ELE 457 Homework # 1 Solutions

1 Prob. 2.12

(a) From diagram (a), $C = G_2(G_1 \cdot E + H \cdot F)$ (1)

From diagram (b), $C = G_a(E + G_b \cdot F)$ (2)

From (1) & (2), $\boxed{G_a = G_2 \cdot G_1}$ and

$$G_a \cdot G_b = G_2 \cdot H \quad \text{or} \quad G_b = \frac{G_2 \cdot H}{G_a} = \frac{\cancel{G_2} \cdot H}{\cancel{G_2} \cdot G_1} = \boxed{\frac{H}{G_1}} \quad (a)$$

$$G_b = G_1^{-1} G_2 H$$

From diagram (c), $C = G_c E + G_d F$ (3)

From (1) & (3), $\boxed{G_c = G_2 \cdot G_1}$ and (b)

$\boxed{G_d = G_2 \cdot H}$

2. (a) $f(t) = -3te^{-t} \rightarrow \boxed{F(s) = \frac{-3}{(s+1)^2}}$

(b) $f(t) = -5\cos t \longrightarrow \boxed{F(s) = \frac{-5s}{s^2+1}}$

(d) $f(t) = 7e^{-.5t}\cos 3t \rightarrow \boxed{F(s) = \frac{7(s+.5)}{(s+.5)^2+9}}$

$\cos(30) = \frac{\sqrt{3}}{2}$, $\sin(30) = \frac{1}{2}$

(e) $f(t) = 5\cos(4t + 30^\circ)$

$$= 5\cos(4t)\cos(30^\circ) - 5\sin(4t)\sin(30^\circ)$$

$$\approx 4.33\cos(4t) - 2.5\sin(4t)$$

$$F(s) = \frac{4.33s}{s^2+16} - \frac{10}{s^2+16} = \boxed{\frac{4.33s - 10}{s^2+16}}$$

(f) $f(t) = 6e^{-2t}\sin(t - 45^\circ)$

$$= 6e^{-2t}\left[\sin(t)\cos(45) - \cos(t)\sin(45)\right]$$

$$\approx 4.24e^{-2t}\left[\sin t - \cos t\right)$$

$$= 4.24\left[\frac{1}{(s+2)^2+1} - \frac{s+2}{(s+2)^2+1}\right]$$

$$= \boxed{\frac{-4.24(s+1)}{s^2+4s+5}}$$

3. $\ddot{y} + 2\dot{y} + 7y = 3\dot{u} + u$

take $\downarrow$ Laplace Transform

$$s^2Y(s) + 2sY(s) + 7Y(s) = 3s\,U(s) + U(s)$$

$$(s^2+2s+7)Y(s) = (3s+1)U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{3s+1}{s^2+2s+7}}$$

4. (a) CLTF

$$\frac{\frac{k\cdot b}{s+a}}{1+\frac{k\cdot b}{s+a}} = \boxed{\frac{k\cdot b}{s+a+k\cdot b}}$$

call it $H(s)$

(b) $R(s) = \frac{1}{s}$; $Y(s) = H(s)\,R(s)$

$$Y_{ss} = \lim_{s\to 0} \cancel{s}\,H(s)\,\frac{1}{\cancel{s}} = H(0) = \boxed{\frac{k\cdot b}{a+k\cdot b}}$$

(c) CLTF

$$\frac{\frac{k_2\cdot b}{s(s+a)}}{1+\frac{k_2\cdot b}{s(s+a)}} = \boxed{\frac{k_2\cdot b}{s^2+as+k_2\cdot b}}$$

$$Y_{ss} = H(0) = \frac{k_2\cdot b}{k_2\cdot b} = \boxed{1}$$

(d) CLTF

$$\frac{b(k_3+k_4/s)/(s+a)}{1+\frac{b(k_3+k_4/s)}{s+a}} = \frac{b(k_3+k_4/s)}{s+a+b(k_3+k_4/s)}$$

$$= \boxed{\frac{b\cdot k_3\cdot s+b\cdot k_4}{s^2+(a+b\cdot k_3)s+b\cdot k_4}}$$

$$Y_{ss} = H(0) = \frac{b\cdot k_4}{b\cdot k_4} = \boxed{1}$$