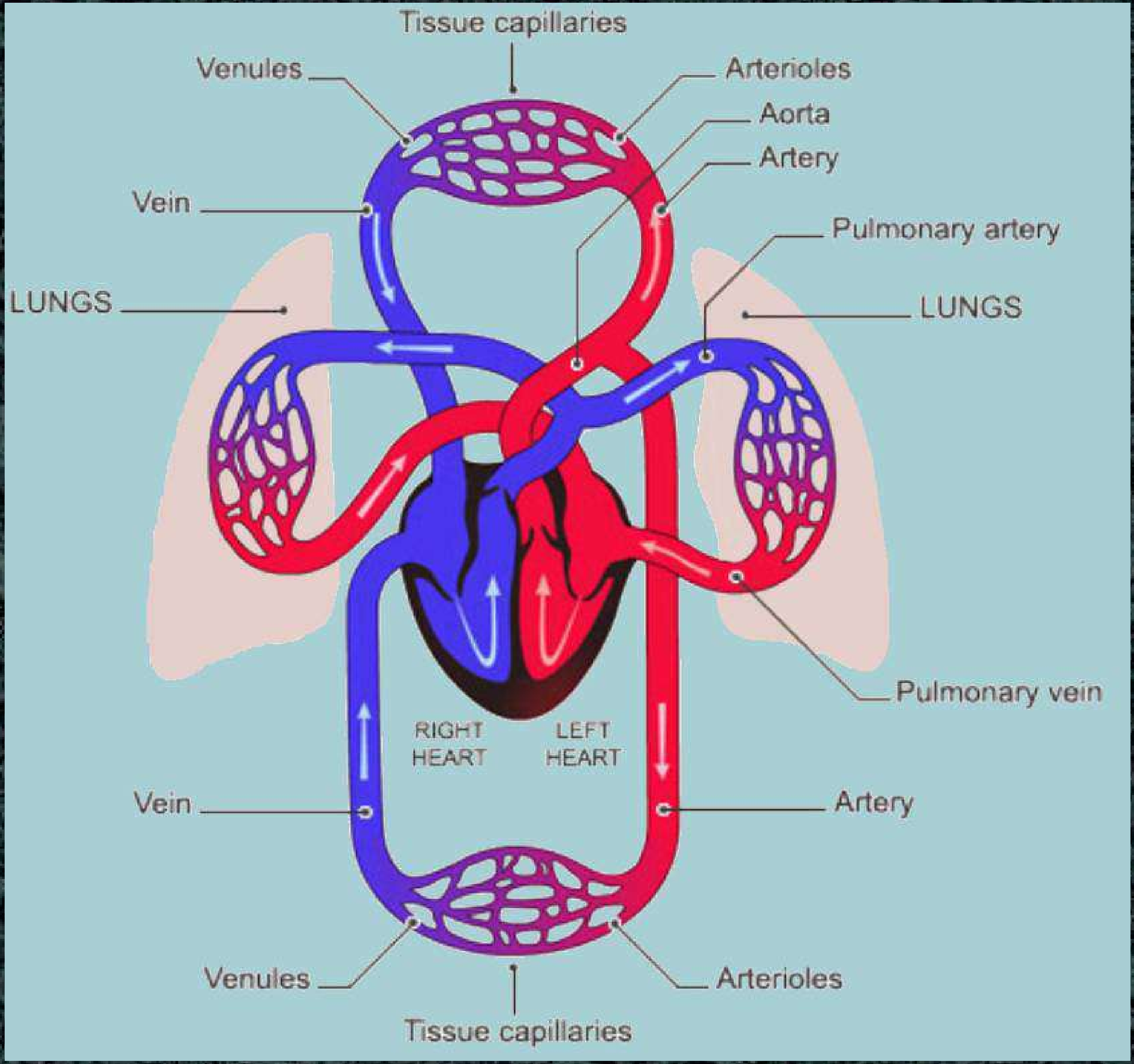
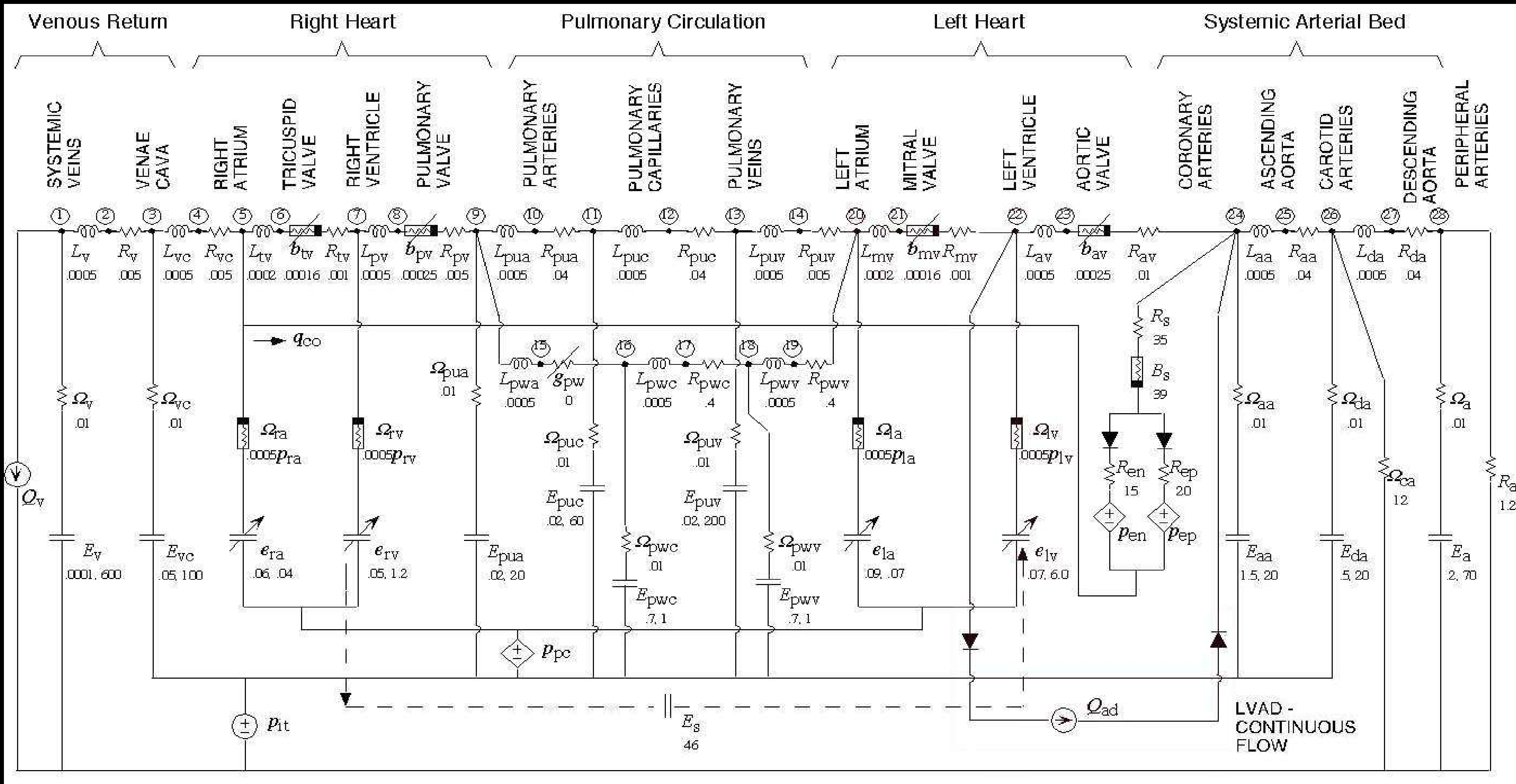
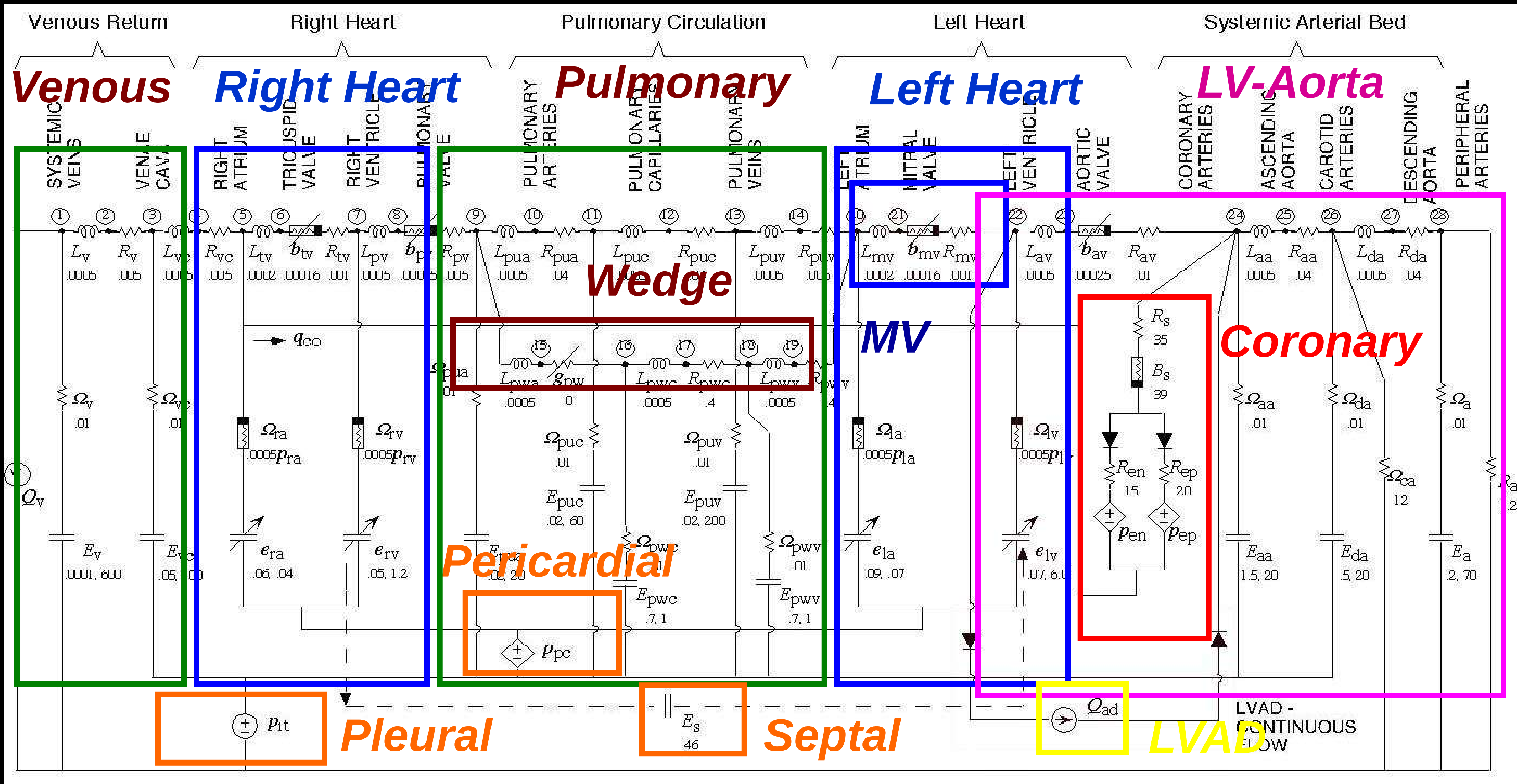
Mathematical Modeling and Simulation of the Cardiovascular System



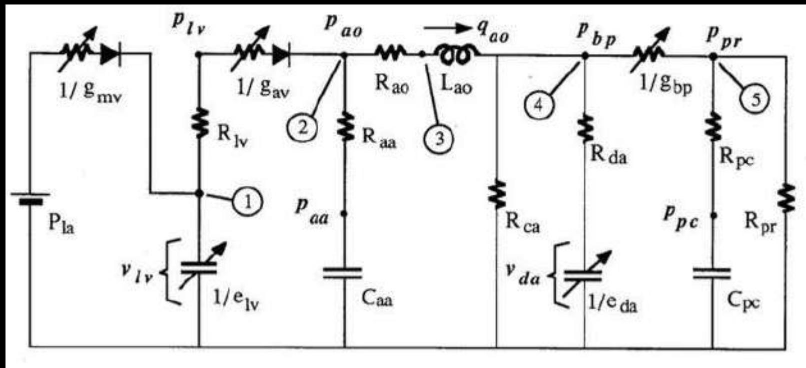
Electrical Analog Model of the Cardiovascular System

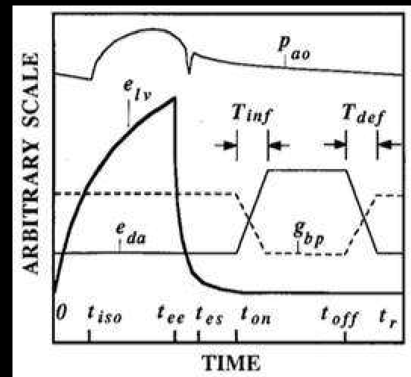


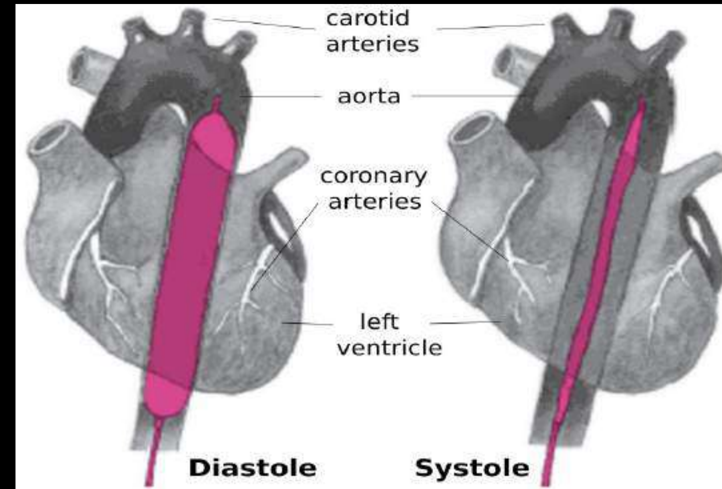
Electrical Analog Model of the Cardiovascular System



Intra-Aortic Balloon Pump (IABP)







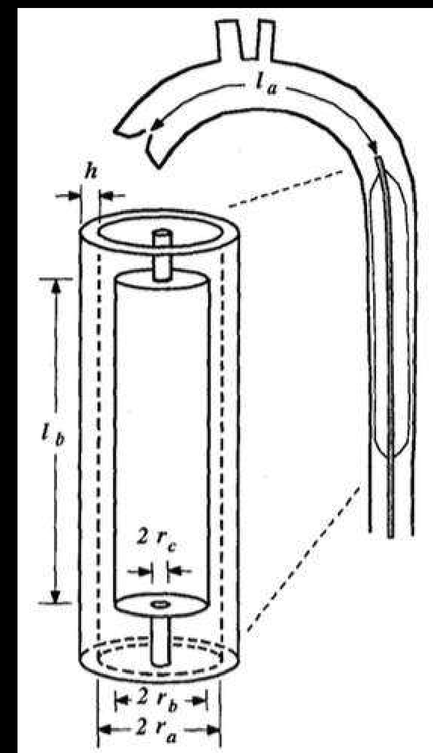
$$\frac{dv_{lv}}{dt} = -\left(g_{mv} + \frac{g_{av}}{1 + g_{av}R_{lv}}\right)e_{lv}v_{lv} + g_{mv}P_{la} + \left(p_{aa} + R_{aa}C_{aa}\frac{dp_{aa}}{dt}\right)\frac{g_{av}}{1 + g_{av}R_{lv}}$$

$$\frac{dp_{aa}}{dt} = \left(\frac{g_{av}}{1 + g_{av}R_{lv}}e_{lv}v_{lv} - \frac{g_{av}}{1 + g_{av}R_{lv}}p_{aa} - q_{ao}\right) \Big/ \left[C_{aa}\left(1 + \frac{g_{av}}{1 + g_{av}R_{lv}}R_{aa}\right)\right]$$

$$\frac{dq_{ao}}{dt} = \frac{p_{aa}}{L_{ao}} - \frac{e_{da}}{L_{ao}}v_{da} - \frac{R_{ao}}{L_{ao}}q_{ao} + \frac{R_{aa}C_{aa}}{L_{ao}}\frac{dp_{aa}}{dt} - \frac{R_{da}}{L_{ao}}\frac{dv_{da}}{dt}$$

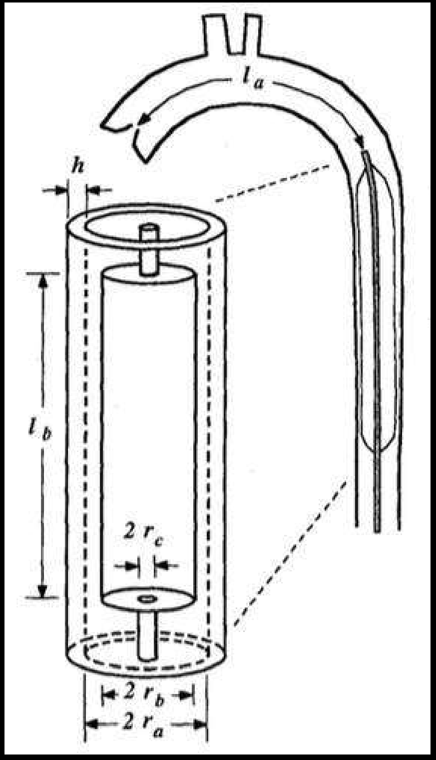
$$\frac{dv_{da}}{dt} = \left[q_{ao} - \left(g_{bp} + \frac{1}{R_{ca}}\right)e_{da}v_{da} + g_{bp}p_{pc} + g_{bp}R_{pc}C_{pc}\frac{dp_{pc}}{dt}\right] \Big/ \left[1 + R_{da}\left(g_{bp} + \frac{1}{R_{ca}}\right)\right]$$

$$\frac{dp_{pc}}{dt} = \left\{\frac{g_{bp}R_{da}q_{ao}}{1 + R_{da}(g_{bp} + 1/R_{ca})} + \left[1 - \frac{R_{da}(g_{bp} + 1/R_{ca})}{1 + R_{da}(g_{bp} + 1/R_{ca})}\right]g_{bp}e_{da}v_{da} - \left[g_{bp} + \frac{1}{R_{pr}} - \frac{g_{bp}^2 R_{da}}{1 + R_{da}(g_{bp} + 1/R_{ca})}\right]p_{pc}\right\} \Big/ \left\{C_{pc}\left[1 + g_{bp}R_{pc} + \frac{R_{pc}}{R_{pr}} - \frac{g_{bp}^2 R_{da}R_{pc}}{1 + R_{da}(g_{bp} + 1/R_{ca})}\right]\right\}$$



Electrical Equivalence for Fluid Dynamic Components

Resistance – Viscous resistance against flows
Inductance – Inertance, inertia of flows (storing kinetic energy)
Capacitance – Compliance, elastic walls (storing static energy)

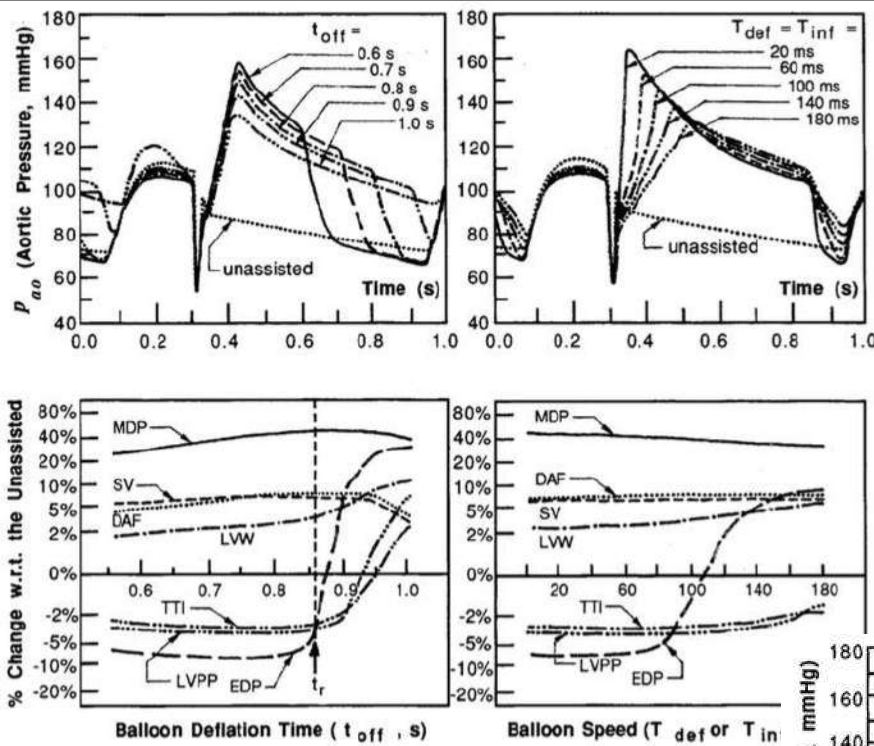


$$R_{ao} = \frac{81\mu l_a}{8\pi r_a^4} \Big/ 1{,}333 \ (\text{mmHg}\cdot\text{s}\cdot\text{ml}^{-1})$$

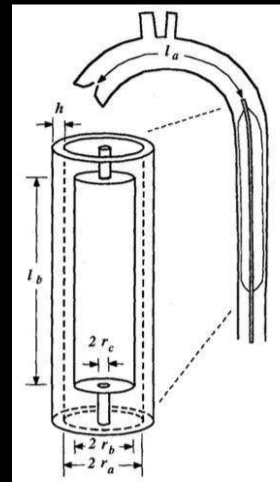
$$L_{ao} = \frac{9\rho l_a}{4\pi r_a^2} \Big/ 1{,}333 \ (\text{mmHg}\cdot\text{s}^2\cdot\text{ml}^{-1})$$

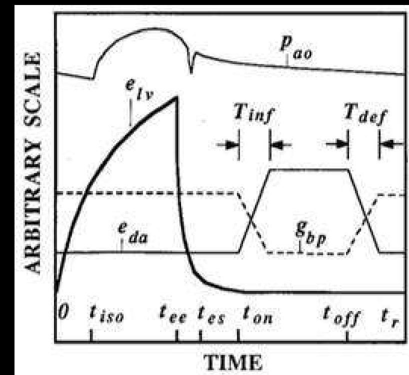
$$C_{ao} = \left(\frac{3\pi r_a^3 l_a}{2Yh}\right) 1{,}333 \ (\text{ml/mmHg})$$

Optimization of IABP Parameters

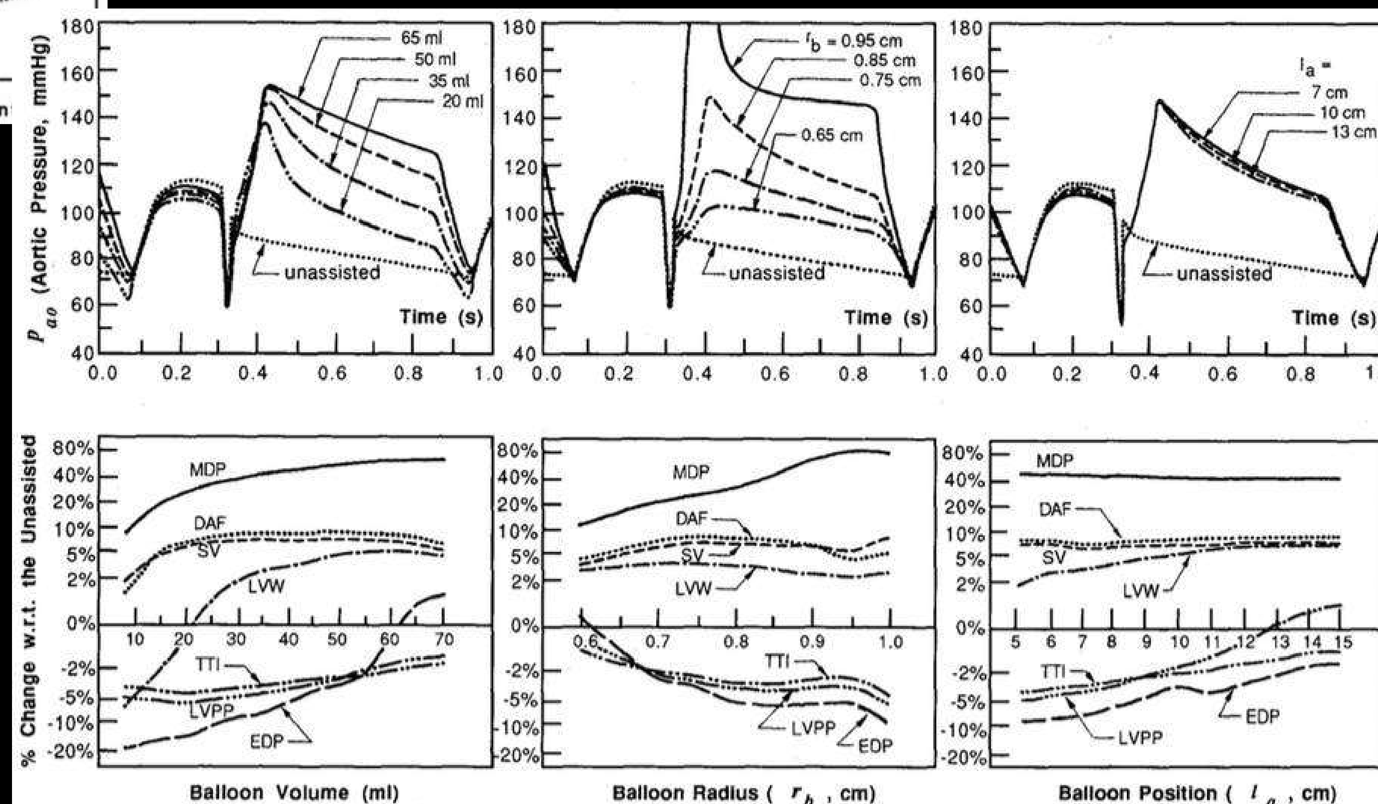


Timing





Balloon Configurations



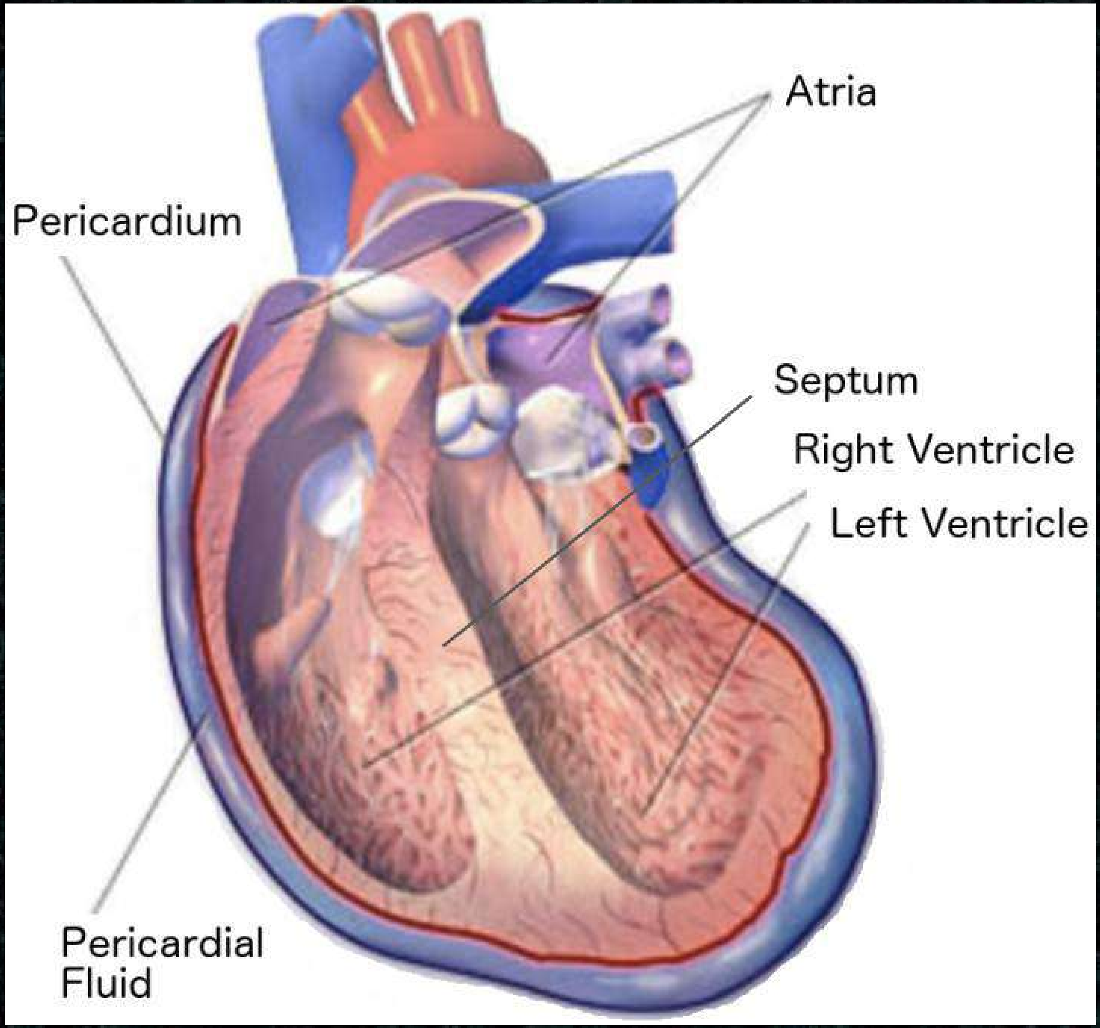
Sun Y. Modeling the dynamic interaction between left ventricle and intraaortic balloon pump. American Journal of Physiology 261(4) (Heart Circulatory Physiology 30): H1300-H1311, 1991.

Coupling Between Right and Left Ventricles

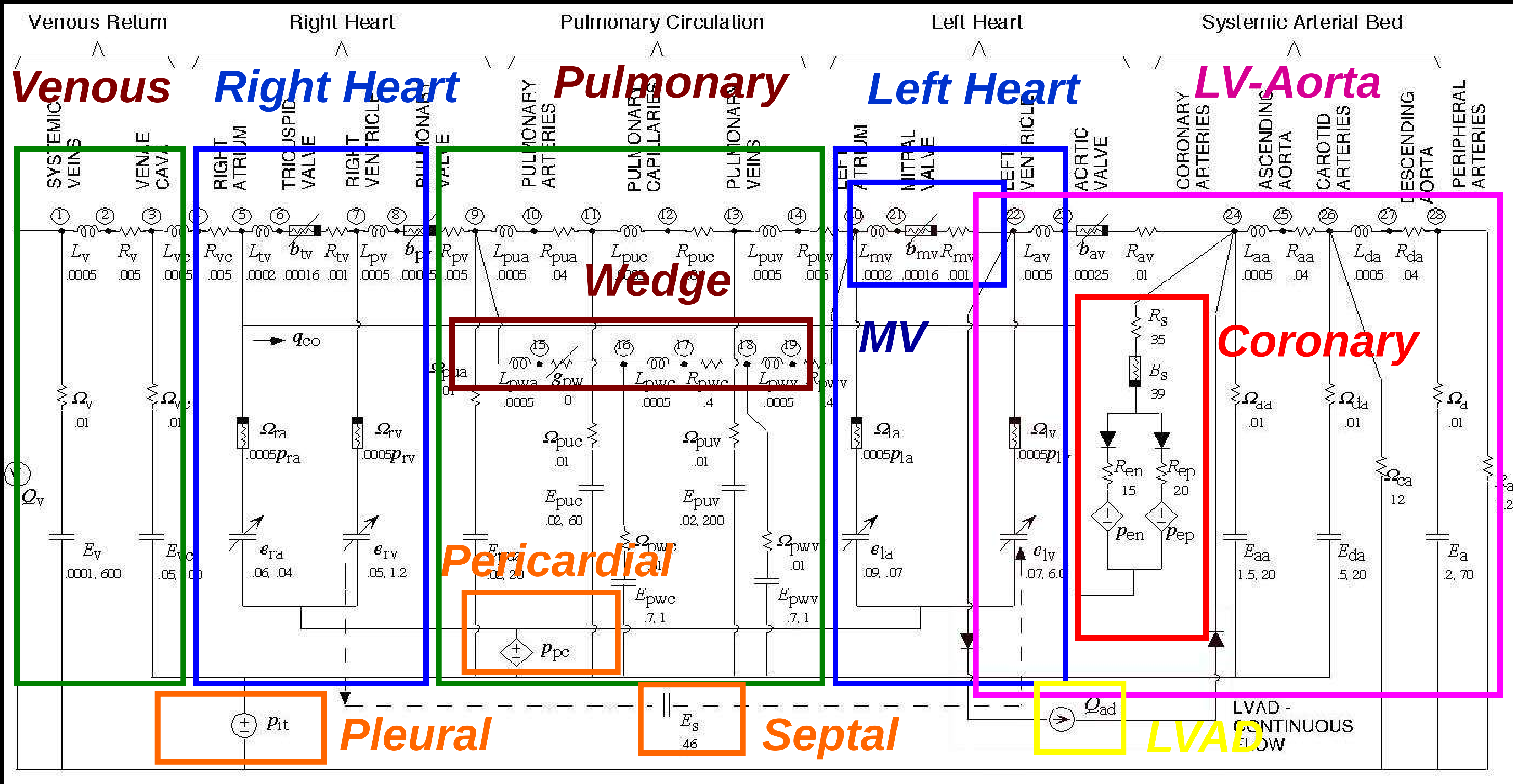
- ***Hemodynamic***
- ***Transseptal***
- ***Pericardial***

Which one is more important?

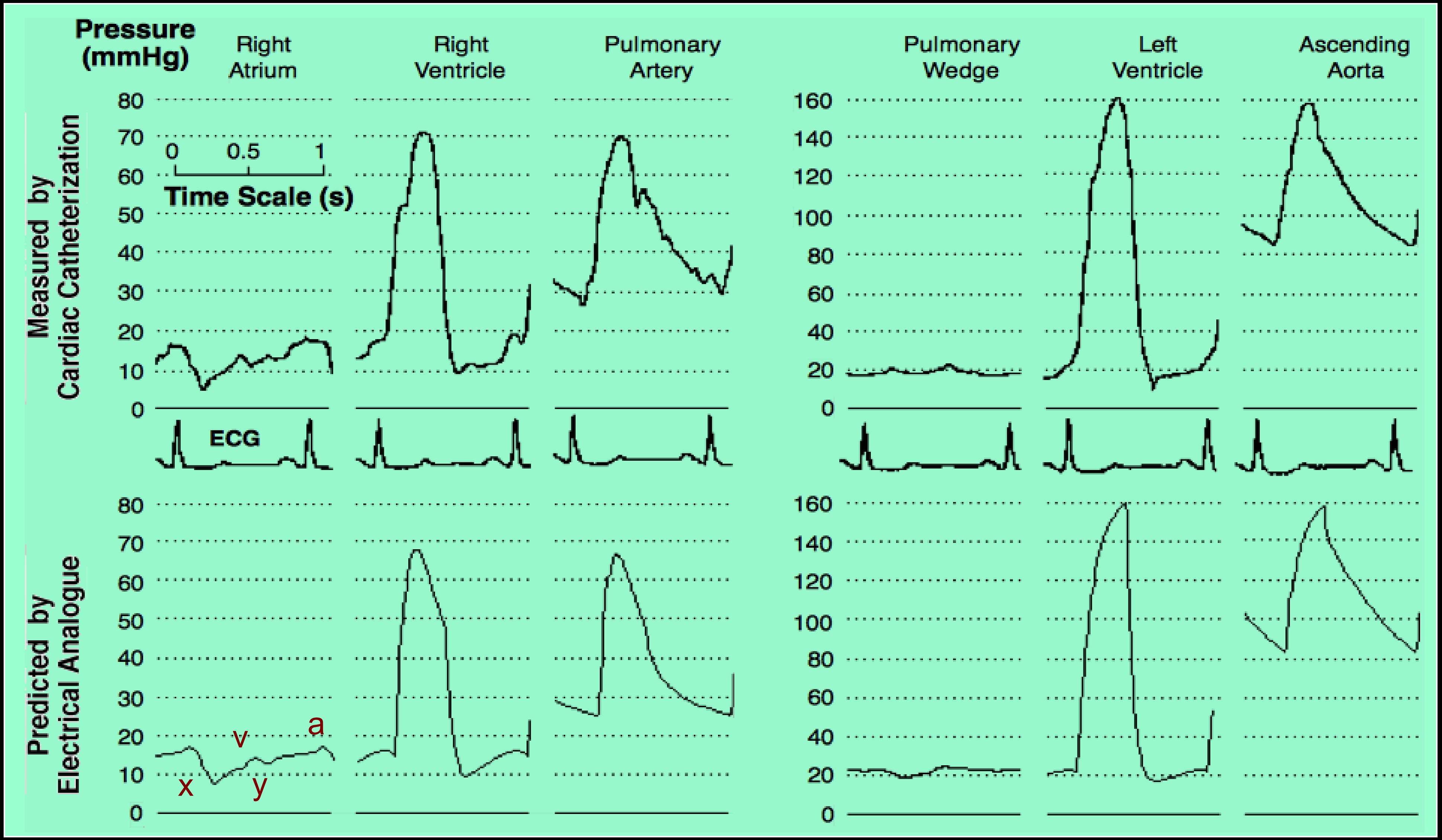
Sun Y, Beshara M, Lucariello RJ, Chiaramida SA. A comprehensive model for right-left heart interaction under the influence of pericardium and intrathoracic pressure. American J. Physiology 272 (3 Pt 2; Heart Circ Physiol 41): H1499-H1515, Mar. 1997.



Electrical Analog Model of the Cardiovascular System

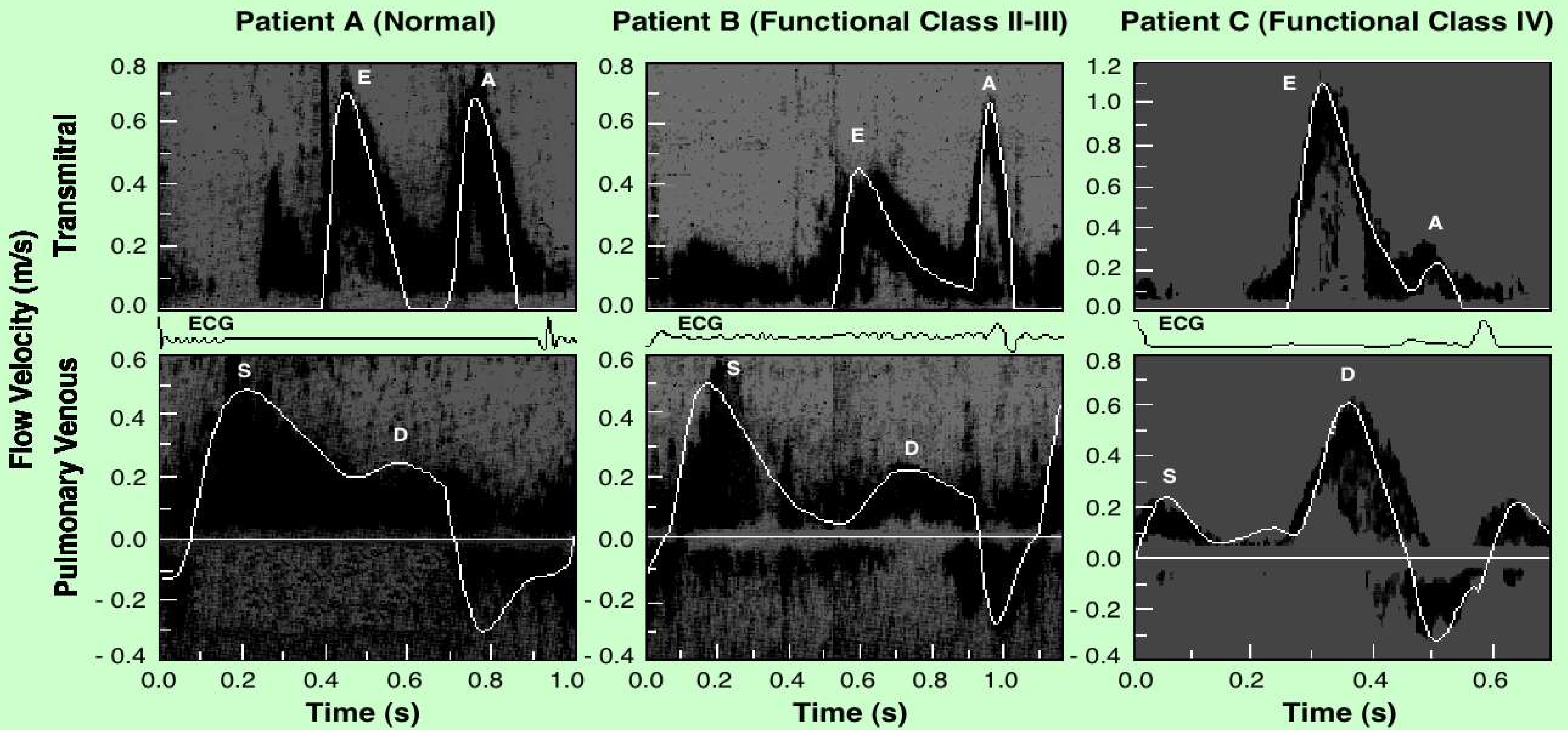


Validation – Cath Lab Data (Pressures)



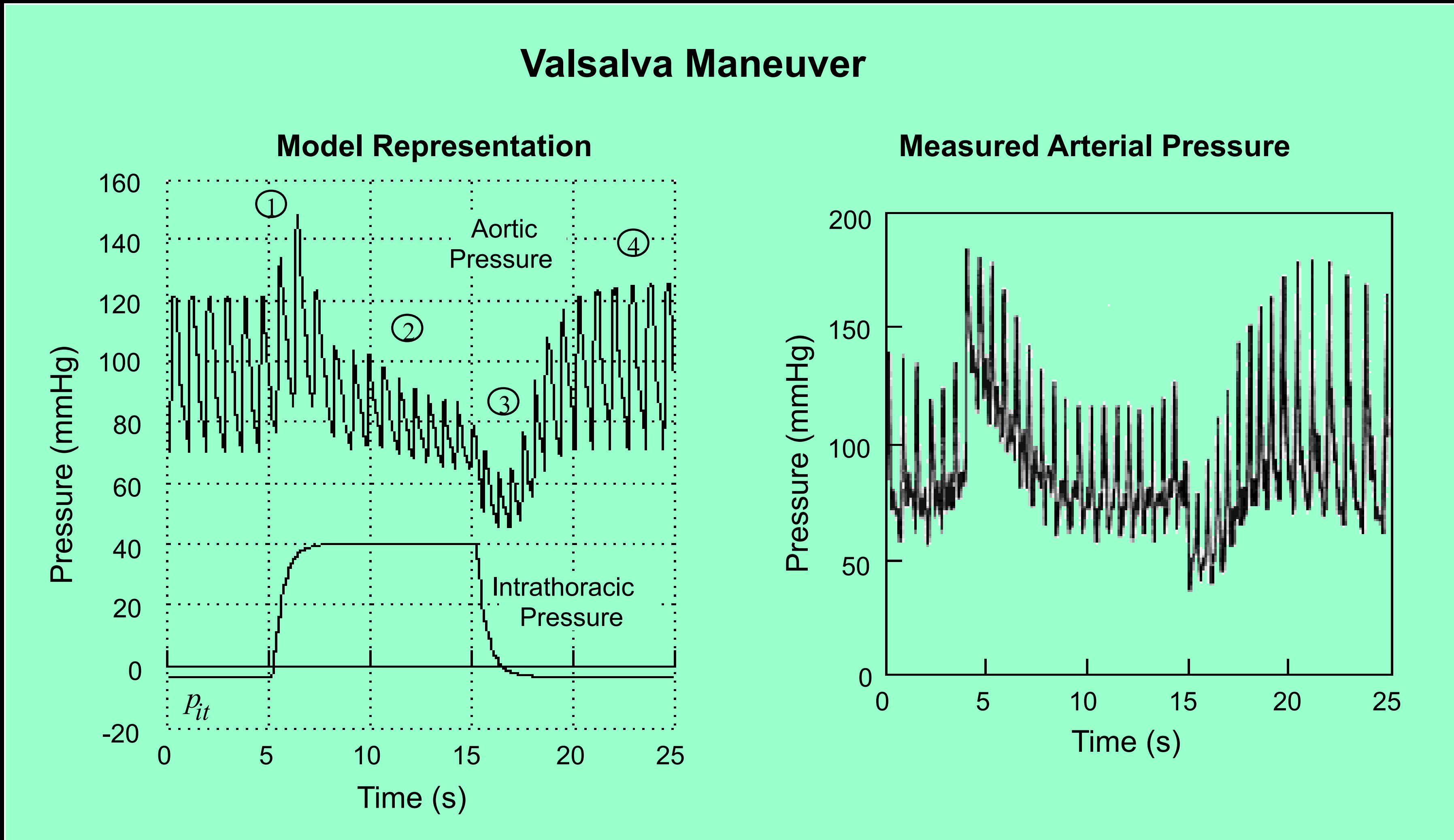
Sun, Chiaramida et al., American J of Physiology 1997

Validation – Doppler Echocardiographic Data



Sun, Chiaramida et al., American J of Physiology 1997

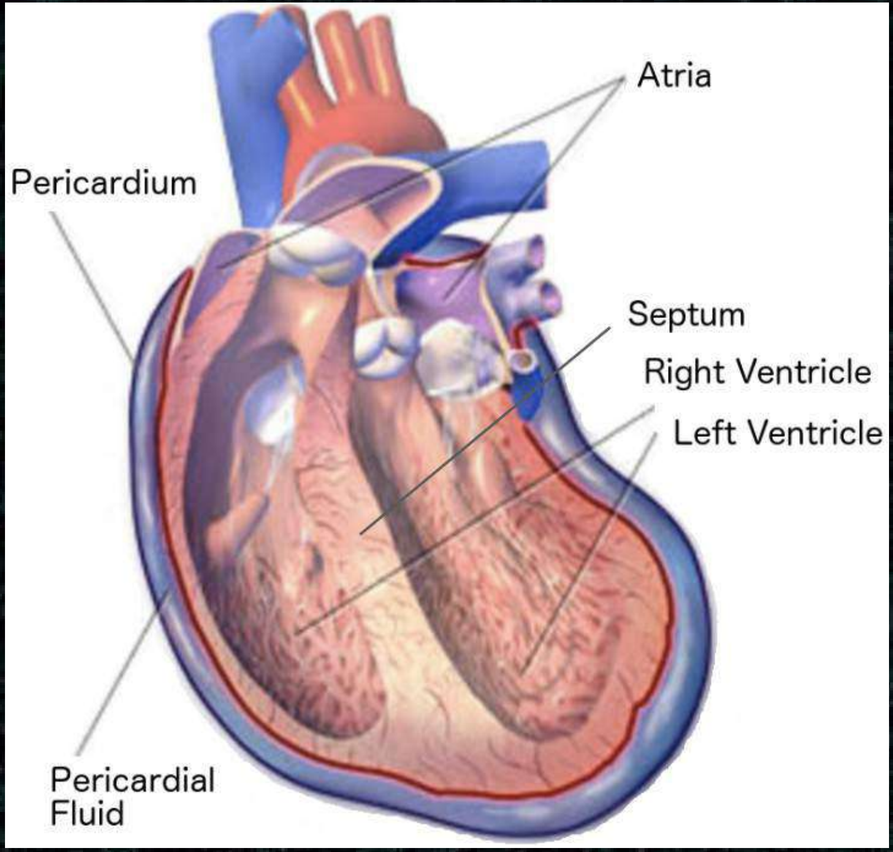
Validation – Valsalva Maneuver



Sun, Chiaramida et al., American J of Physiology 1997

Coupling Between Right and Left Ventricles

- *Hemodynamic*
- *Transseptal ~2%*
- *Pericardial ~20%*



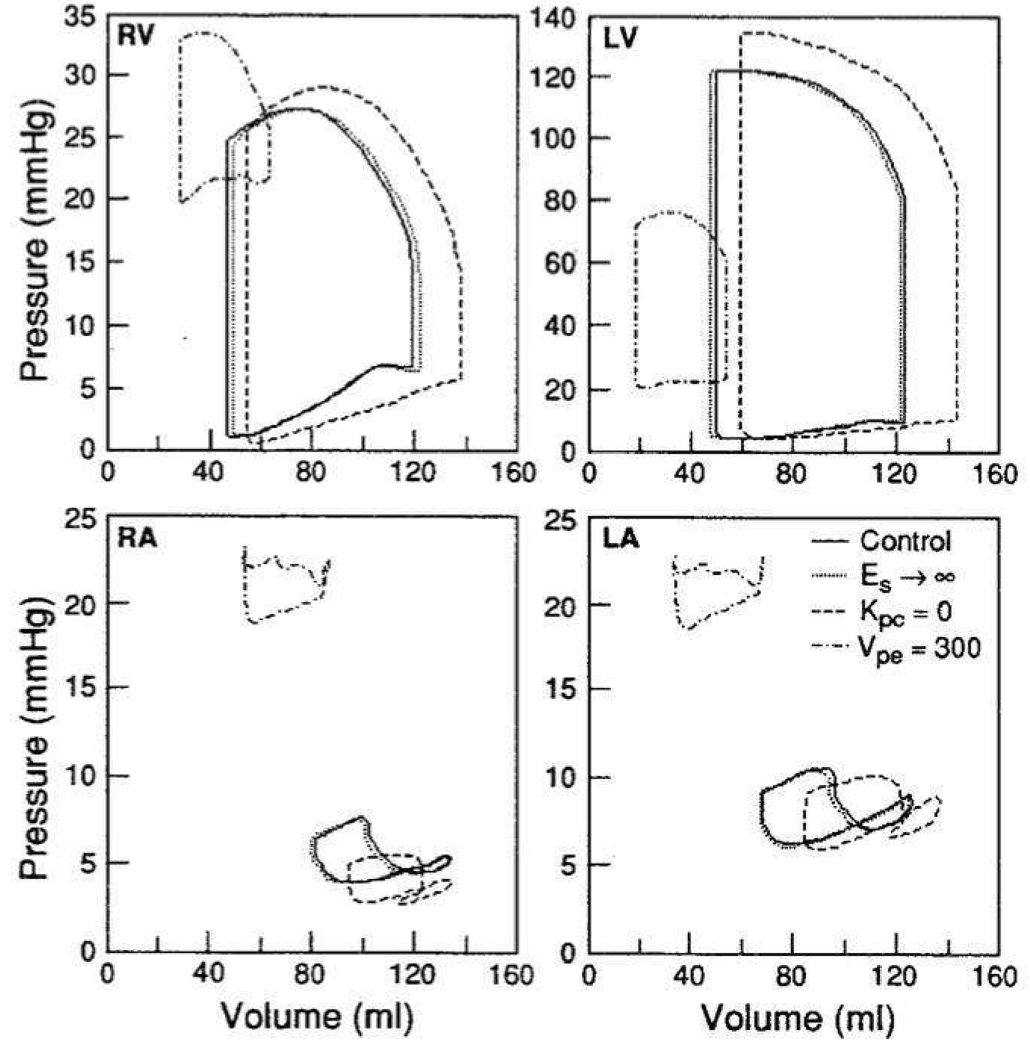
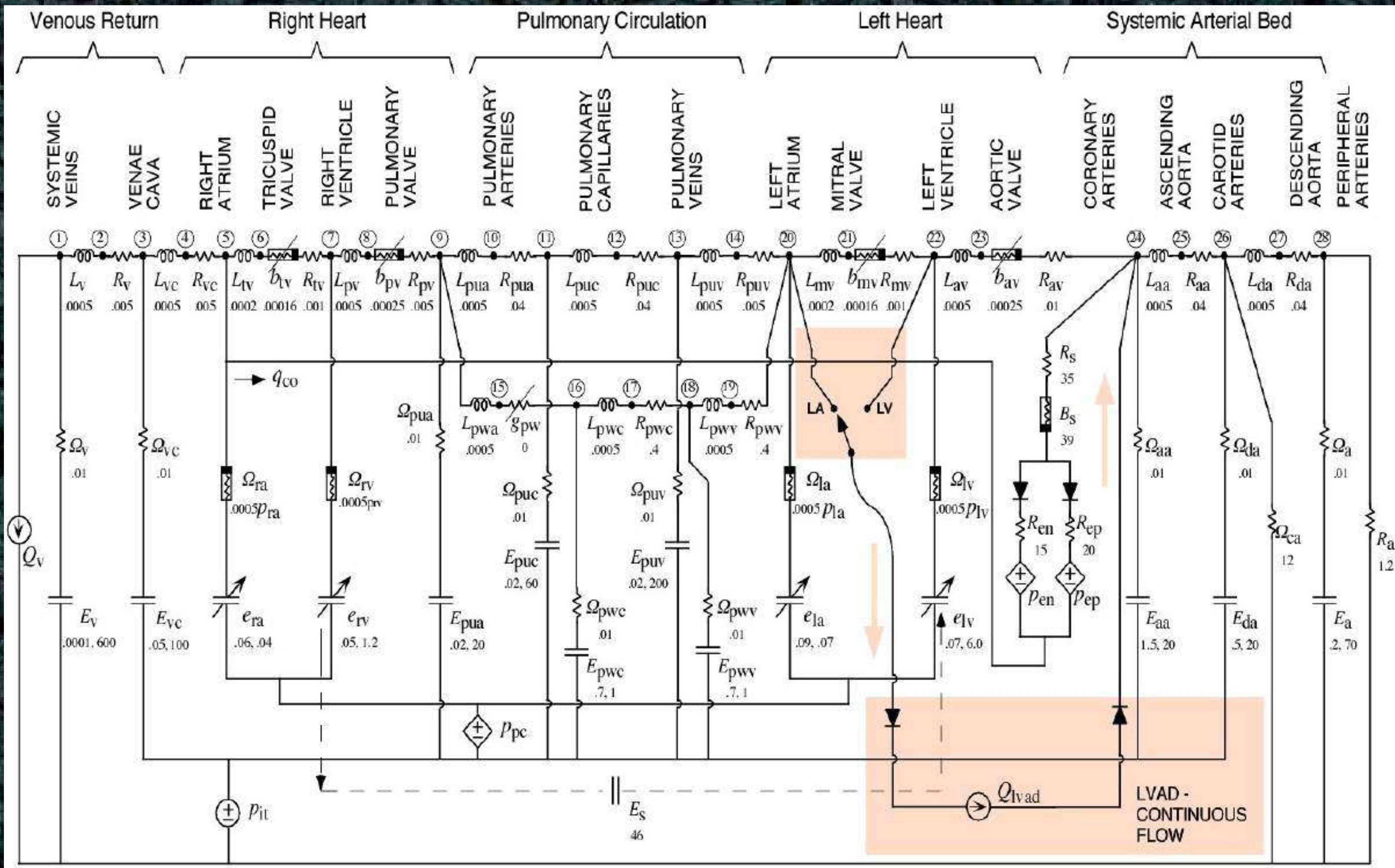
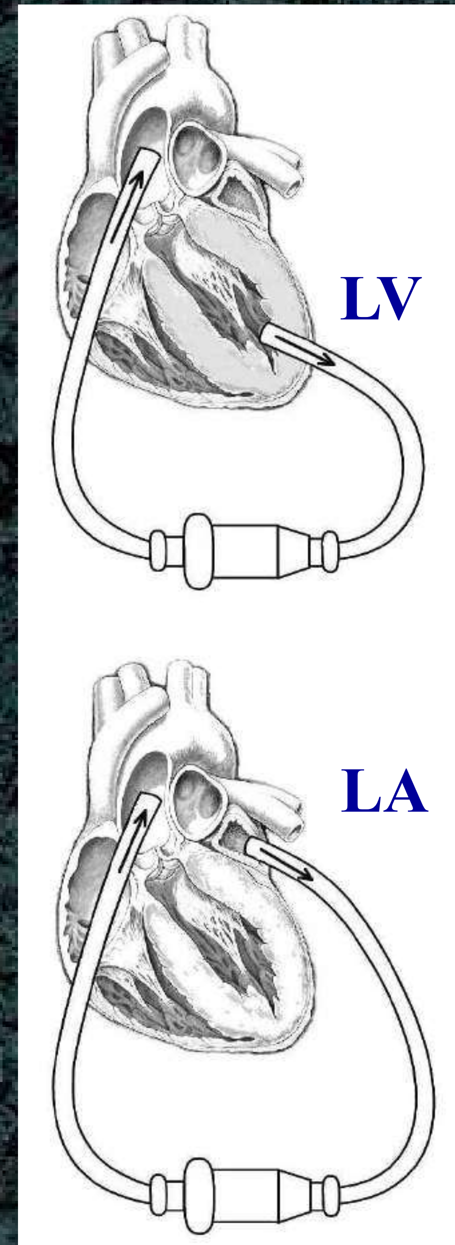


Fig. 11. Effects of removing transseptal coupling ($E_s \to \infty$), removing pericardial coupling ($K_{pc} = 0$), and cardiac tamponade ($V_{pe} = 300$ ml) on pressure-volume loops in 4 cardiac chambers. See *Glossary* for definition of abbreviations.

Left Ventricular Assist Device (LVAD)

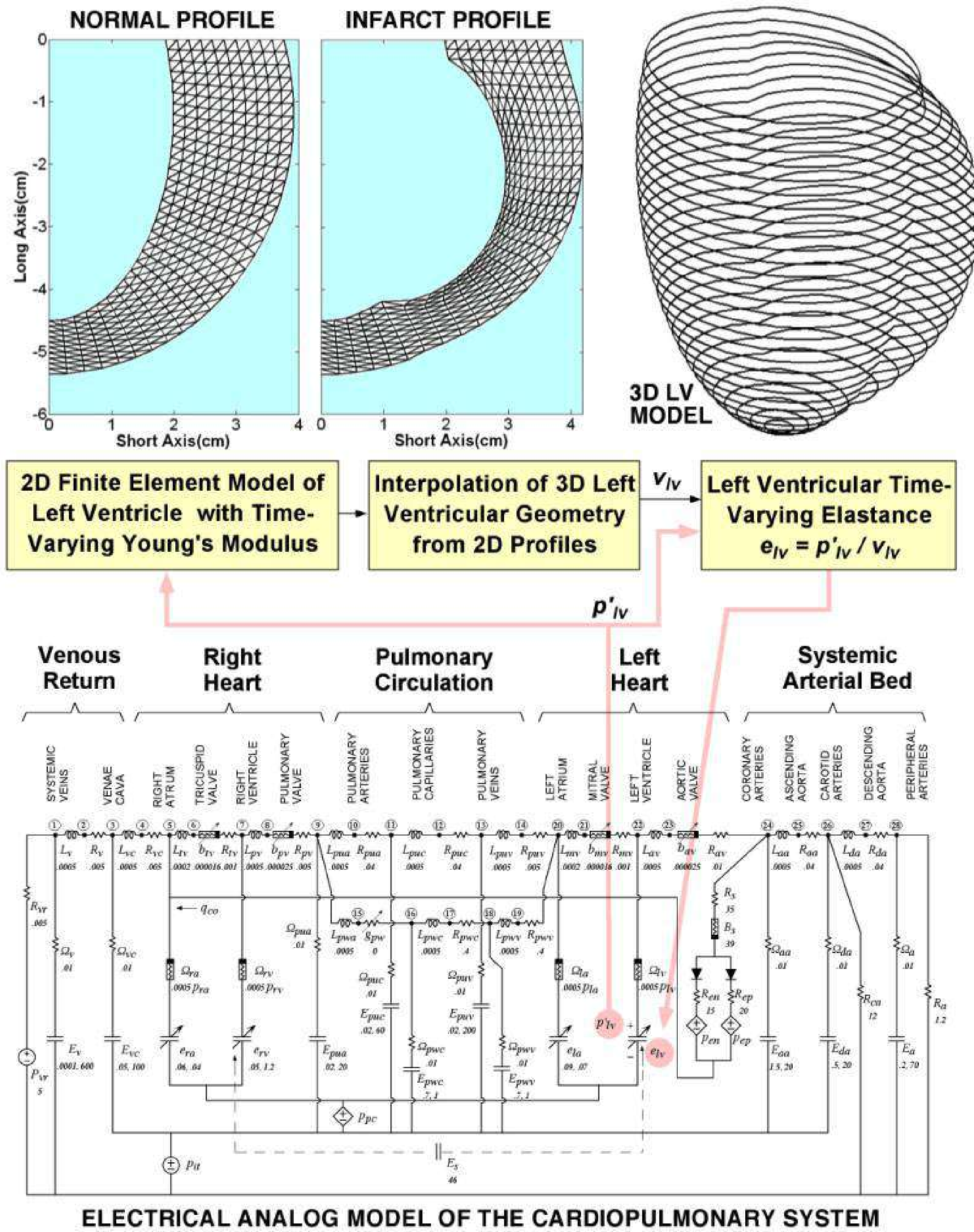




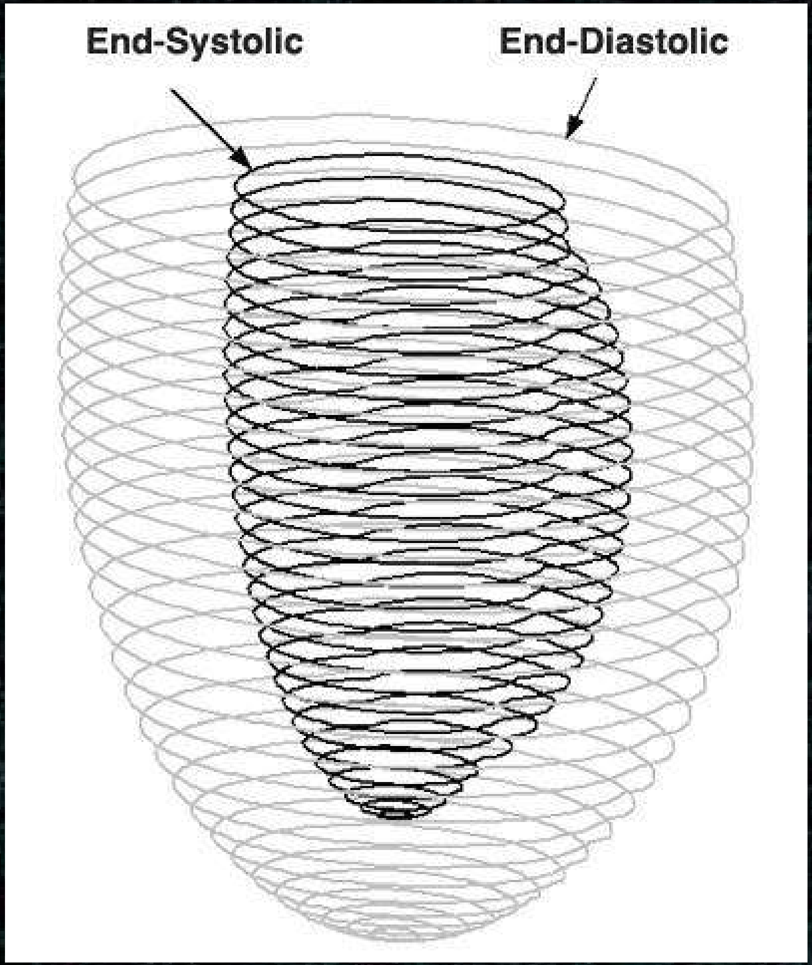
3D Model of the Left Ventricle with Infarction

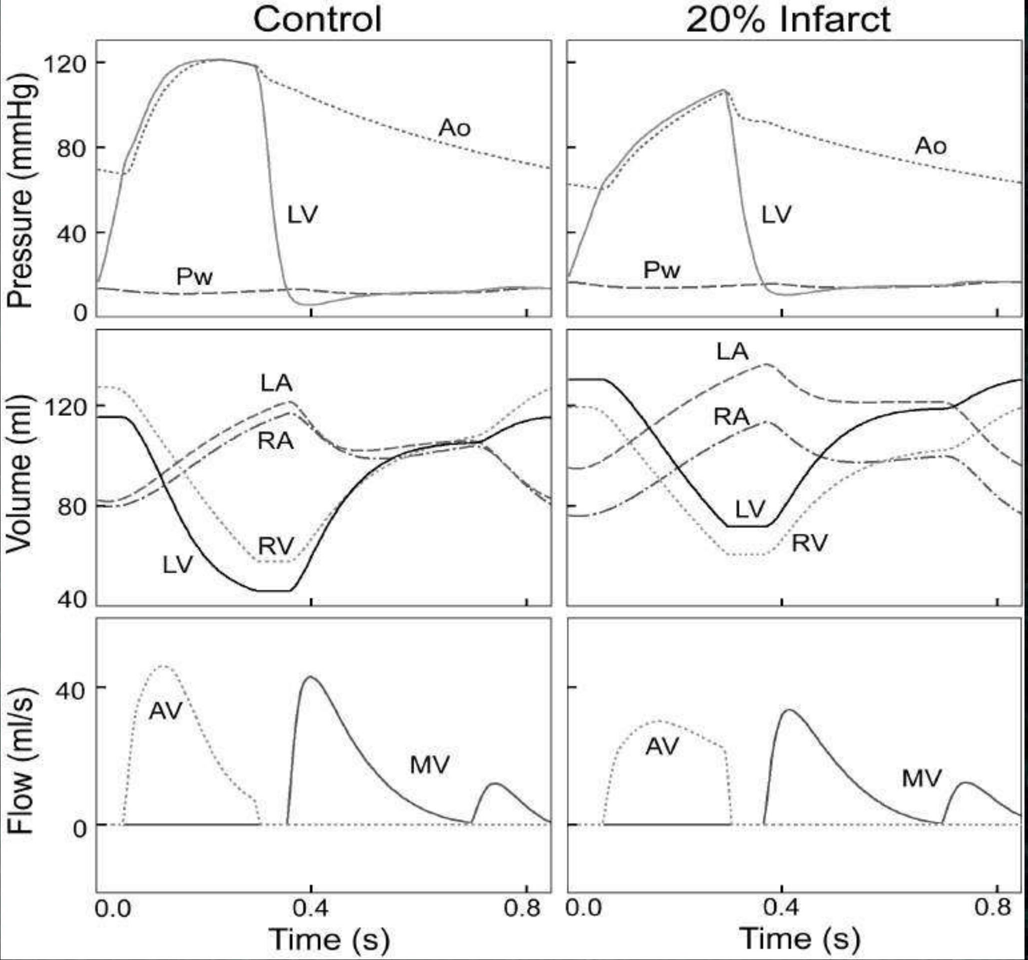
A finite element model of the left ventricle interacts dynamically with the circulatory model at a 5-ms time step.

US Patent No. US 8,295,907 B2, October 23, 2012



3D LV Model and Hemodynamic Waveforms

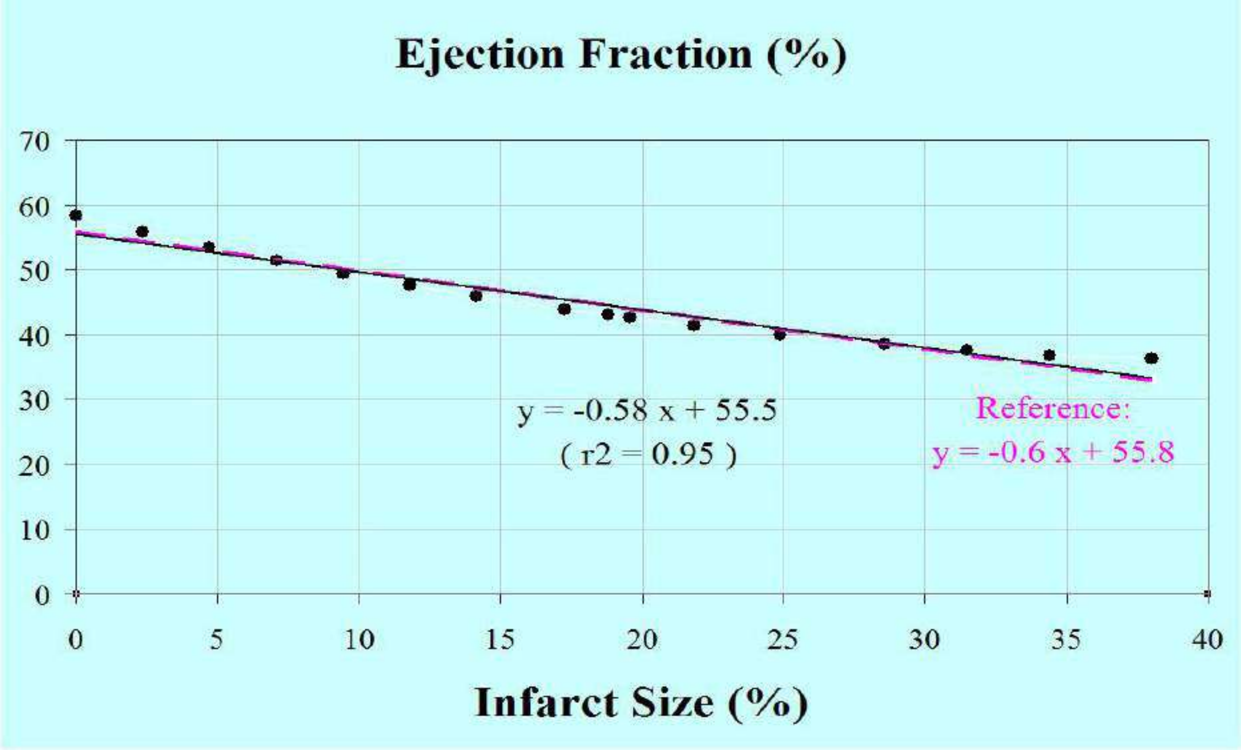


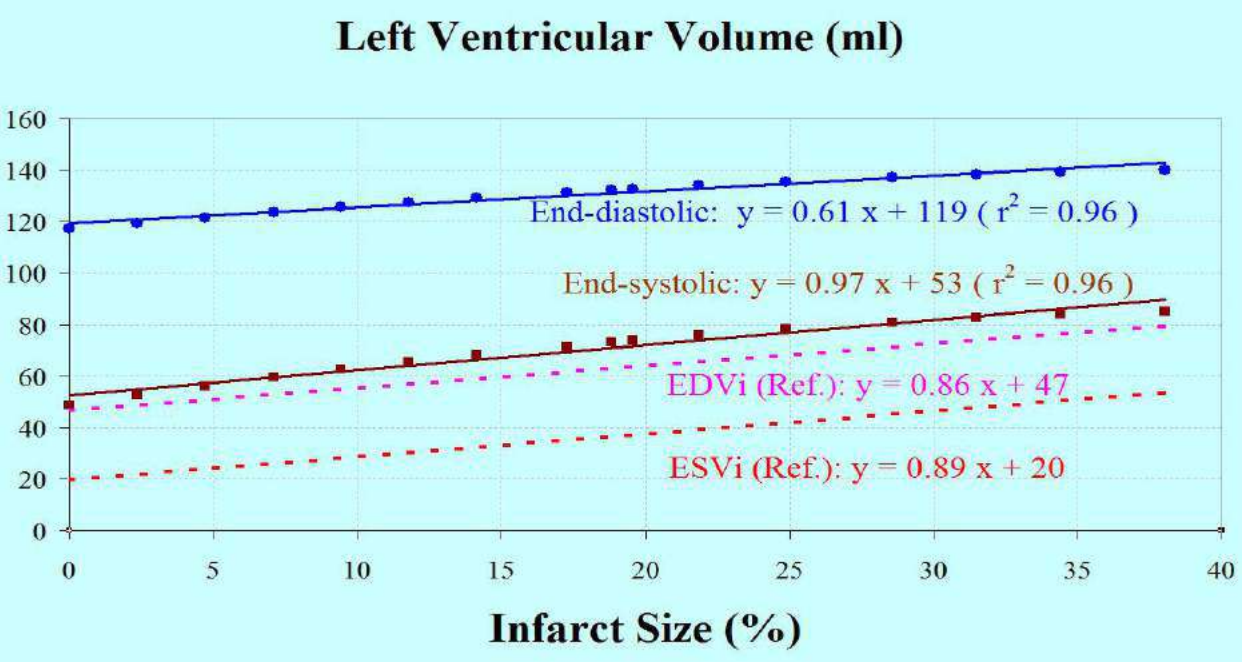


For an infarct size of 20% of the total left ventricular mass, the LV ejection fraction reduces from 59% to 41%.

Validation

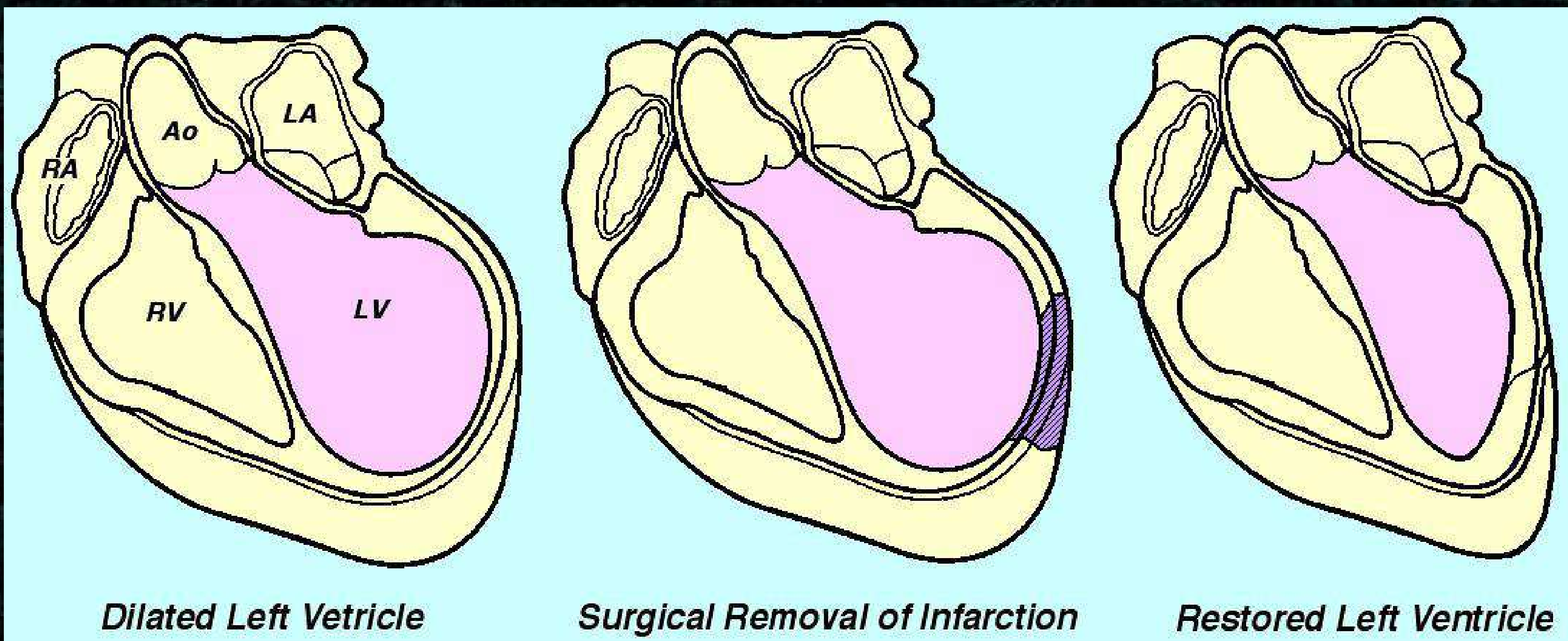
The integrated model predicts the decrease of LV ejection fraction and the increase of LV volumes as infarct size increases in consistence with clinical data (Sciagra et al. European J Nuclear Medicine & Molecular Imaging 31: 969-974, 2004).



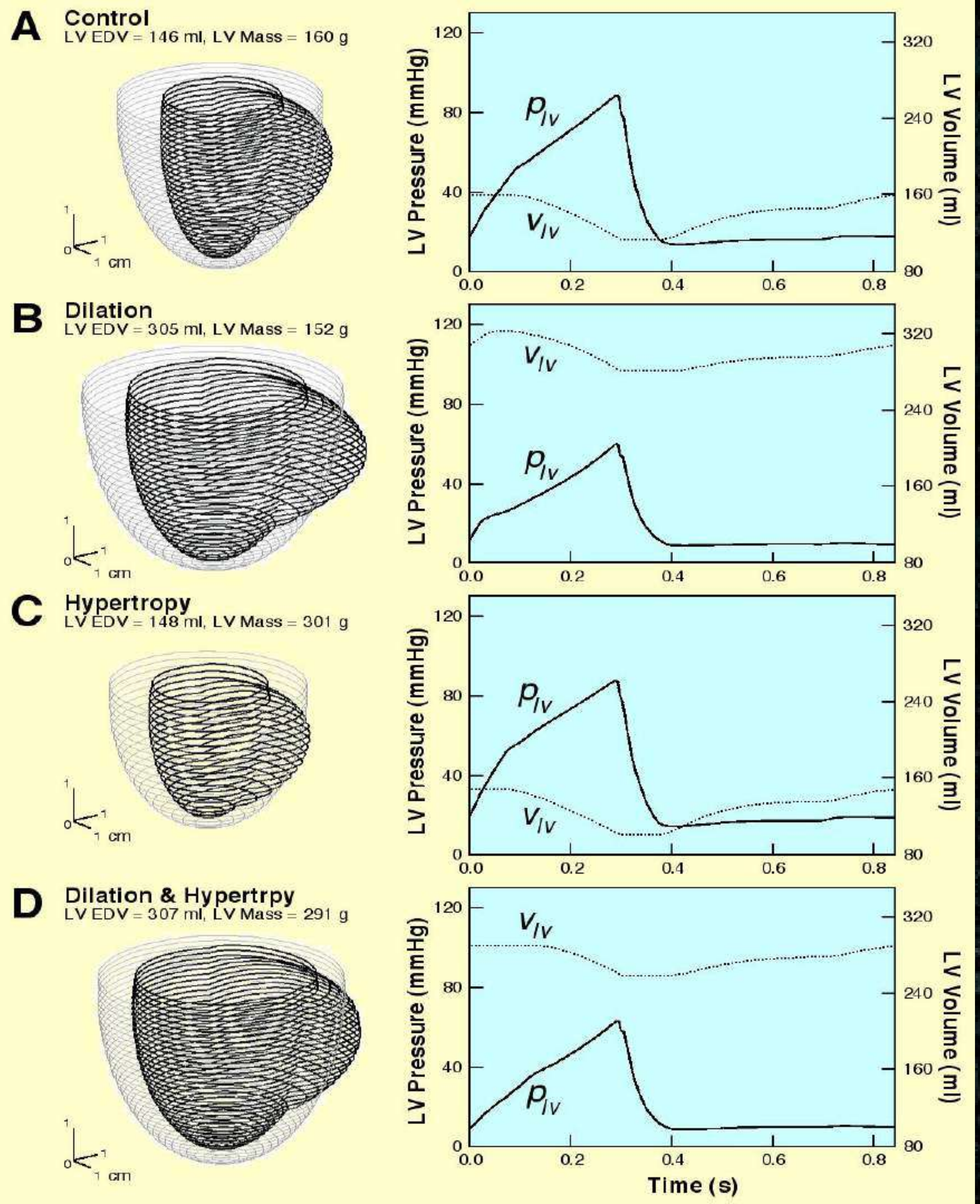


Surgical Ventricular Restoration

To remove infarction and to restore a remodeled left ventricle to its optimal size and shape for a patient with congestive heart failure



Model Predictions of LV pressure and volume waveforms for a 40% infarction under various preoperative LV geometries



Optimal Infarct Removal for Various LV Geometries with a 40% Preoperative Infarction

LV EDV / LV Mass	150 ml	225 ml	300 ml
150 g	**50%**	**76%**	**100%**
225 g	**40%**	**75%**	**100%**
300 g	**39%**	**76%**	**88%**

Summary

Mathematical models and simulations can be useful for biomedical research for the following purposes:

- ***Data regression*** **數據內插**
- ***Mechanism explanation*** **解釋機制**
- ***Hypothesis formulation*** **創造假設**
- ***Outcome prediction*** **預測結果**