ELE 457 Homework 3 Solutions Fall '09

5.11

$\boxed{1.}$ (a) $G_1(s) = \dfrac{10K}{5s+1}$ $\qquad d(t) = 5$ (step)

$$Y_d(s) = \underbrace{\boxed{\frac{1}{1+G_1(s)}}}_{Y_d(s)/R(s)} \underbrace{\boxed{\frac{5}{s}}}_{R(s)} ; \quad \underset{\text{steady state}}{y_d} = \left.\frac{5(5s+1)}{5s+1+10K}\right|_{s=0}$$

$$y_d = \frac{5}{1+10K} = .05 \quad \text{or} \quad 4.95 = .5K$$

$$\boxed{K = 9.9}$$

(b) $r(t) = 10$ (step) $\qquad Y(s) = \dfrac{G_1(s)\,R(s)}{1+G_1(s)} = \dfrac{10K}{5s+1+10K} \cdot \dfrac{10}{s}$

$$y_{ss} = \frac{100 \times 9.9}{1+99} = 9.9. \text{ thus}$$

$$e_{ss} = 10 - 9.9 = \boxed{0.1}$$

This error is 1% of the commanded value

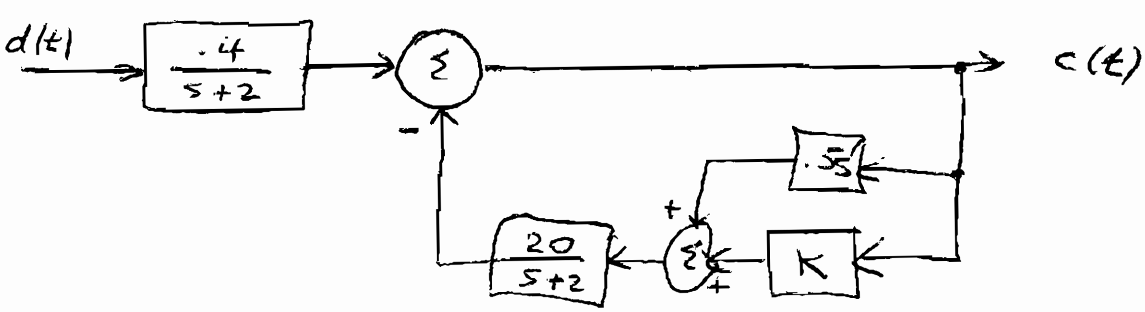
$\boxed{2.}$ (5.13)

(a) From the diagram, $V(s) = 0.5s \; C(s)$

Take inverse Laplace Transform to get

$$\boxed{v(t) = \tfrac{1}{2}\dot{c}(t)}$$ ($\dot{}$ = derivative wrt time)

(b)



The feedback path transfer function is $\dfrac{(20)(.5s+K)}{s+2}$

$$= \frac{10s+20K}{s+2}$$

$$\frac{C(s)}{D(s)} = \frac{.4}{s+2} \frac{1}{1 + \frac{10s+20K}{s+2}} = \frac{.4}{11s+2+20K}$$

$$\underset{\text{(steady state}}{C_d} = \frac{.4D}{2+20K} < .01\,D$$

$$.4D < .02 + .2K \qquad K > \frac{.38}{.2}$$

$$\boxed{K > 1.9}$$

(c) The TF from $e(t)$ to $c(t)$ is

$$\frac{K\frac{20}{s+2}}{1+\frac{10s}{s+2}} = \frac{20K}{11s+2} \leftarrow \text{no factor of } s$$

$\therefore$ $\boxed{\text{Type-0 system}}$

(d) In steady-state $(s=0)$, $0.5s = 0$ so $\boxed{\text{output of sensor 2 has no effect on steady-state error}}$

$\boxed{3.}$ (5.14) (a) Type 0, CLTF $\frac{(s+1)(s+3)}{(s+1)(s+3)+10} = \frac{E(s)}{R(s)}$

$$e_{ss}\,(\text{step}) = \frac{3}{3+10} = \boxed{\frac{3}{13}}$$

$$e_{ss}\,(\text{ramp}) = \lim_{s\to 0} s \cdot \frac{3}{3+10}\frac{1}{s^2} = \lim_{s\to 0}\frac{3}{s\cdot 13} \rightarrow \boxed{\infty}$$

$\therefore$ $\boxed{e_{ss} \text{ is infinite}}$

(b). Type 1 $\quad \frac{E(s)}{R(s)} = \frac{s(s+1)(s+6)}{s(s+1)(s+6)+10}$

$$e_{ss}(\text{step}) = \frac{0}{10} = \boxed{0}$$

$$e_{ss}(\text{ramp}) = \lim_{s\to 0} \frac{s^2(s+1)(s+6)}{s(s+1)(s+6)+10}\frac{1}{s^2}$$

$$= \frac{6}{10} = \boxed{.6}$$

(c). Type 2 $\quad \frac{E(s)}{R(s)} = \frac{s^2(s+6)}{s^2(s+6)+7(s+2)}$

$$e_{ss}(\text{step}) = \frac{0}{14} = \boxed{0}$$

$$e_{ss}(\text{ramp}) = \lim_{s\to 0} \frac{s^3(s+6)}{s^2(s+6)+7(s+2)}\frac{1}{s^2}$$

$$= \lim_{s\to 0} \frac{s(s+6)}{14} = \boxed{0}$$

(d). Type 1 $\quad \frac{E(s)}{R(s)} = \frac{s(s^2+4)}{s(s^2+4)+6s^2+2s+10}$

$$e_{ss}(\text{step}) = \boxed{0}$$

$$e_{ss}(\text{ramp}) = \frac{4}{10} = \boxed{.4}$$