Chapter 6

Designing a Multistage FIR Filter

6.1 Introduction

It has been shown in the previous chapter that a significant effort must be made to meet the specifications with a single-stage decimation filter. In order to minimize the silicon utilization, it is useful to implement cascaded filter structures. This approach becomes the more interesting the higher the filter specifications are. In this chapter, lowpass decimation filters are realized in a cascade of two or more filters. Unfortunately, cascading the decimation filter leads to multi-bit signals. Thus, a greater complexity in the intermediate stages is expected. The sampling rate of the input signal x(n) must be reduced from 5MHz to 156.25kHz. The stopband attenuation is 100dB. The front-end is always a comb filter, which performs the "bulk decimation". The back-end performs the final decimation and compensates the passband droop of the comb filter. Due to the need of linear phase, as mentioned before, only FIR filters are used for the purpose of compensation. Furthermore, some efforts must be made, to eliminate initial values in the comb structure, which will occure at low frequencies or DC input signals.

6.2 Proposed Structure for this Design

6.2.1 Specifications

The specifications for the decimation filter are again: (ref. chapter 5)

Stopband Attenuation:	$A_s = 100$ dB	
Passband Ripple:	$\delta_p < 0.1$ dB	
Passband Frequency:	$f_p = 78125.0$Hz,	$f'_p = 0.5$ [1]
Stopband Frequency:	$f_s = 85937.5$Hz,	$f'_s = 0.55$ [1]
Decimation:	D = 32	

At first some general multitage structures are presented. Possible cascaded filter structures for this design would consist of two or three stages. The signal is downsampled after each stage. Each single stage can be split into two filters with different objectives. We can determine the filter order as a function of the specifications by using (1.16).

6.2.2 Two-Stage Decimation

Referring to (1.15), Δf strongly depends on the sampling frequency at the input of the filter stage. In the two-stage filter cascade, we have an intermediate sampling frequency of 312.5kHz. Figure 6.1 illustrates the process of decimation for a two-stage cascade. Table 6.1 shows the expected filter lengths for a two-stage cascade. All possible cases are listed, assuming that the desired decimation ratio is a power-of-two. Obviously, the smallest overall filter length can be achieved by performing the decimation in two steps of 8 and 4.

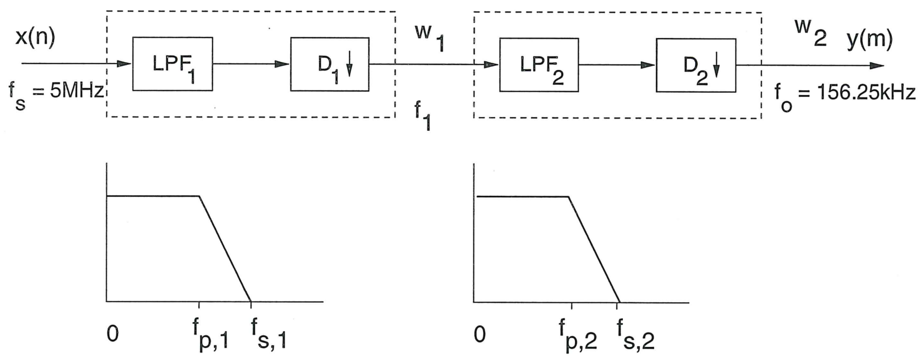

Figure 6.1: Two-Stage Decimation

Stage	D_i	$f_{p,i}$ [kHz]	$f_{s,i}$ [kHz]	f_i [kHz]	Δf_i	N_i
1	16	78.125	156.25	5000.0	0.015625	383
2	2	78.125	85.9375	312.5	0.025	240
1	2	78.125	1250	5000.0	0.234375	27
2	16	78.125	85.9375	2500	0.003125	1908
1	8	78.125	312.5	5000.0	0.046875	129
2	4	78.125	85.9375	625	0.0125	478
1	4	78.125	625	5000.0	0.109375	56
2	8	78.125	85.9375	1250	0.00625	955

Table 6.1: Parameters for the Two-Stage Filter Cascade $f_{sa} = 5\text{MHz}$

6.2.3 Three-Stage Decimation

As mentioned above, Δf_i depends very much on the sampling frequency at the input of every filter stage. In the three-stage filter cascade, two intermediate sampling frequencies of 625kHz (f_1) and 312.5kHz (f_2) appear. Figure 6.2 illustrates the process of decimation for a three-stage cascade. Table 6.2 shows the expected filter order for every single stage. Due to the relatively low decimation ratio of 32, just the presented case of $8 \cdot 2 \cdot 2$ decimation is suggested. The first stage in such a

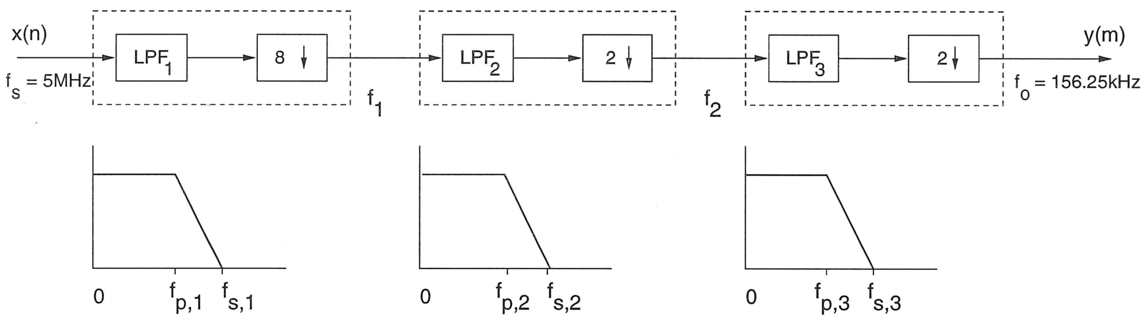

Figure 6.2: Three-Stage Decimation

Stage	$f_{p,i}$ [kHz]	$f_{s,i}$ [kHz]	f_{sa} [kHz]	Δf_i	N_i
1	78.125	625	5000.0	0.109375	56
2	78.125	250.0	625.0	0.275	23
3	78.125	85.9375	312.5	0.025	240

Table 6.2: Parameters for the Three-Stage Filter Cascade $f_{sa} = 5\text{MHz}$

filter cascade is usually a multiplier-free comb or $sinc^K$ filter. We save costly multiply and accumulate units at the expense of a nonideal frequency response, i.e., a loss in attenuation near the the passband cutoff frequency. The main purpose of the following FIR filter is to compensate for this loss in attenuation. The useful comb filter configuration is applied in the following approaches.

6.3 The Comb - FIR Filter Cascade

6.3.1 Realization of the First Stage

Multistage comb filters, also called $sinc^K$ filters, have several advantages for the design of a decimation filter with narrow transition band (ref. chapter 1). A simple implementation can be realized at the cost of an increased number of add and delay units. The attempt to compare the multistage comb filter with the straightforward one-stage FIR filter is the main objective of this chapter. Another goal is to investigate and compare two different comb filter ralizations. This is the conventional comb filter and a modified structure for sharpening the frequency response. As previously pointed out, the decimation ratio of the first stage should be chosen with respect to the overall decimation ratio.

In the process of designing a decimation filter, care has to be taken to prevent aliasing. The aliased bands $\frac{k \cdot f_{sa}}{D} - f_p < f_n < \frac{k \cdot f_{sa}}{D} + f_p$ must be sufficiently attenuated. For this design, a stopband attenuation of $A_s = 100$dB is demanded. In order to prevent alias noise from the $\Sigma\Delta$-modulator, Candy [1] pointed out that the order of the comb filter should be at least equal the order of the modulator plus 1. The considered $\Sigma\Delta$-modulator is the fifth-order IFLF modulator as depicted in Figure 3.7.

Ten comb filters are required to achieve sufficient alias rejection at a decimation

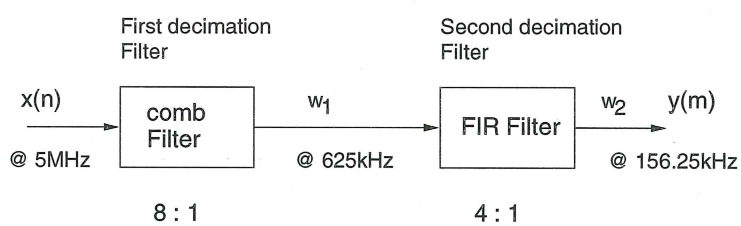

Figure 6.3: Block Diagram of the Comb - FIR Cascade

ratio of 16 and a passband frequency of 78.125kHz. Due to this decimation ratio, the first notch appears at a relatively low frequency, which makes this filter order necessary. The intervall edges for the frequencies which will be fold back are $f_{fb} = \frac{k \cdot \pi}{8} \pm f_p$, where the first lobe (k=1) is the most critical one. In order to reduce the hardware requirements, a maximum decimation ratio of 8 has been chosen for all following approaches.

The designed comb filter has to achieve enough alias rejection in the aliased bands. Having a passband frequency of $f_p = 78.125$kHz, the worst case aliasing will occure at

$$\Omega_{\mathrm{fb}} = \frac{2 \cdot \pi}{\mathrm{D}} - \frac{\mathrm{f_p}}{\mathrm{f_s}/2} \cdot \pi \qquad (6.1)$$

Table 6.3 lists the minimum alias rejection for the specified passband frequency. In the straightforward comb filter structure, the left lobe of the first notch determine the maximum achievable alias rejection. The results listed in Table 6.3 are obtained by applying the equation for the length-D comb cascade (1.36)

$$\mathrm{H(z)} = \left[\frac{1}{\mathrm{D}} \cdot \frac{1 - \mathrm{z}^{-\mathrm{D}}}{1 - \mathrm{z}^{-1}}\right]^{\mathrm{K}} \qquad (6.2)$$

where K is the number of comb filters.

Another option is to use non-D length comb filter in the stages of the comb filter cascade. We obtain additional zeros between $[0;\ 1/D]$ by replacing one length-D comb with one length-(D+k) comb filter. The fundamentals are shown in chapter 1. The conventional comb filter is constrasted to the length-(D+1) and length-(D+2) below. The transfer function for the length-(D+k) comb filter is

$$\mathrm{H(z)} = \mathrm{a_s} \cdot \frac{(1 - \mathrm{z}^{-\mathrm{D}})^{\mathrm{K}-1} \cdot (1 - \mathrm{z}^{-(\mathrm{D}+\mathrm{k})})}{(1 - \mathrm{z}^{-1})^{\mathrm{K}}}. \qquad (6.3)$$

where a_s is the scaling factor.

It depends predominatly on the design specifications to find out which comb cascade is most suitable.

Possible filter cascades are:

- A cascade of five length-8 and one length-9 comb filters for a front-end decimation of $D_1 = 8$ (Figure 6.7), or
- A cascade of seven length-16 and one length-22 comb filters for a front-end decimation of $D_1 = 16$ (Figure 6.12).

Table 6.4 lists the alias rejection we are able to reach with the modified comb cascade. The results are better than those achieved by the conventional structure (consider Table 6.3 in contrast to Table 6.4). In the following sections, the

K	A_{fb} [dB] D = 4	A_{fb} [dB] D = 8	A_{fb} [dB] D = 16	A_{pd} [dB] D = 4	A_{pd} [dB] D = 8	A_{pd} [dB] D = 16
1	23	17	10	0.05	0.22	0.91
2	46	34	21	0.10	0.44	1.82
3	68	51	31	0.16	0.66	2.73
4	91	68	42	0.21	0.88	3.63
5	**114**	85	52	**0.26**	1.10	4.54
6	137	**102**	63	0.31	**1.33**	5.45
7	159	119	73	0.37	1.55	6.36
8	182	136	83	0.42	1.77	7.27
9	205	153	94	0.47	1.99	8.18
10	228	170	**104**	0.52	2.21	**9.09**
Ω	0.46875π	0.21875π	0.09375π	0.03125π	0.03125π	0.03125π

Table 6.3: Maximum Alias Attenuation and Passband Droop of the conventional Comb Filter ($f_p = 78.125$kHz, $f_{sa} = 5$MHz)

results of the simulations of several filter structures are presented with respect to the specifications for this design. Basically, the frequency responses of the conventional comb filter 5*th*, 6*th*, 7*th* and 8*th* order are plotted. Furthermore, every conventional comb filter is compared to its modified structure according to (6.3).

K	A_{fb} [dB] length-4/-3 D = 4	A_{fb} [dB] length-8/-9 D = 8	A_{fb} [dB] length-16/-22 D = 16	A_{pd} [dB] len-4/-3 D = 4	A_{pd} [dB] len-8/-9 D = 8	A_{pd} [dB] len-16/-22 D = 16
2	30.6	33.5	31.3	0.078	0.490	2.60
3	53.2	52.6	46.1	0.130	0.705	3.49
4	75.9	71.8	58.4	0.181	0.921	4.37
5	98.5	90.9	70.0	0.232	1.137	5.26
6	**121.1**	**110.0**	81.3	**0.282**	**1.352**	6.14
7	143.7	129.2	92.2	0.334	1.568	7.03
8	166.4	148.4	**103.0**	0.385	1.784	**7.92**

Table 6.4: Maximum Alias Attenuation and Passband Droop of the modified Comb Filter ($f_p = 78.125\text{kHz}$, $f_{sa} = 5\text{MHz}$)

Comb Filter 5*th* order

Figure 6.4 depicts the conventional comb filter without any intermediate notches. The comb cascade in Figure 6.5 consists of four length-8 and one length-10 comb filters. The single order zero is shifted by two to lower frequencies. The normalized frequency 1 (π) refers to $\frac{f_{sa}}{2}$. Obviously, the specifications are not met by this comb filter. The transfer function for the conventional comb filter is

$$\mathrm{H(z)} = \left[\frac{1}{8} \cdot \frac{1 - \mathrm{z}^{-8}}{1 - \mathrm{z}^{-1}}\right]^5 \tag{6.4}$$

while Figure 6.5 is obtained by applying

$$\mathrm{H(z)} = \mathrm{a_s} \cdot \frac{(1 - \mathrm{z}^{-8})^4 \cdot (1 - \mathrm{z}^{-10})}{(1 - \mathrm{z}^{-1})^5} \tag{6.5}$$

where a_s is the scaling factor.

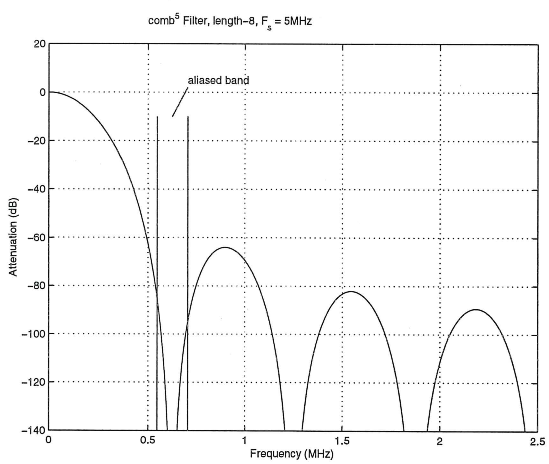

Figure 6.4: 5*th* order Comb Filter with Decimation D = 8

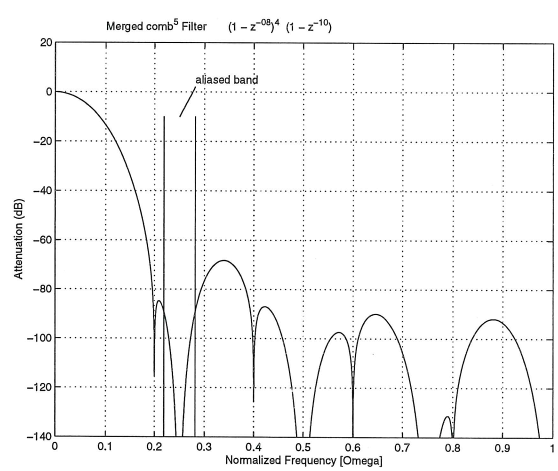

Figure 6.5: Cascade of four length-8 and one length-10 comb Filter

Comb Filter $6th$ order

Figure 6.6 shows the conventional $6th$ order comb filter without any intermediate notches. The comb cascade in Figure 6.7 consists of five length-8 and one length-9 comb filters. The single order zero is shifted by one to lower frequencies. The normalized frequency 1 (π) refers to $\frac{f_{sa}}{2}$. The transfer function for the conventional comb filter is

$$\mathrm{H(z)} = \left[\frac{1}{8} \cdot \frac{1 - \mathrm{z}^{-8}}{1 - \mathrm{z}^{-1}}\right]^6 \tag{6.6}$$

while Figure 6.7 is obtained by applying

$$\mathrm{H(z)} = \mathrm{a_s} \cdot \frac{(1 - \mathrm{z}^{-8})^5 \cdot (1 - \mathrm{z}^{-9})}{(1 - \mathrm{z}^{-1})^6} \tag{6.7}$$

where a_s is the scaling factor.

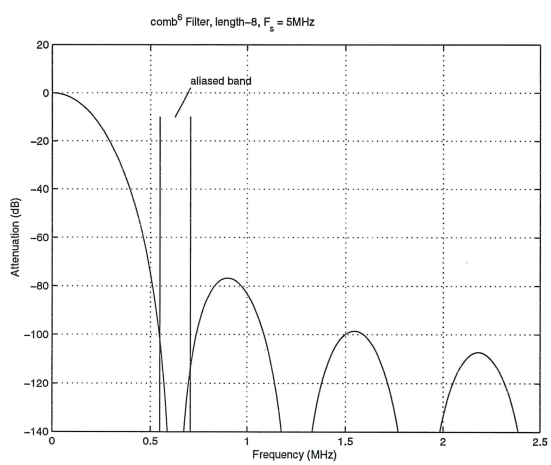

Figure 6.6: $6th$ order Comb Filter with Decimation D = 8

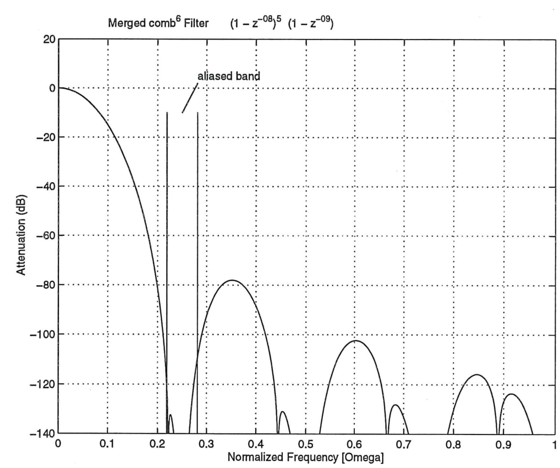

Figure 6.7: Cascade of five length-8 and one length-9 comb Filter

Figure 6.8 presents the length-8 and length-10 comb cascade. The transfer function is

$$H(z) = a_s \cdot \frac{(1 - z^{-8})^5 \cdot (1 - z^{-10})}{(1 - z^{-1})^6} \tag{6.8}$$

where a_s is the scaling factor.

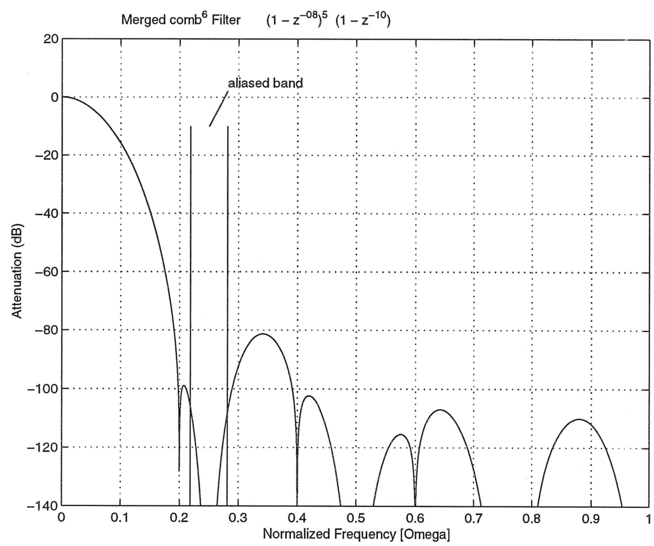

Figure 6.8: Cascade of five length-8 and one length-10 comb Filter

Comb Filter 7*th* order

Due to the sufficient attenuation for these filters, a decimation rate of 16 is chosen. Figure 6.9 shows the conventional comb filter without any intermediate notches. The comb cascade in Figure 6.10 is a structure consisting of six length-16 and one length-22 comb filters. The single order zero is shifted by six to lower frequencies. The normalized frequency 1 (π) refers to $\frac{f_{sa}}{2}$. The transfer function for the conventional comb filter is

$$\mathrm{H(z)} = \left[\frac{1}{16} \cdot \frac{1 - \mathrm{z}^{-16}}{1 - \mathrm{z}^{-1}}\right]^7 \tag{6.9}$$

while Figure 6.10 is obtained by applying

$$\mathrm{H(z)} = \mathrm{a_s} \cdot \frac{(1 - \mathrm{z}^{-16})^6 \cdot (1 - \mathrm{z}^{-22})}{(1 - \mathrm{z}^{-1})^7} \tag{6.10}$$

where a_s is the scaling factor.

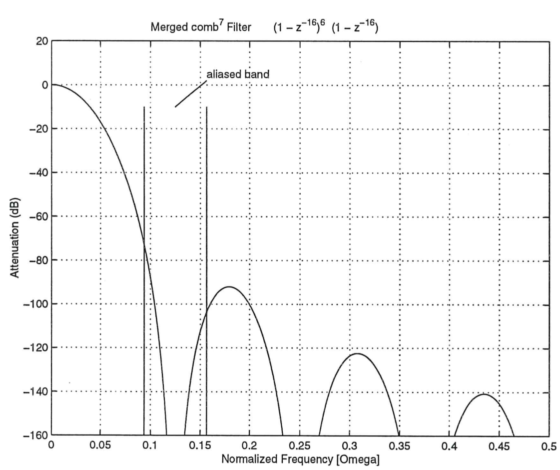

Figure 6.9: 7*th* order Comb Filter with Decimation D = 16

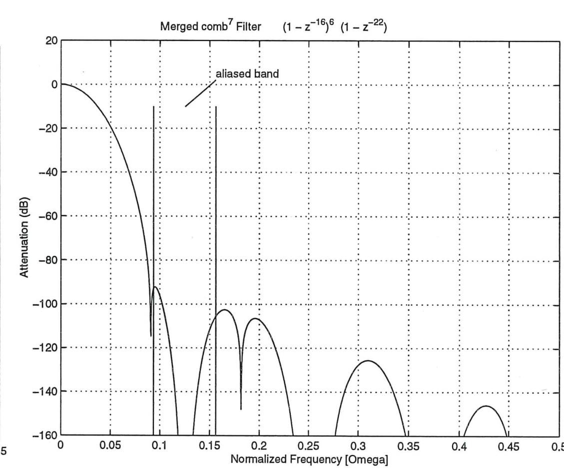

Figure 6.10: Cascade of six length-16 and one length-22 comb Filter

Comb Filter 8*th* order

Figure 6.11 depicts the conventional comb filter having no intermediate notches. The comb cascade in Figure 6.12 consists of seven length-16 and one length-22 comb filters. The single order zero is shifted by six to lower frequencies. The normalized frequency 1 (π) refers to $\frac{f_{sa}}{2}$. The transfer function for the conventional comb filter is

$$H(z) = \left[\frac{1}{16} \cdot \frac{1 - z^{-16}}{1 - z^{-1}}\right]^8 \qquad (6.11)$$

while Figure 6.12 is obtained by applying

$$H(z) = a_s \cdot \frac{(1 - z^{-16})^7 \cdot (1 - z^{-22})}{(1 - z^{-1})^8} \qquad (6.12)$$

where a_s is the scaling factor.

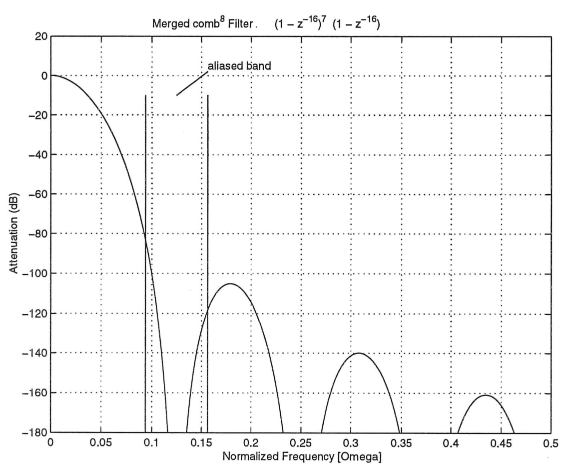

Figure 6.11: 8*th* order Comb Filter with Decimation D = 16

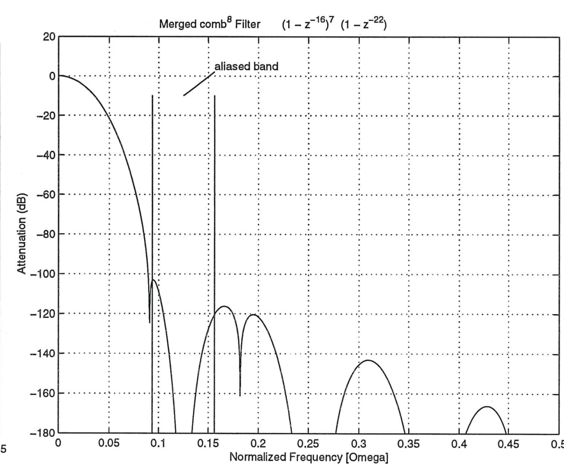

Figure 6.12: Cascade of seven length-16 and one length-22 comb Filter

6.4 The Comb - FIR Filter Cascade Design Example 1

6.4.1 Filter Properties

The first topology to be considered is the $6th$ order length-8 and $5th$ order length-4 comb filter cascade. This is the straightforward realization of the recursive filter structure based on a moving average filter. The general properties are described in chapter 1. The transfer function for this example is

$$\mathrm{H(z)} = \left[\frac{1}{8} \cdot \frac{1 - \mathrm{z}^{-8}}{1 - \mathrm{z}^{-1}}\right]^6 \cdot \left[\frac{1}{4} \cdot \frac{1 - \mathrm{z}^{-4}}{1 - \mathrm{z}^{-1}}\right]^5 \qquad (6.13)$$

Every stage of the two stage comb cascade is followed by a decimator. After the first stage, a decimation of 8 is performed, while after the second stage, the signal is downsampled by 4. Figure 6.13 illustrates the decimation process. The total stopband attenuation achieved by this structure is slightly lower than in the modified merged structure. Inserting additional zeros slightly increases the stopband attenuation as shown in Figures 6.5, 6.7, 6.8 and 6.12. Examples for the modified structure are presented in [26]. Both, the modified merged and the conventional architecture, require the same length to achieve 100dB attenuation. The modified structure does not reduce hardware complexity. Consequently, the conventional comb filter is chosen for this design example. The impulse response is recursively performed and can be computed in two operations. According to (1.32), the impulse response for the first stage is given by

$$\mathrm{y(n)} = \mathrm{y(n-1)} + \mathrm{x(n)} - \mathrm{x(n-8)}. \qquad (6.14)$$

A new output sample is obtained by adding the previous output sample to the new input sample and subtracting the input value that occured 8 samples ago. In [32], a multistage comb cascade is realized with similar properties. Figure 6.13 also reveals, that the IIR part as well as the FIR part of the comb filter run at the high input sampling frequency. Therefore, a shift register of length D_i is required for every stage. The width of the shift register increases according to the output word length w_i of the preceding stage.

Figure 6.14 presents the composed frequency response of the comb filter stages. The final compensation filter is realized as a one-stage FIR filter using conventional multiply and accumulate units. The FIR filter must satisfy three specifications at the same time. It has to compensate the loss in attenuation in the band of interest, guarantee a narrow transition band and yield sufficient stopband attenuation. This leads to a prohibitively high filter order of approximately $N_{FIR} \approx 690$. This is a coarse approximation, obtained via simulations. A proper solution is to split the compensator into two FIR filter sections. The purpose of the first stage is to compensate the passband droop while the second stage emulates a lowpass filter with a narrow transition band. This approach is presented in the following example.

Figure 6.15 shows the desired filter response versus the real response where 1 refers to f_{Ny}=156250Hz. Figure 6.16 shows the composite frequency response at the output of the comb stages and the frequency response of the compensation filter. Basically, this is just a transformation of the frequency responses depicted in Figure 6.14 and 6.15 to a common reference frequency. Finally, Figure 6.17 presents the overall frequency response at the output of the entire filter.

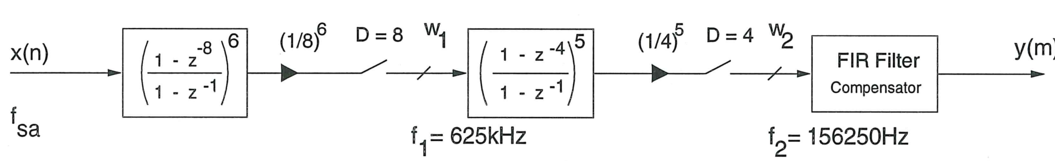

Figure 6.13: Block Diagram of the Comb Filter Cascade followed by the FIR Compensator

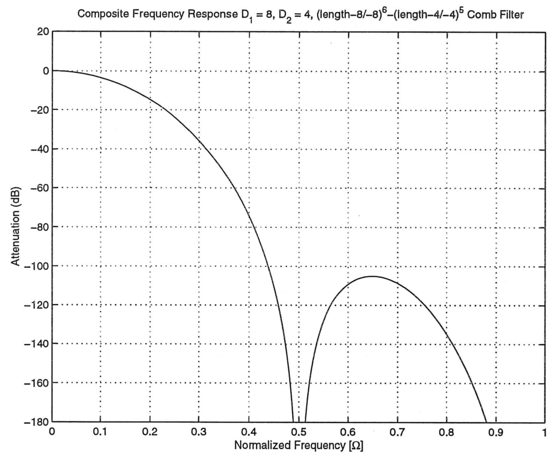

Figure 6.14: Composite Frequency Response at the output of the second Comb Filter (1 refers to $f_1/2$=312.5kHz)

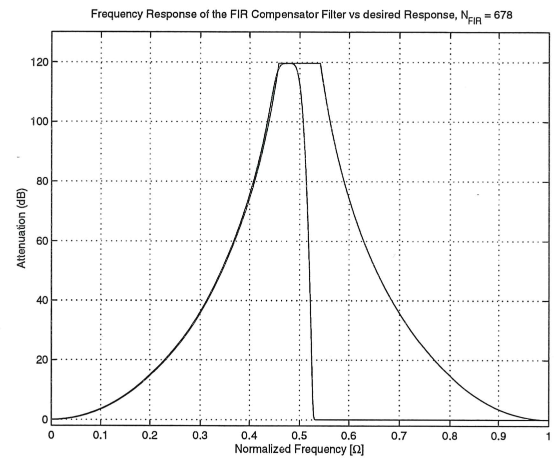

Figure 6.15: The desired FIR Compensator Frequency Response for the two stage Comb Cascade

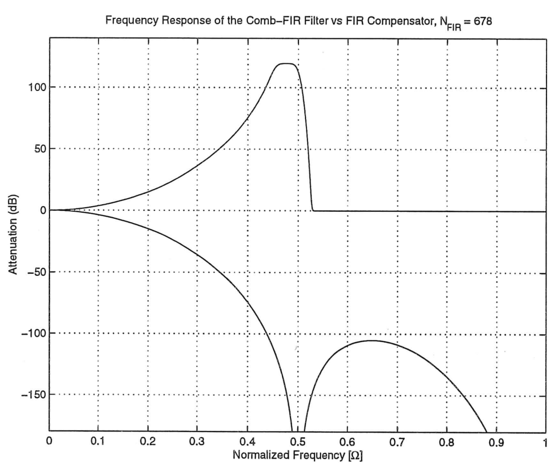

Figure 6.16: Frequency Response of the Comb Stages and Compensation Filter

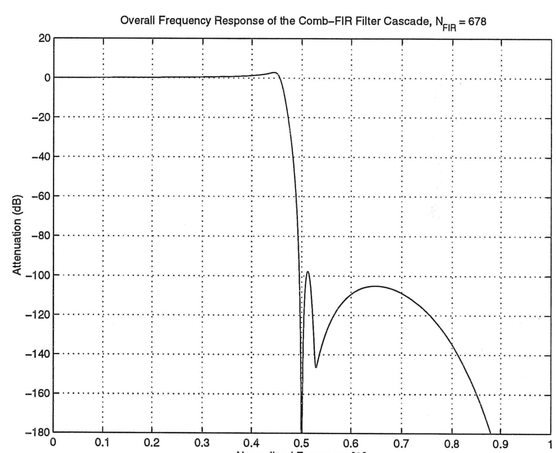

Figure 6.17: Overall Frequency Response normalized to $f_1/2$=312.5kHz

6.4.2 Hardware Requirements

In order to estimate the cost in hardware, let us consider briefly the impulse response derived from the moving average filter. The transfer function of the moving average filter for this example is $y(n) = \sum_{i=0}^{7} x(n-i)$. Applying the assumptions of chapter 1, the transfer function can be written as

$$\mathrm{Y} = \frac{1}{\mathrm{D}}\mathrm{X} + \mathrm{Y} \cdot \mathrm{z}^{-1} - \frac{1}{\mathrm{D}}\mathrm{X} \cdot \mathrm{z}^{-\mathrm{D}} \tag{6.15}$$

$$\rightarrow \mathrm{H(z)} = \frac{1}{\mathrm{D}} \cdot \frac{1-\mathrm{z}^{-\mathrm{D}}}{1-\mathrm{z}^{-1}} \tag{6.16}$$

Figure 6.19 illustrates the realization of the moving average filter and the recursive filter structure. The savings in computation using the recursive form is obvious. The depicted block diagram can be implemented using one registered add/subtract unit and one multiplexer controlled via a timing unit [32]. Figure 6.18 shows the schematic of the single comb filter as presented by Adams [32],[33]. The filter is realized using an ALU, in this case a simple add/subtract unit, which performs the process descibed by (6.15). Input B is fed back from the register output of the ALU. Input A is supplied alternatingly by the actual input signal or the 8 times delayed input signal. With this architecture, the entire filter operates at the high input sampling frequency. Therefore, as mentioned in chapter 1 (ref. Figure 1.12), no saving in power consumption is achieved. A 8 bit shift register is located before every add/subtract block. The width of the shift register increases by the number of output bits for every stage.

The word length increases for every single stage by 3 bits in the first cascade and 2 bits in the second cascade. Hence, a theoretical word length of $\mathrm{w}_1 = 18$ bit ($\mathrm{K}_1 = 6$) and $\mathrm{w}_2 = 28$ bit ($\mathrm{K}_2 = 5$) is obtained. The required width of the simple registers and the shift registers increases accordingly for every stage. Note that this word length is not the true resolution, it is rather the resolution determined by the dynamic range of the modulator (SNR+THD). In Figure 6.21, the overall comb cascade is shown. The output word length of 28 bit is due to the large number of comb filters. Using the IFLF 5 $\Sigma\Delta$-modulator, we obtain a dynamic range of 90dB, which is equivalent to 15 bit resolution. Hence, the output of the decimator is truncated to 16 bit.

The number of Full-Adders required to realize the add/subtract unit for stage i is equal to the output word length $w_{out,i-1}$ of the previous stage. The size of the shift registers is determined by the input word length to $8 \times w_{in,i}$. Figure 6.18 shows furthermore, that for every stage i, a Latch with a word length of $w_{out,i}$ is required. The requirements in logic blocks are summarized in the lower part of Table 6.5. Note, that no overflow considerations are made in this example.

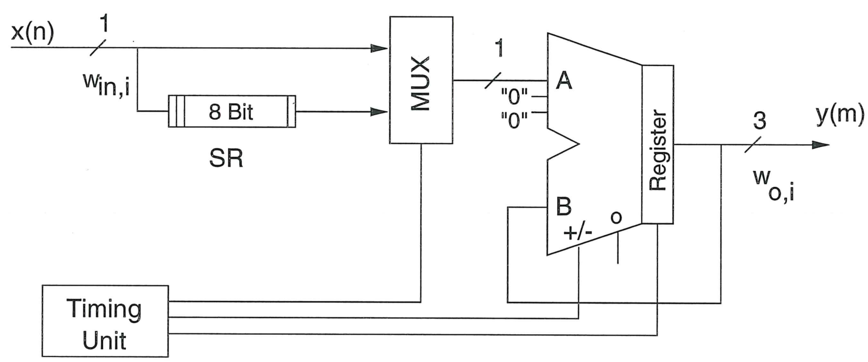

Figure 6.18: Realization of the Comb Filter

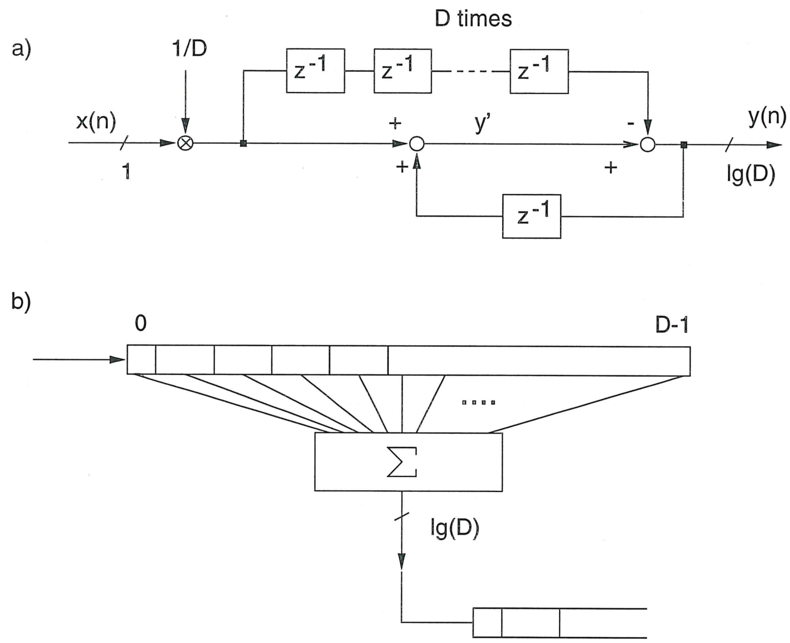

Figure 6.19: (a) Block Diagram of the recursive Comb Filter
(b) Block Diagram of the Moving Average Filter

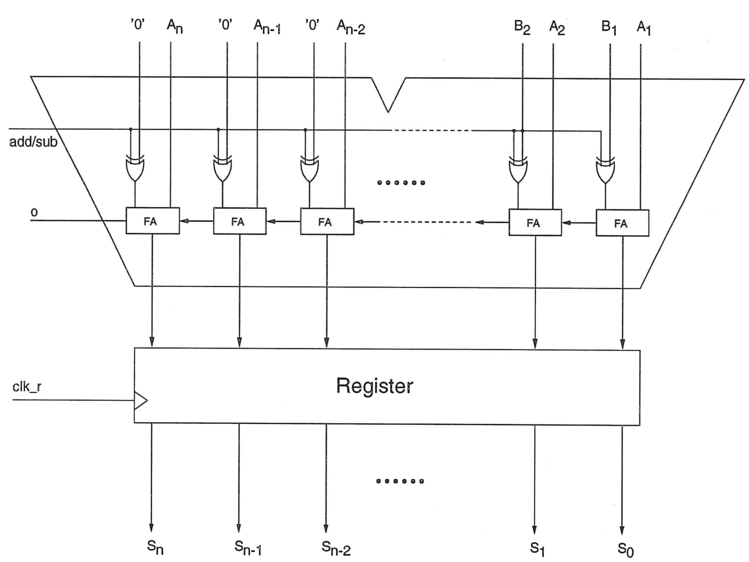

Figure 6.20: Block Diagram of the Add/Subtract Unit

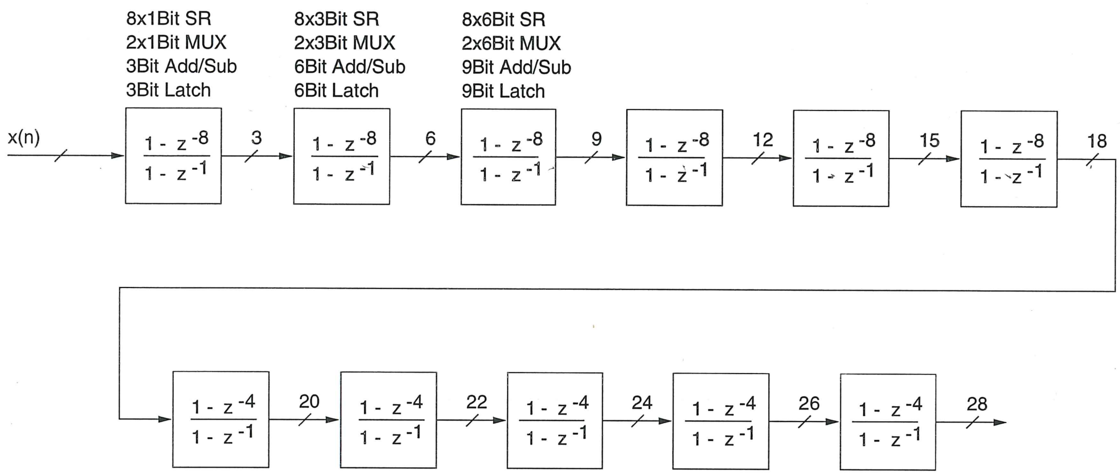

Figure 6.21: Overall Structure of the Comb Filter, Decimation is not shown

Comb Filter Stage i	$w_{in,i}$ [bit]	$w_{out,i}$ [bit]	Full Adder	Shift Register	Register	XOR
1	1	3	3	8 × 1 bit	3 bit	3
2	3	6	6	8 × 3 bit	6 bit	6
3	6	9	9	8 × 6 bit	9 bit	9
4	9	12	12	8 × 9 bit	12 bit	12
5	12	15	15	8 × 12 bit	15 bit	15
6	15	18	18	8 × 15 bit	18 bit	18
7	18	20	20	8 × 18 bit	20 bit	20
8	20	22	22	8 × 20 bit	22 bit	22
9	22	24	24	8 × 22 bit	24 bit	24
10	24	26	26	8 × 24 bit	26 bit	26
11	26	28	28	8 × 26 bit	28 bit	28
Total	—	—	183	8 × 156 bit	—	183
D Flip Flops	—	—	—	1248	183	—
AND	—	—	366	—	—	—
OR	—	—	915	—	—	—
XOR	—	—	—	—	—	183

Table 6.5: Hardware Requirements for the length-8, length-4 standard Comb Filter K_1=6, K_2=5