

## BME 360 sample exams – solutions

**Exam #1:** 1. C; 2. A; 5. C; 6. B; 7. D; 8. D; 9. A; 10. B

3. Monitoring ECG — 0.5 Hz – 40 Hz  
 Diagnostic ECG — 0.05 Hz – 150 Hz  
 EEG — 0.1 Hz – 100 Hz  
 EMG — 25 Hz – 5 KHz  
 Neuronal action potentials — DC – 10 KHz

4. Ventricular contraction — QRS wave  
 Ventricular relaxation — T wave  
 Atrial contraction — P wave  
 Atrial fibrillation — absence of P wave  
 Bundle branch block — broadened QRS

**Exam #1a:** (B) 75 bpm; (B) 1.4 mV; See below: (A) a = 0; (C) b = 15;

i	a	b	a = b % 3
0	0	0 → 5	2
1	2	5 → 13	1
2	1	13 → <u>15</u>	<u>0</u>

**Exam #2:** 2. B; 3. A; 4. D; 5. A; 6. C; 7. B; 8. B; 9. C; 10. A

1. Differential gain =  $\infty$   
 Common mode gain = 0  
 Common mode rejection ratio =  $\infty$   
 Input impedance =  $\infty$   
 Output impedance = 0  
 Bandwidth =  $\infty$

**Exam #3a:** 1. B; 2. A; 3. A; 4. C; 5. C; 6. B; 7. C; 8. D; 9. B; 10. B

2. i = 0; temp = 0 → 127  
 i = 1; temp = 127 → 254  
 i = 2; temp = 254 → 381 → 190  
 i = 3; temp = 190 → 317 → 158  
 i = 4; temp = 158 → 285 → 142

**Exam #3b:** 1. E; 2. D; 3. B; 4. A; 5. B; 6. C; 7. D; 8. C; 9. B; 10. A;  
 11. E; 12. E or B.

**Exam #3b**

1. ( **E** ) According to the paper "Microprocessor-based real-time QRS detection," which of the following statements is incorrect? (A) A real-time algorithm must be causal. (B) The detection algorithm can be made more robust when more samples from the QRS complex are used, but at the sacrifice of the response time. (C) Some instruments may require triggering on the premature ventricular contractions, while others don't. (D) The receiver operating characteristics (ROC) is concerned with the tradeoff between false positives and false negatives. (E) A narrow dynamic range means that the detection algorithm is more accurate and robust.

Dynamic range is the maximum over the minimum signal magnitude that the QRS can still be detected. A **wide** dynamic range means that the detection algorithm is more accurate and robust.

2. ( **D** ) According to the paper "Microprocessor-based real-time QRS detection," a refractory period starts immediately after the detection of the onset of the QRS complex in order to prevent multiple detections on the same QRS. The detection threshold is set to the maximum of the MOBD signal immediately after the refractory period. Then, the threshold is halved when a period (called the decay time constant) elapses. Based on the results shown in this paper, the refractory period and the decay time constant, respectively, were set to: (A) 50 ms and 50 ms, (B) 100 ms and 50 ms, (C) 50 ms and 100 ms, (D) 100 ms and 100 ms, (E) none of the above.

The refractory period is 100 ms (see page 479, right column, line 2 from the bottom). The value for the decay time constant was not specifically mentioned in the text. But from Fig. 3, it can be seen that the decay time constant is the same as the refractory period.

3. ( **B** ) After the execution of the C code on the right, what is the value of  $x$ ? (A) 0, (B) 1, (C) 2, (D) 3, (E) none of the above. (Hint: % is the mod operator, which takes the remainder of a division.)

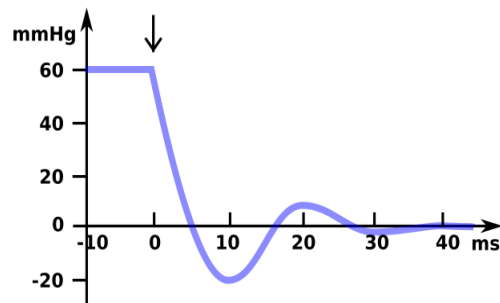
i	x	$x = x \% 3$
4	$2 \Rightarrow 2 + 7 = 9$	$9 \% 3 = 0$
3	$0 \Rightarrow 0 + 4 = 4$	$4 \% 3 = 1$
2	$1 \Rightarrow 1 + 1 = 2$	$2 \% 3 = 2$
1	$2 \Rightarrow 2 + 7 = 9$	$9 \% 3 = 0$
0	$0 \Rightarrow 0 + 4 = 4$	$4 \% 3 = 1$

```
int i, x = 2;
for (i=4; i>= 0; i--) {
    switch (x) {
        case 0:
            x += 4; break;
        case 1:
            x += 1; break;
        case 2:
            x += 7; break;
    }
    x = x % 3;
}
```

4. ( **A** ) Which of the following statements regarding *pressure* is incorrect? (A) The cgs unit for pressure is dyne, which is  $g \cdot cm/s^2$ . (B) The conversion between mmHg and dyne is  $1 \text{ mmHg} = 1333 \text{ dyne/cm}^2$ . (C) The conversion between pound per square inch and mmHg is  $1 \text{ psi} = 51.7 \text{ mmHg}$ . (D) The average human arterial pressure is about 100 mmHg and venous pressure about 5 mmHg. (E) none of the above.

The cgs unit for pressure is  $dyne/cm^2$ , which is  $g/(cm \cdot s^2)$ .

5. ( **B** ) A "pop test" is conducted to evaluate the frequency response of a pressure catheter and transducer system. The waveform on the right shows the step response of the system as the "pop" is introduced at time 0 (indicated by arrow). What is the damping factor ( $\alpha$ )? (A) 100 radians/s, (B) 110 radians/s, (C) 120 radians/s, (D) 130 radians/s, (E) none of the above.



$$\alpha = -\ln OS / T_p ; OS = 20/60 = 1/3, T_p = 0.01s \Rightarrow \alpha = -\ln(1/3)/0.01 = 110$$

6. ( **C** ) For the above problem, what is the natural frequency ( $f_n$ ) in Hz? (A) 45 Hz, (B) 50 Hz, (C) 53 Hz, (D) 62 Hz, (E) none of the above.

$$\omega_d = \pi/T_p = 3.1416/0.01 = 314.14 \Rightarrow \omega_n = \sqrt{\omega_d^2 + \alpha^2} = \sqrt{314^2 + 110^2} = 333$$

$$\Rightarrow f_n = \omega_n/\pi = 333/6.28 = 53 \text{ Hz}$$

7. ( **D** ) For the above problem, what is the transfer function of the 2nd-order system?

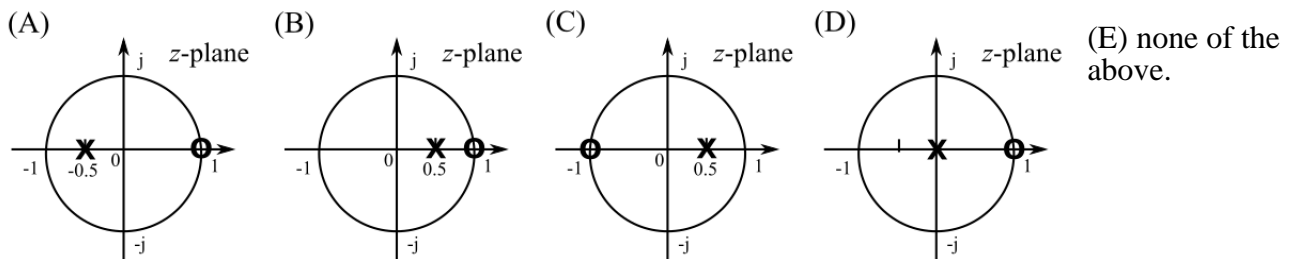
(A)  $H(s) = \frac{98696}{s^2 + 110s + 98696}$ , (B)  $H(s) = \frac{98696}{s^2 + 220s + 98696}$ ,  
 (C)  $H(s) = \frac{110765}{s^2 + 110s + 110765}$ , (D)  $H(s) = \frac{110765}{s^2 + 220s + 110765}$ ,  
 (E) none of the above.

$$H(s) = \frac{\omega_n}{s^2 + 2\alpha s + \omega_n^2} = \frac{110765}{s^2 + 220s + 110765}$$

8. ( **C** ) We use a catheter to transfuse blood. The catheter is 10 cm long and has an inner lumen radius of 1 mm. The viscosity of blood = 0.025 poise. The flow rate of the blood through the catheter is 1 cm<sup>3</sup>/s. What is the pressure gradient across the catheter? (A) 2.9 mmHg, (B) 3.2 mmHg, (C) 4.8 mmHg, (D) 5.3 mmHg, (E) none of the above. (Hint: Don't forget to convert dyne/cm<sup>2</sup> to mmHg.)

$$\text{Poiseuille's Law: } \Delta P = \frac{8\mu L}{\pi r^4} Q = \frac{(8)(0.025)(10)}{(3.14)(0.1)^4} (1) / 1333 = 4.8 \text{ mmHg}$$

9. ( **B** ) We implement a digital filter according to:  $y[n] = x[n] - x[n-1] + y[n-1] / 2$ . The pole-zero plot of this filter should be:

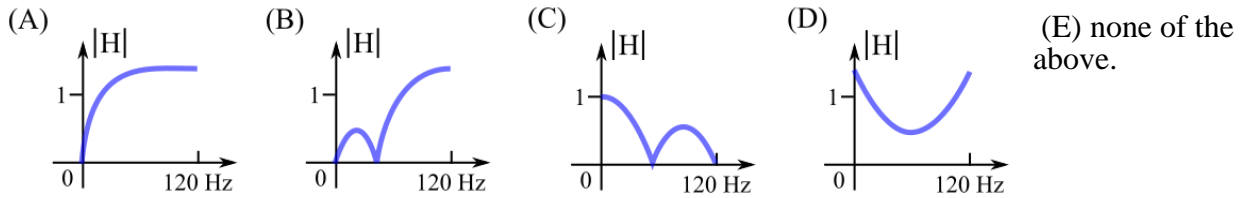


Take ZT on the filter equation.

$$Y(z) = X(z) - z^{-1}X(z) + z^{-1}Y(z)/2 \Rightarrow (1 - z^{-1}/2)Y(z) = (1 - z^{-1})X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - z^{-1}/2} = \frac{z - 1}{z - 1/2} \Rightarrow \text{zero at } 1, \text{ pole at } 1/2 \Rightarrow \text{B}$$

10. (A) For the above problem, if the sampling rate is 240 Hz, the magnitude of the Fourier transform should look like:

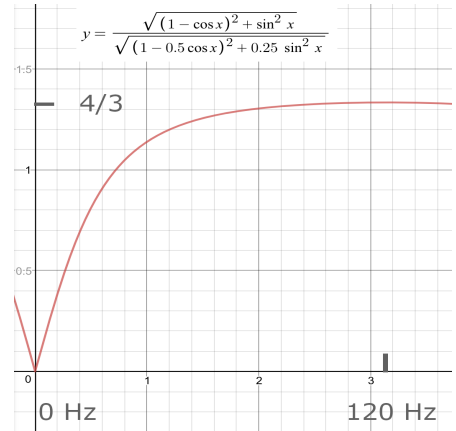


By inspection, A is the most likely answer. The magnitude of the FT is 0 Hz at DC and cannot be 0 again before 120 Hz. The magnitude at 120 Hz is 4/3 (let  $z = -1$ ). To obtain a closed-form solution. Replace  $z$  by  $e^{j\omega}$ .

$$H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{1 - e^{-j\omega}/2} = \frac{1 - \cos \omega - j \sin \omega}{1 - 0.5 \cos \omega - 0.5 j \sin \omega} \Rightarrow$$

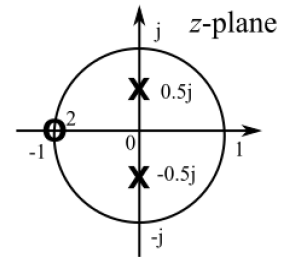
$$|H(e^{j\omega})| = \frac{\sqrt{(1 - \cos \omega)^2 + \sin^2 \omega}}{(1 - 0.5 \cos \omega)^2 + 0.25 \sin^2 \omega}$$

Plot this at <<https://www.desmos.com/calculator>>, as shown on the right.



11. (E) The pole-zero plot of another digital filter is shown on the right. There are two complex conjugate poles and a double zero at  $z = -1$ . The gain of the filter at DC is unity. What is the filter equation:

- (A)  $y[n] = (5/4) (x[n] + x[n-1] + x[n-2] + y[n-2])$ ,
- (B)  $y[n] = (5/4) (x[n] + x[n-1] + x[n-2] - y[n-1])$ ,
- (C)  $y[n] = (5/4) (x[n] + 2x[n-1] + x[n-2] - y[n-2])$ ,
- (D)  $y[n] = (5/4) (x[n] + 2x[n-1] - x[n-2] - y[n-1])$ ,
- (E) none of the above.



The transfer function is given by  $H(z) = K \frac{(z+1)^2}{(z-0.5j)(z+0.5j)} = K \frac{(z+1)^2}{z^2+1/4}$ .

For DC ( $z = 1$ ),  $H(z) = 1$ . Thus,  $1 = K \frac{4}{1+1/4} = K \frac{16}{5} \Rightarrow K = 5/16$

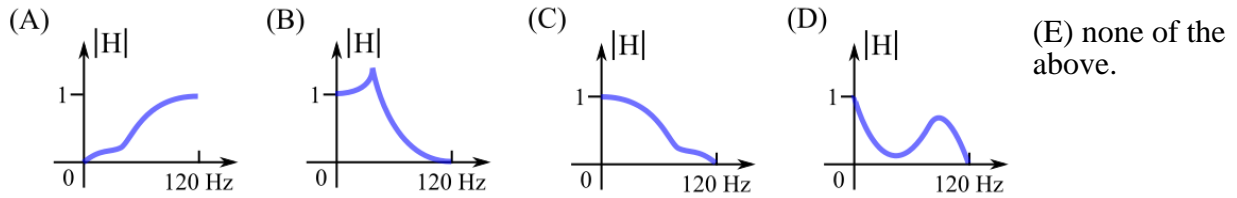
$$H(z) = \frac{Y(z)}{X(z)} = \frac{5}{16} \frac{(z+1)^2}{z^2+1/4} = \frac{5}{16} \frac{1+2z^{-1}+z^{-2}}{1+1/4z^{-2}} \Rightarrow$$

$$Y(z)(1+1/4z^{-2}) = \frac{5}{16} [X(z)+2z^{-1}X(z)+z^{-2}X(z)]$$

$$Y(z) = \frac{5}{16} [X(z)+2z^{-1}X(z)+z^{-2}X(z)] - \frac{1}{4} z^{-2} Y(z) \quad \text{The the inverse ZT:}$$

$$y[n] = \frac{5}{16} (x[n]+2x[n-1]+x[n-2]) - \frac{1}{4} y[n-2] \Rightarrow \text{(E) none of the above.}$$

12. ( E ) For the above problem, if the sampling rate is 240 Hz, the magnitude of the Fourier transform should look like:



$$H(z) = \frac{5}{16} \frac{1+2z^{-1}+z^{-2}}{1+1/4z^{-2}} \Rightarrow$$

$$H(e^{j\omega}) = \frac{5}{16} \frac{1+2e^{-j\omega}+e^{-2j\omega}}{1+1/4e^{-2j\omega}} =$$

$$\frac{5}{16} \frac{1+2\cos\omega-2j\sin\omega+\cos 2\omega-j\sin 2\omega}{1+1/4\cos 2\omega-1/4j\sin 2\omega} \Rightarrow$$

$$|H(e^{j\omega})| = \frac{\sqrt{(1+2\cos\omega+\cos 2\omega)^2+(-2\sin\omega-\sin 2\omega)^2}}{\sqrt{(1+1/4\cos 2\omega)^2+1/16\sin^2 2\omega}}$$

