High-Pass Two-Dimensional Ladder Network Resonators for MRI
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Introduction:
Multi-element radio frequency resonator designs are of interest for magnetic resonance imaging (MRI) due to their sensitivity for anatomical structures near the surface of the body. In this article, a novel two-dimensional (2-D) coil design consisting of inductively coupled planar elements is presented. It has been theoretically shown that the array produces a high-frequency resonant mode that can be used to generate the traditional quadrature B1 fields widely used in MRI.

Two-D Ladder Networking Resonators:
Low-pass 2-D ladder networks offer a homogeneous mode for circularly polarized MRI applications. However, higher field applications of these structures are limited by the fact that the Eigen value of the most homogeneous normal mode is lowest in frequency. The high-pass configuration differs in that the elements are only inductively coupled, i.e., they do not share a common leg; the resonator is a collection of inductively coupled resonators as shown in fig 1(d) above. Each element of the resonator array is represented by four conducting strips having a self inductance $L$ and a capacitor $C$ in each leg. A recursion relation for Kirchhoff’s Voltage equations on the meshes of the structure of interest has been used. It can be written as:

$$\left(\omega L - \frac{1}{\omega C}\right)I_{m,n} = -\kappa^2 L(I_{m+1,n} + I_{m-1,n} + I_{m,n+1} + I_{m,n-1}) - \alpha^2 L(I_{m+1,n+1} + I_{m-1,n+1} + I_{m-1,n-1} + I_{m+1,n-1} - I_{m,n}) = 0$$

where $I_{m,n}$ are the current amplitudes in the $m$, $n$th element $(m=0..M+1, n=0..N+1)$. The constants $k$ and $\alpha$ represent the coefficients of mutual inductance between nearest neighbors having parallel legs.

$$I_{m,n}(\omega) = A_{m,n} \sin \frac{\pi \omega \Gamma}{M+1} \sin \frac{\pi \omega \Gamma}{N+1}$$

yield the dispersion relation

$$\omega^2 = \frac{1}{LC} \left(1 - 2\kappa \left(\cos \frac{\pi \omega \Gamma}{M+1} + \cos \frac{\pi \omega \Gamma}{N+1}\right) - 4\alpha^2 \cos \frac{\pi \omega \Gamma}{M+1} \cos \frac{\pi \omega \Gamma}{N+1}\right)^{-1}.$$ 

In the weak coupling limit $k$ and $\alpha$ approach zero, and each element operates independently at the frequency $\omega = (LC)^{-1/2}$ as expected. In the strong coupling limit $\alpha$ and $k$ are non-negligible and the individual circuit elements combine to produce normal modes of oscillation. The circuit as a whole exhibits a high-pass behavior in that the lowest odd numbers corresponding to the most homogeneous eigenfunctions appear at the highest frequencies. Several normal modes for two representative arrays are shown in figure 2. In figure 2(b) the complete set of 25 modes for a 5 * 5 lattice is shown. From (2) it is evident that the set of modes for an element array forms an orthogonal set of $N^2$ basis functions. In figure 2(c) the lowest 25 modes of a 30*30 array are shown to better illustrate the eigenfunctions. As the matrix size increases, the ratio of the imaging mode to the single element resonant frequency increase, but eventually plateaus asymptotically, although the current distribution remains the same.

III. Experiments:
To test the above hypothesis, a 5*5 mesh inductively coupled planar and head resonators were constructed for magnetic resonance imaging.

a) Planar Array: Each element had dimensions of 8*8 cm with $C = 40$ pF. The 4 capacitors were placed in the four corners of the elements. The 25 elements were then attached and were tuned to a resonant frequency of 117 MHz. The experimental eigenvalues are compared with theoretical values. For MRI experiments, the coil was driven inductively by a drive loop of the same size as the coil elements, which included a matching capacitor and was placed over a central element located on an edge of the array. Image acquisition was performed on a General Electric Signa LX 3 T magnetic resonance scanner.

Head Array: A second array was constructed on a hemispherical substrate 22 cm in diameter and 13 cm deep. However, in contrast to the planar array, each mesh had only one capacitor in the center of one leg. For MRI, the coil was driven by direct capacitive coupling using three capacitors in the matching circuit (b) and images were obtained as shown in figure 2(d).

Conclusion: Other possible configurations under current research are cylindrical 2-D high pass ladder network resonators for small animal and whole-body imaging. Another goal of this work was to gain and understanding of the underlying physics of these structures for applications to high-field MRI.