ELE 457   **Homework 2 Solutions**   Fall '09

$\boxed{1.}$ From the graph, the steady-state value is 0.5, and the time at which $y(t)$ crosses $.63 \times 0.5 = .32$ is approximately 0.18 sec.

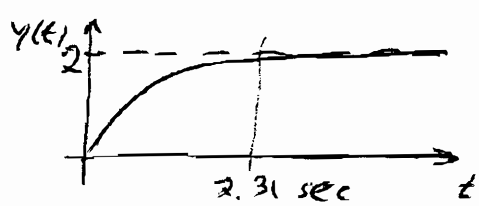
Thus $a = \dfrac{1}{.18} = 5.55$ and

$$\frac{H \cdot b}{a} = \frac{3 \cdot b}{5.55} = 0.5 \Rightarrow b = .925$$

$$\boxed{\frac{b}{s+a} = \frac{.925}{s+5.55}}$$

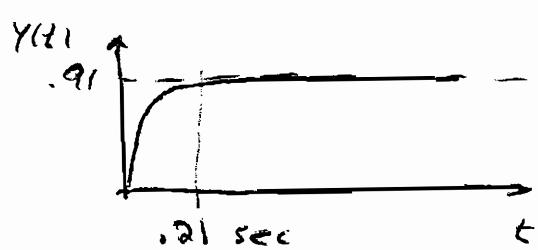
$\boxed{2.}$ $\dfrac{4}{s+2}$ $y_{ss} = \dfrac{4}{2} = 2$, $T_s = \dfrac{4.62}{2} = 2.31$

(a)

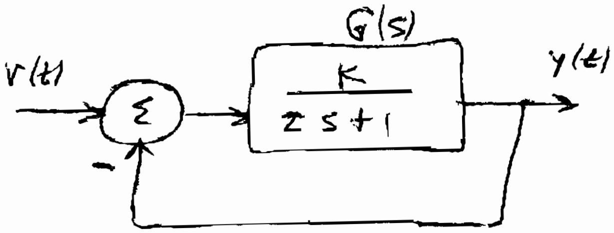


(b) $G = \dfrac{20}{s+2}$.   CLTF $\dfrac{G}{1+G} = \dfrac{20}{s+2+20} = \dfrac{20}{s+22}$

$$y_{ss} = \frac{20}{22} = .91, \quad T_s = \frac{4.62}{22} = .21 \text{ sec}$$



3. Prob 4.2 a



$$\frac{G}{1+G} = \frac{K}{\tau s + 1 + K} = \frac{K/\tau}{s + \left(\frac{K+1}{\tau}\right)} = \frac{b}{s+a}$$

$$y_{ss} = \frac{K}{K+1} = \underbrace{.96}_{\text{from graph}} \Rightarrow K = .96K + .96$$

$$.04K = .96 \quad \boxed{K = 24}$$

From the graph, $y(1) = .60$

$$y(t) = \frac{b}{a}(1-e^{-at}) = \frac{K}{K+1}(1-e^{-at})$$

$$y(1) = .6 = \frac{24}{25}(1-e^{-a})$$

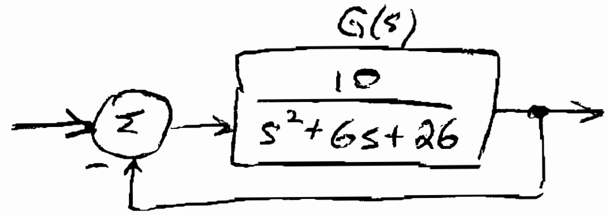
$$\text{or} \quad \frac{(.6)(25)}{24} = 1-e^{-a}, \quad e^{-a} = .375$$

$$-a = \ln(.375) = -.9808$$

$$a = 0.9808 = \frac{K+1}{\tau} = \frac{25}{\tau}; \quad \boxed{\tau = 25.5}$$

Note: the picture in the book cannot be correct because the settling time should be $\frac{4.62}{.98} \approx 4.7$ sec and the picture shows the settling time to be around 2 sec.

4. Prob 4.4 (a)



CLTF $\frac{G}{1+G} = \frac{10}{s^2+6s+36} = H(s)$

Steady-state value is $H(0) = \frac{10}{36} \approx .2778$

$\omega_n^2 = 36 \Rightarrow \omega_n = 6$

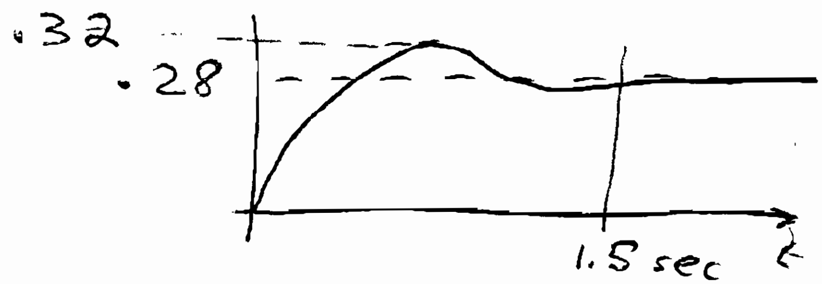
$2\zeta\omega_n = 6 \Rightarrow \zeta = 6/(2\cdot 6) = 0.5$

$\zeta\omega_n = 3 \Rightarrow$ Settling time is $\frac{4.62}{3} \approx 1.54$ sec

$P.O. = 100\, e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 16.3\ \%$

So $\frac{\Delta}{y_{ss}} = \frac{\Delta}{.2778} = .163$ or $\Delta = .0453$

Peak $= y_{ss} + \Delta = .2778 + .0453 = .3231$



5. 4.14 (a) $T_s < 2 \Rightarrow \frac{4.62}{\zeta\omega_n} < 2$ or $\boxed{\zeta\omega_n > 2.31}$

$P.O. < 10 \Rightarrow 100\, e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} < 10$

$e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} < .1 \Rightarrow \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} < \underbrace{-2.3}_{\ln(0.1)}$

Multiply both sides by minus 1 and change sense of inequality.

$\zeta\pi > 2.3\sqrt{1-\zeta^2}$ $\qquad 2.3^2 = 5.29$

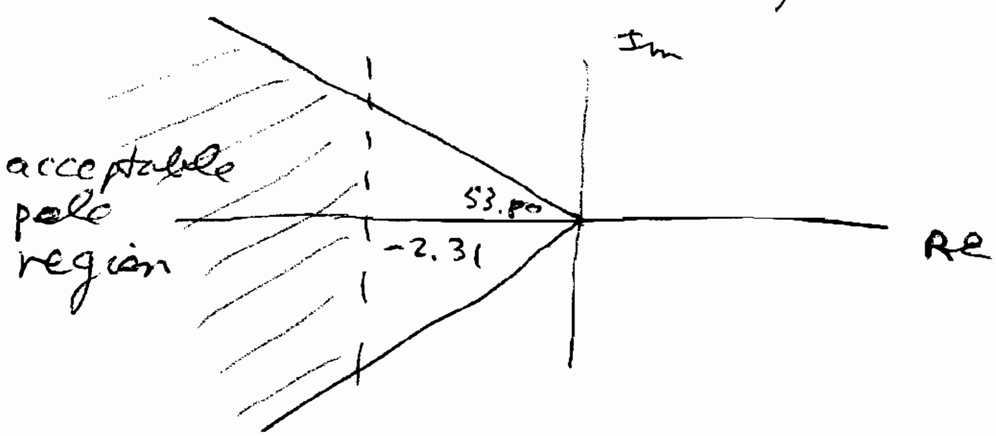
$\zeta^2\pi^2 > 5.29(1-\zeta^2)$

$\zeta^2(\pi^2+5.29) > 5.29$ $\qquad \zeta^2 > \dfrac{5.29}{\pi^2+5.29}$ $\quad .349$

or $\zeta > \sqrt{.349} = .59$

$\boxed{\zeta > .59}$

$\theta = \cos^{-1}(.59) = 53.8°$, $\quad \theta < 53.8°$



$\boxed{6.}$ (a) CLTF $\dfrac{4K}{s^2+2s+4K}$

$\omega_n = 2\sqrt{K}$

$2\zeta\omega_n = 2$

or $\omega_n = \dfrac{1}{\zeta} = \dfrac{1}{1.2}$

$= .833$

$.833 = 2\sqrt{K}$

$$\boxed{K = \left(\frac{.833}{2}\right)^2 = .174}$$

Denominator is $s^2+2s+.696$, whose roots are

$$-1 \pm \sqrt{\left(\tfrac{2}{2}\right)^2 - .696} = -1 \pm .55$$

Poles are at $\boxed{-1.55 \ \& \ -.45}$

Approximate settling time is $\dfrac{4.62}{.45} \approx \boxed{10.3 \text{ sec}}$

(b) We choose $\boxed{\beta = 2}$ to cancel the stable plant pole at $s=-2$. This plant pole is effectively replaced by a pole at $s=-\alpha$ and the closed-loop system remains second order.

To get $\frac{1}{2}$ the settling time as in (a), we need $\zeta\omega_n$ to be twice as large, but keep ζ at a value of 1.2. Thus $\omega_n = 2(.833) = 1.666$

The CLTF is $\dfrac{4K}{s^2 + \underbrace{\alpha s}_{2\zeta\omega_n} + 4K}$

$2 \cdot \zeta \cdot \omega_n$
$= 2(1.2)(1.666)$
$\boxed{\alpha = 4}$

$4K = \omega_n^2 = (1.666)^2 \Rightarrow \boxed{K = .694}$

So $\boxed{C(s) = .694 \dfrac{(s+2)}{s+4}}$

Closed-loop poles are the roots of $s^2 + 4s + 2.776$ which are $-2 \pm \sqrt{\left(\frac{4}{2}\right)^2 - 2.776} = -2 \pm 1.1063$

Poles: $-3.1063, \; -.8937$

Approximate settling time $\dfrac{4.62}{.8937} = \boxed{5.17 \text{ sec}}$ which is half that obtained in part (a).