Due Tuesday, November 10

Here are some Matlab instructions that make root-locus design easy. If the forward-path transfer function of a negative-unity-feedback control system is given by

$$G(s) = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

then the following instructions will create a root locus plot. Note that all denominator coefficients must be entered in the command, including those that are zero. For the numerator, the first nonzero coefficient and all coefficients of lower powers of $s$ must be entered, but leading coefficients of zero do *not* have to be entered. In other words, the coefficient vector for the numerator starts with the first nonzero coefficient.

```
G=tf([b0 b1 ... bn],[1 a1 ... an]);
sisotool('rlocus',G)
```

Note that if you are given the denominator in factored form, you can use Matlab to multiply out the roots to obtain the polynomial coefficients using the `poly` command. For example, if the denominator of $G(s)$ is given as $(s + p_1)(s + p_2) \cdots (s + p_n)$ then the first Matlab command given above can be replaced by: `G=tf([b0 b1 ... bn],poly([-p1 -p2 ... -pn]);`

1. Problem 7.8 from the book. Show hand sketches of the root locus plots.

2. Problem 7.10 (a), (b). Replace the remaining parts of the problem with the following.

   (c) Keep $K$ fixed at 0.377. Design PI compensator to achieve a critically damped step response. What is the approximate settling time of this control system? What is it about the root-locus plot that tells you that you cannot make this PI control system have a smaller settling time?

   (d) Use Simulink to simulate the unit step response of the closed-loop system from part(b).

   (e) Simulate the unit step response for the system with the PI compensator from part (c).

   (f) In order to improve the settling time, use a lead-lag compensator with two poles and two zeros. Let the poles of the compensator be at 0 and -11. (The pole at zero is an integrator which will result in zero steady-state error to a step input.) Let the zeros of the compensator be at -1 and -2. Notice that these zeros cancel two of the plant poles. Perform this cancelation *before* giving the transfer function to `sisotool`. Choose the compensator gain to get a critically damped response. What is the approximate settling time of this control system?

   (g) Simulate the unit step response for the control system designed in the previous part. Note that Simulink does *not* require that you cancel the plant poles and the compensator zeros. Thus, your Simulink model should contain a compensator with two poles and two zeros and the plant with all three of its poles.

3. Consider the system described in Problem 7.12. For this system, do the following:

   (a) Find the value of $K_D$ that makes the root locus contain a double pole at $s = -0.25$. [Hint: The polynomial $s^2 + 2as + b$ has roots at $s = -a \pm \delta$. The sum of these roots is $2a$. In this problem, we want $a = 0.25$ so that the two roots will meet at $s = -0.25$.

   (b) Let $K_D$ have the value found in part (a). Find the value of $K_P$ that results in a critically damped control system. [Hint: use `sisotool`.]

   (c) Use Simulink to obtain the response of this system to a step input of 10°.