HW# 1

2.3)

$$y(n) = \sum_{k=-\infty}^{\infty} a^k u(k)\, b^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} a^k b^{n-k} = b^n \sum_{k=0}^{n} \left(a/b\right)^k$$

$$= b^n \left(\frac{1-(a/b)^{n+1}}{1-(a/b)} \right) u(n) = \frac{b^{n+1} - a^{n+1}}{b-a}\, u(n)$$

$a \neq b$

For $a = b$, $y(n) = \sum_{k=0}^{n} a^n = (n+1)\, a^n u(n)$.

2.4)

$$h(n_T) = a^{n_T} \approx e^{-1} \Rightarrow n_T \ln a = -1$$

$$\therefore\ n_T \approx -1/\ln a \approx \frac{1}{1-a} \quad \text{for } \tfrac{1}{2} < a < 1$$

example: $a = 0.9 \Rightarrow n_T = 10$ samples

2.7) (a)

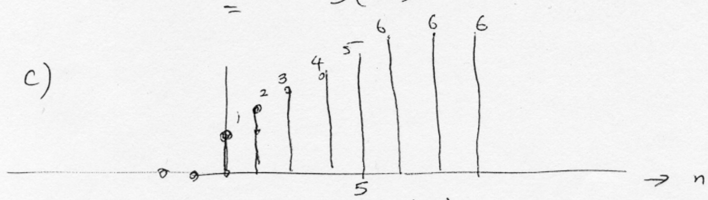
$$s(n) = \sum_{k=0}^{\infty} h(k)\, u(n-k) = \sum_{k=0}^{n} h(k)$$

b)

$$h(n) = \sum_{k=0}^{n} h(k) - \sum_{k=0}^{n-1} h(k)$$

$$= s(n) - s(n-1)$$

c)



d)

$$s(n) = \frac{1-\left(\frac{-1}{2}\right)^{n+1}}{1-(-1/2)} = \frac{1}{3}\left(2 + \left(-\tfrac{1}{2}\right)^n\right)$$

$$\therefore\ s(n) \to \frac{2}{3} \quad \text{as} \quad n \to \infty$$

2.14)

$A \cos(w_0 n + \phi) = A \cos(w_0 (n + n_d) + \phi) \Rightarrow n_d$ is period

a) $n_d = 4 \Rightarrow w_0 \cdot 4 = 2\pi \Rightarrow w_0 = \pi/2$, $\phi = -3\pi/2$, $A = 1$

e) $A = 2$, $w_0 = \pi/3$, $\phi = -\pi/3$.