ELE 457 — HW #4 Solutions — Fall 2009

1 (a)

$$\begin{array}{|cc} 1 & 3 \\ 2 & 1 \\ \frac{5}{2} & \\ 1 & \end{array} \qquad -\frac{1}{2}(1-6) = \frac{5}{2}$$

$\boxed{\text{stable}}$

(b)

$$\begin{array}{|ccc} 1 & 5 & 4 \\ 1 & 2 & 0 \\ 3 & 4 & \\ \frac{2}{3} & 0 & \\ 4 & & \end{array} \qquad -\frac{1}{3}(4-6) = \frac{2}{3}$$

$\boxed{\text{stable}}$

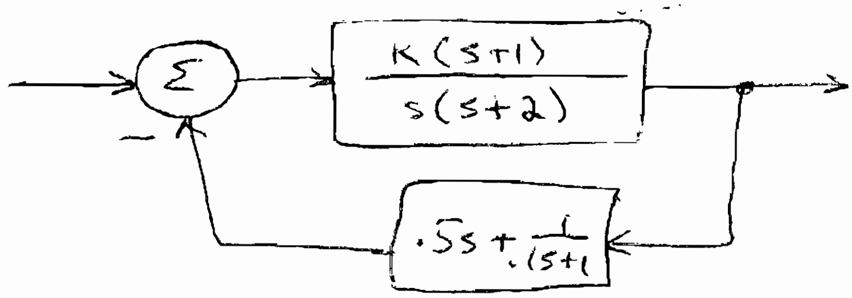
2. Prob 6.5

(a) @ $s=0$, sensor 1 has a $\boxed{\text{gain of } 1}$

(b) @ $s=0$, sensor 2 has a $\boxed{\text{gain of } 0}$

(c) Type 1 (1 integrator in forward path)

(d)



$$CLTF = \frac{G}{1+GH}$$

$$= \frac{\dfrac{K(s+1)}{s(s+2)}}{1 + \dfrac{K(s+1)}{s(s+2)} \cdot \dfrac{.5s^2+5s+10}{s+10}} \cdot \frac{s(s+2)(s+10)}{s(s+2)(s+10)}$$

$$\frac{.5s(.1s+1)+1}{.1s+1}$$

$$\frac{.5s^2+5s+10}{s+10}$$

$$\text{CLTF} = \frac{K(s+1)(s+10)}{s(s+2)(s+10) + K(s+1)(.5s^2+5s+10)}$$

denominator is $s^3+12s^2+20s+.5Ks^3+5Ks^2+10Ks$
$\qquad + .5Ks^2 \quad +5Ks + 10K$

$$(1+.5K)s^3 + (12+5.5K)s^2 + (20+15K)s + 10K$$

$$\begin{array}{l|ll}
 & 1+.5K & 20+15K \\
 & 12+5.5K & 10K \\
 & \frac{240+280K+77.5K^2}{12+5.5K} & \\
 & 10K &
\end{array}
\qquad \frac{(240+290K+82.5K^2)-(10K+5K^2)}{12+5.5K}$$

$10K \rightarrow$ implies $\boxed{K>0}$

with $K>0$, all entries in first column are positive

$\boxed{3.}$ 6.7 (a) System is Type 1 so steady-state error for a step input is $\boxed{0.}$

(b) CLTF is $\dfrac{K}{s(s+1)(s+3)+K} \rightarrow s^3+4s^2+3s+K$

$$\begin{array}{l|ll}
 & 1 & 3 \\
 & 4 & K \\
 & \frac{12-K}{4} & \longrightarrow K<12 \\
 & K & \longrightarrow K>0
\end{array}
\qquad \boxed{0<K<12}$$

$\boxed{4.}$ CLTF is
$$\frac{k(s+1)(s+6)}{\underbrace{s(s+2)(s-1)}_{s^3+s^2-2s} + \underbrace{k(s+1)(s+6)}_{ks^2+7ks+6k}}$$

denominator is $s^3 + (k+1)s^2 + (7k-2)s + 6k$

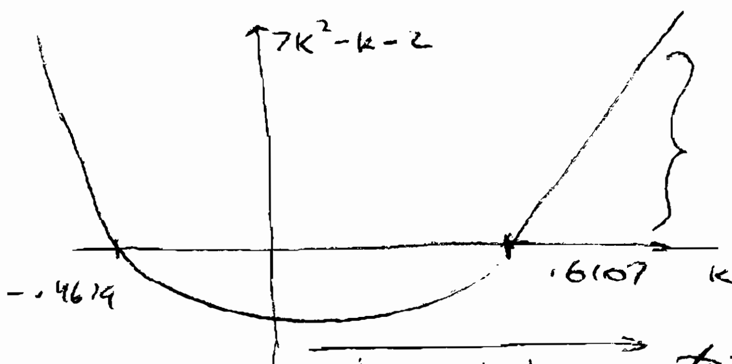
$$\begin{array}{l|ll} & 1 & 7k-2 \\ & 1+k & 6k \\ & \alpha & 0 \\ & 6k & \longrightarrow k>0 \end{array} \qquad \alpha = \frac{(k+1)(7k-2) - 6k}{k+1}$$

for α to be greater than zero we need (with $k>0$)

$7k^2 - k - 2 > 0$ [Positive coeff on k^2 means parabola opens upward.]

roots: $\frac{1}{14} \pm \frac{\sqrt{1+56}}{14}$

$= .0714 \pm .5393 = -.4679,\ 0.6107$



need $\boxed{k > .6107}$

k must be greater than zero from element 4 of Routh Array first column