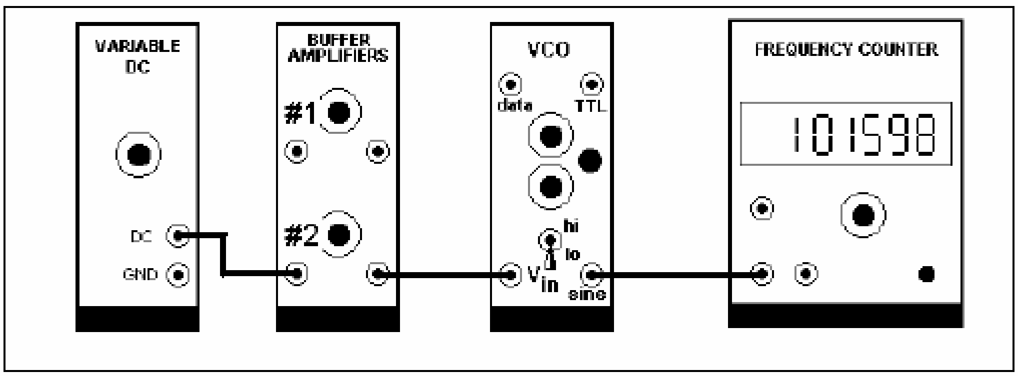

Figure 3: The FM generator

For this experiment you will need to measure the sensitivity of the frequency of the VCO to an external control voltage, so that the frequency deviation can be set as desired.

The mean frequency of the VCO is set with the front panel control labelled fo.

The mean frequency can as well be varied by a DC control voltage connected to the Vin socket. Internally this control voltage can be amplified by an amount determined by the setting of the front panel GAIN control. Thus the frequency sensitivity to the external control voltage is determined by the GAIN setting of the VCO.

A convenient way to set the sensitivity (and thus the GAIN control, which is not calibrated), to a definable value, is described below.

T1 *before plugging in the VCO, set the mode of operation to 'VCO' with the onboard switch SW2. Set the front panel switch to 'LO'. Set the front panel GAIN control fully anti-clockwise.*

T2 *patch up the model of Figure 3.*

T3 *use the FREQUENCY COUNTER to monitor the VCO frequency. Use thefront panel control f_o to set the frequency to 10 kHz.*

T4 *set the VARIABLE DC module output to about +2 volt. Connect this DC voltage, via BUFFER #2, to the* V_{in} *socket of the VCO.*

T5 *with the BUFFER #2 gain control, set the DC at the VCO* V_{in} *socket to exactly -1.0 volt. With the VCO GAIN set fully anti-clockwise, this will have no effect on* f_0*.*

T6 *increase the VCO GAIN control from zero until the frequency changes by 1 kHz. Note that the direction of change will depend upon the polarity of the DC voltage.*

The GAIN control of the VCO ***is now set*** *to give a 1 kHz peak frequency deviation for a modulating signal at* V_{in} *of 1 volt peak.*

The gain control setting will now remain unchanged.

For this setting you have calibrated the sensitivity, ***S***, of the VCO for the purposes of the work to follow.

Here: ***S*** = 1000 Hz/volt .

Deviation Linearity:

The linearity of the modulation characteristic can be measured by continuing the above measurement over a range of input DC voltages. If a curve is plotted of DC volts versus frequency deviation the linear region can be easily identified.

T7 *take a range of readings of frequency versus DC voltage at* V_{in} *of the VCO, sufficient to reveal the onset of non-linearity of the characteristic. This is best done by producing a plot as the readings are taken.*

Viewing the FM:

T8 *in the FM generator model of Figure 2 replace the variable DC module with an AUDIO OSCILLATOR, tuned to 500 Hz.*

T9 *view the output of the VCO for the FM signal.*

T10 *Use the spectrum analyzer to view the FM spectra. Sketch the FM signal and the FM spectra.*

First Bessel Zero:

Since β = Δf / fm and Δf = Vin.***S***

We have already set the sensitivity ***S*** to 1000 Hz/volt.

Hence Vin.***S***=1000 Hz.

T11 *adjust, with the AUDIO OSCILLATOR supplying the message, via the BUFFER amplifier, for β = 2.45.*

Check your Bessel tables and confirm that J0(β) = 0 when β = 2.45. You can also check this from figure 1. This Bessel coefficient controls the amplitude of the spectral component at carrier frequency, so with β = 2.45 there should be a carrier null.

T12 *use the SPECTRUM ANALYSER to confirm the amplitude of the carrier has fallen very low.*

T13 *find one of the adjacent sidebands. Its amplitude should be* $J_1(\beta)$ *times the amplitude of the unmodulated carrier. since* $J_1(2.54) = 0.5$

Special Case – β = 1.45:

For β = 1.45 the amplitude of the first pair of sidebands is equal to that of the carrier; and this will be J0(1.45) times the amplitude of the unmodulated carrier

T14 *set β = 1.45 and confirm that the carrier component, and either or both of the first pair of sideband, are of similar amplitude.*

FM Demodulation:

A simple FM demodulator, if it reproduces the message without distortion, will provide further confirmation that the VCO output is indeed an FM signal.

A scheme for achieving this result is shown modelled in Figure 4.

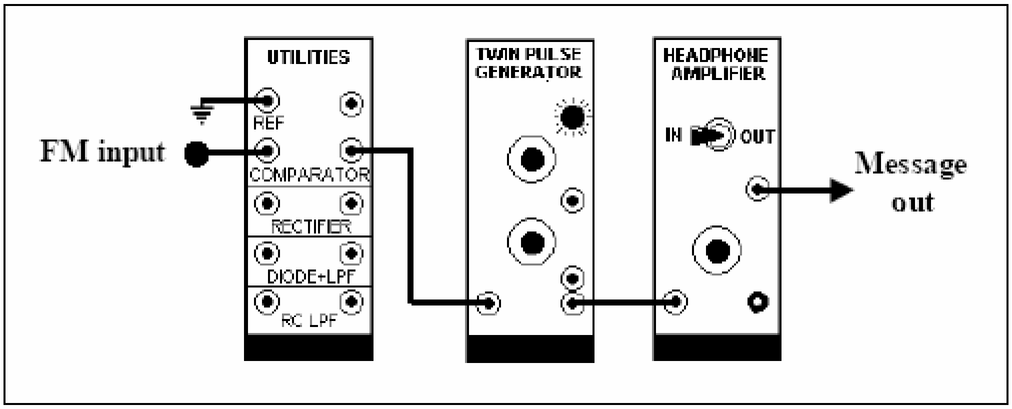

Figure 4: an FM demodulator using a zero-crossing demodulator

The TWIN PULSE GENERATOR is required to produce a pulse at each positive going zero crossing of the FM signal. To achieve this the FM signal is converted to a TTL signal by the COMPARATOR, and this drives the TWIN PULSE GENERATOR.

note:

Note that you can also use the Tunable LPF module instead of the Headphone Amplifier Module.

T15 *before plugging in the TWIN PULSE GENERATOR set the on-board MODE switch SW1 to SINGLE. Patch up the demodulator of Figure 4.*

T16 *set the frequency deviation of the FM generator to zero, and connect the VCO output to the demodulator input.*

T17 *observe the demodulator output from Head Phone Amplifier module or the Tunable LPF module.*

Questions:

Q1 State mathematically the difference between an FM signal and PM signal.

Q2 What can you say about the zero crossings of an FM and PM signal as compared to AM signal?

Q3 Given a modulating signal m(t), how would you generate an FM signal using a phase modulator?

Q4 What does the modulation index signify? Define narrow band FM and wide band FM with respect to the modulation index.