

Due Tuesday, November 3

1. The purpose of this problem is to use Matlab to investigate the accuracy of a recently derived formula for the settling time of a second-order system. The formula given in class for settling time is: $T_{S1} = 4.62/\alpha$, where α is the negative of the real part of the pole(s) closest to the $j\omega$ axis. For second-order systems, this formula is exact only for $\zeta < 0.9$. It is an approximation for all other values of ζ . However, we can find a “fudge-factor”, f , such that the settling time is given by $T_{S2} = 4.62f/\alpha$. The formula for f is given below:

$$f = \begin{cases} 1 & \zeta < 0.9 \\ 4.466\zeta - 3.024 & 0.9 < \zeta < 1 \\ \frac{0.008891}{\zeta - 0.98} + 1.0014 & 1 < \zeta < 1.7 \\ 1 & \zeta > 1.7 \end{cases}$$

Consider the following system:

$$\frac{21.344}{s^2 + 9.24\zeta s + 21.344}$$

For systems that do not have too much overshoot (e.g. for $\zeta > 0.9$) the following Matlab code may be used to find the settling time of a given second-order system:

```
>> time=[0:1.e-4:4]; % create a vector of time points
>> zeta=xxx; % set the value of zeta
>> y=step(tf(21.344,[1 9.24*zeta 21.344]),time); % compute step response
>> t=find(y>=0.99);
>> st=time(min(t)); % this is the actual settling time
```

Create a table with the following columns: ζ , actual settling time (from Matlab), $4.62/\alpha$, f , $4.62f/\alpha$, percent error for T_{S1} , percent error for T_{S2} . Note that the percent error is:

$$\frac{T_S - \text{actual settling time}}{\text{actual settling time}}$$

Fill out the rows of the table for the following values of ζ : 0.9, 0.95, 1, 1.3, 1.5, 1.7.

2. Use Simulink to obtain the step response of the system shown in Fig. P5.13 (page 186 in the book). Replace the Sensor 2 transfer function with the following: $0.05s/(0.1s+1)$. Replace the gain K with the following transfer function: $0.02(s+3)/s$. Include a printout of the Simulink diagram and plots from the scope on the plant output signal for the following two cases:

- (a) Let $r(t)$ be a unit step applied at $t = 0$ and $d(t) = 0$.
- (b) Let $r(t)$ be a unit step applied at $t = 0$ and $d(t)$ be a unit step of height -2 applied at $t = 2$.

Include a printout of the Simulink diagram and a plot from the scope.

3. Obtain a hand sketch of the root locus plot for the following systems. Calculate the center and angle of asymptotes for each plot.

(a) $\frac{1}{s(s+1)(s+4)}$

(b) $\frac{s+3}{s(s+1)(s+4)}$

(c) $\frac{(s+1.5)(s+5.5)}{(s+1)(s+5)}$

(d) $\frac{s+.5}{s(s+1)(s+4)}$