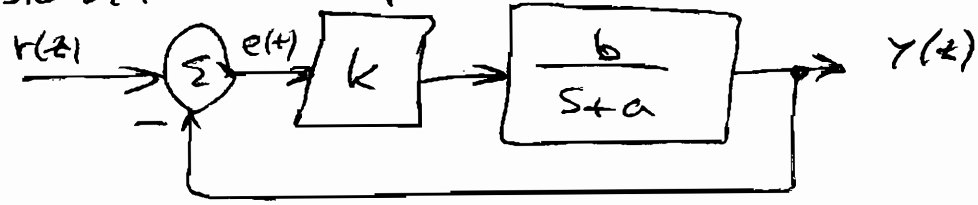
Steady-State Error and Disturbance Rejection

Example 1

Consider:

Proportional control



CLTF : $\dfrac{kb}{s+a+kb} \triangleq H(s)$

for a step input of height R, ~~the~~ the steady-state output is

$$Y_{ss} = H(0) \cdot R = \frac{kb}{a+kb} \cdot R$$

this number is < 1, for large k it approaches 1

There is a steady-state error. Y_{ss} does not exactly equal R, $\therefore$ $e_{ss} = Y_{ss} - R$

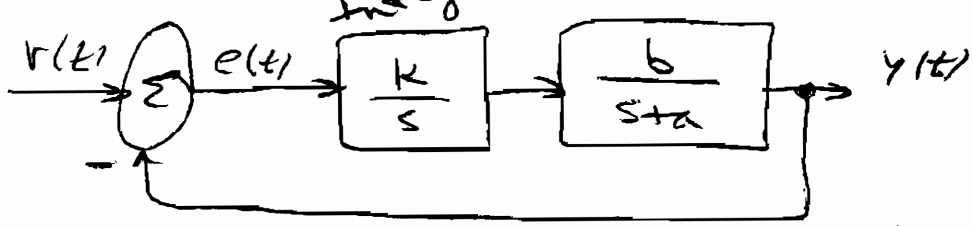
$$= \left(1 - \frac{kb}{a+kb}\right) R$$

$$= \frac{a}{a+kb} \cdot R$$

Example 2

Consider now another control system

Integral Control



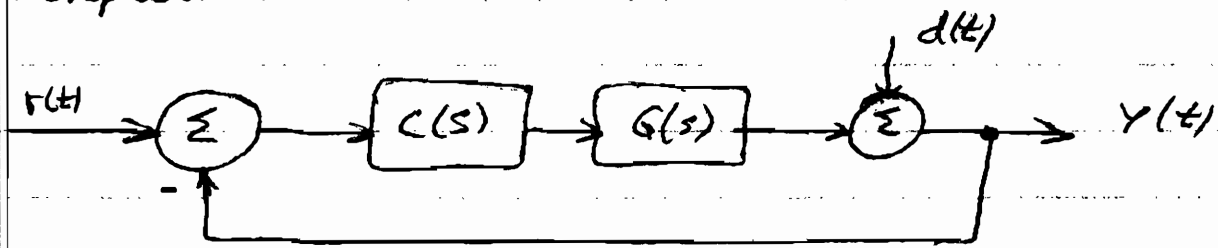
$H(0) = \frac{k \cdot b}{k \cdot b} = 1$

CLTF : $H(s) = \dfrac{k \cdot b}{s^2 + as + k \cdot b}$

$Y_{ss} = 1 \cdot R = R$

$e_{ss} = 0$

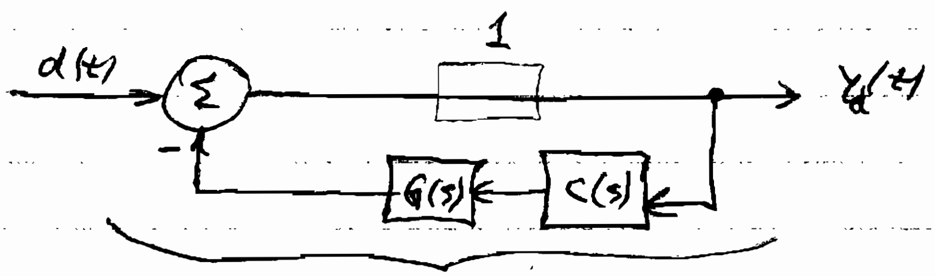
Consider now an arbitrary control system with a reference input and a disturbance input:



By superposition, consider one input at a time (set the other to zero) and add the results:

$$(1) \quad Y(s) = \underbrace{\frac{C(s)\,G(s)}{1+C(s)G(s)}}_{\frac{Y(s)}{R(s)}} R(s) + \underbrace{\frac{1}{1+C(s)G(s)}}_{\frac{Y(s)}{D(s)}} D(s)$$

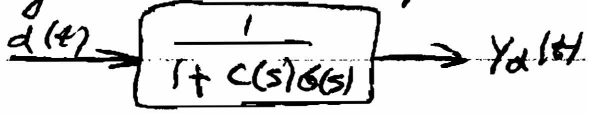
To get this transfer function, redraw picture



(Part of y(t) due to d(t))

CLTF is: $\dfrac{1}{1+C(s)G(s)}$

Consider the effect of a disturbance step input d(t) of height Δ on the plant output. Using eq. (1)



From previous Example 1, $C(s)G(s) = \frac{K \cdot b}{s+a}$

From previous Example 2, $C(s)G(s) = \frac{K \cdot b}{s(s+a)}$

$\boxed{\text{In steady state:}}$

Example 1: $y_d = \left.\frac{1}{1+C(s)G(s)}\right|_{s=0} \cdot D = \boxed{\frac{a \cdot D}{a+kb}}$

Example 2: $y_d = \left.\frac{s(s+a)}{s(s+a)+k \cdot b}\right|_{s=0} \cdot D = 0 \cdot D = \boxed{0}$

Note that a steady-state (step) disturbance has a nonzero effect on the Example 1 control system.
However, a step disturbance in the Example 2 system has <u>zero</u> effect in steady state.
This is called <u>disturbance rejection</u>.

<u>Steady-State Error and System Type</u>

Look at a tracking system with no disturbance.
The error signal is $e(t) = r(t) - y(t)$, so

$$E(s) = R(s) - \frac{C(s)G(s)}{1+C(s)G(s)} R(s) = \frac{1}{1+C(s)G(s)} R(s)$$

Let $C(s)\,G(s) = \frac{1}{s^N} F(s)$, where $F(s)$ is what is "left over" after taking $\frac{1}{s^N}$ out of $C(s)\,G(s)$. Note that $\boxed{F(0) \text{ is a nonzero real number.}}$

N could equal $0, 1, 2, \ldots$ depending on how many factors of $\frac{1}{s}$ are contained in $C(s)G(s)$

Consider $N=0$, called Type-0 system

Step input: $\left(R(s)=\frac{R}{s}\right)$

$$e_{ss} = \lim_{s\to 0} \frac{1}{1+F(s)} \cdot R = \boxed{\frac{R}{1+F(0)}}$$

nonzero steady-state error

$y_{ss} \neq R$

Ramp input:

$r(t) = \alpha \cdot t$

$R(s) = \frac{\alpha}{s^2}$

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+F(s)} \cdot \frac{\alpha}{s^2}$$

$$= \lim_{s\to 0} \frac{1}{(1+F(0))s} = \boxed{\infty}$$

$y(t)$ gets further and further away from $r(t)$ as t increases

N=1, Type 1 System

Step input:

$$e_{ss} = \lim_{s\to 0} \frac{1}{1+\frac{F(s)}{s}} = \lim_{s\to 0} \frac{s}{s+F(s)}$$

$$= \boxed{0}$$

zero steady-state error

Ramp input:

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+\frac{F(s)}{s}} \cdot \frac{\alpha}{s^2} = \lim_{s\to 0} \frac{\alpha}{s+F(s)} = \boxed{\frac{\alpha}{F(0)}}$$

nonzero but finite steady-state error