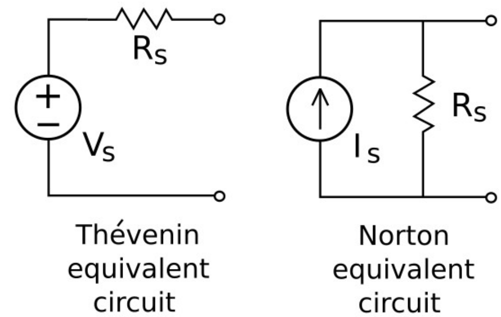
Impedance Matching

BME 360 Lecture Notes *Ying Sun*

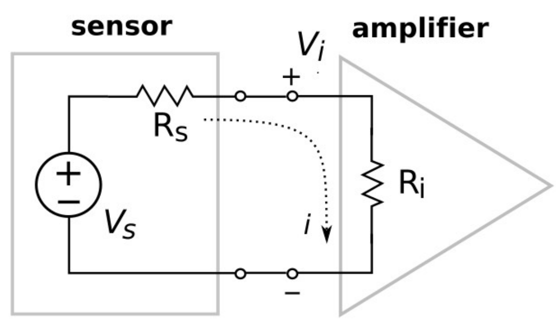
Equivalent Circuit of the Biosensor

At the front-end of a biomeasurement system, the biosensor converts a physical/physiological quantity to a voltage signal. This voltage signal is usually of a small amplitude and very vulnerable to noise, such as the 60 Hz line noise. The electrical properties of the sensor can be modeled by use of either the Thévenin's equivalent circuit or the Norton equivalent circuit as shown. The internal impedance of the sensor is an intrinsic property of the sensor and cannot be changed for a given sensor. For simplicity, we represent the internal impedance with the internal resistance (R_S). In other words, we ignore the frequency-dependent characteristics for now.



Interface between the Sensor and the Amplifier

Figure on the right shows that the sensor represented by its Thévenin's equivalent circuit. The sensor is interfaced with the first stage of an amplifier. We can optimize the sensor-amplifier interface by changing the input resistance (R_i) of the amplifier.



To achieve the best signal-to-noise (SNR), we need the maximum power transfer from the sensor to the amplifier. Let *i* denote the current through the loop and V_i denote the input voltage of the amplifier.

$$i = \frac{V_S}{R_i + R_S} ; \quad V_i = V_S \frac{R_i}{R_i + R_S} .$$

The power transferred from the sensor to the amplifier is:

$$P = V_i \, i = V_S^2 \frac{R_i}{(R_i + R_S)^2} .$$

For maximum voltage transfer, we set $R_i \to \infty$:

$$V_i = \lim_{R_i \to \infty} V_S \frac{R_i}{R_i + R_S} = V_S . \quad \text{But } i = \lim_{R_i \to \infty} \frac{V_S}{R_i + R_S} = 0 .$$

So, the power transfer is zero, $P = 0$.

For maximum current transfer, we set $R_i = 0$:

$$i = \left[\frac{V_S}{R_i + R_S} \right]_{R_i = 0} = \frac{V_S}{R_S} . \text{ But } V_i = \left[V_S \frac{R_i}{R_i + R_S} \right]_{R_i = 0} = 0 .$$

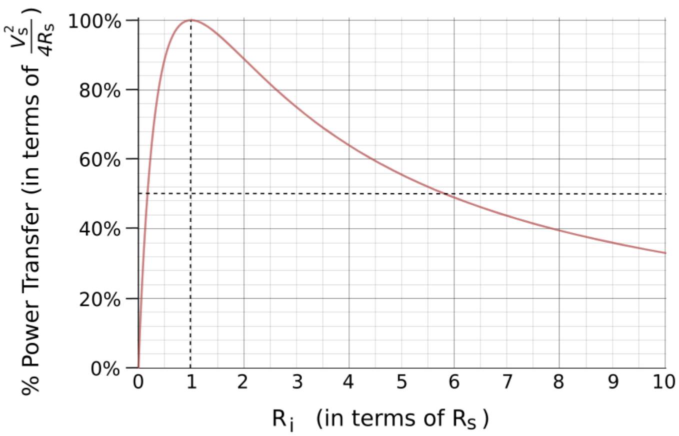
So, the power transfer is also zero, $P = 0$.

For maximum power transfer, R_i should match R_S .:

$$i = \left[\frac{V_S}{R_i + R_S}\right]_{R_i = R_S} = \frac{V_S}{2R_S} \; ; \; V_i = \left[V_S \frac{R_i}{R_i + R_S}\right]_{R_i = R_S} = \frac{V_S}{2} \; .$$

The power transfer is $P = V_i \; i = \left[V_S^2 \frac{R_i}{(R_i + R_S)^2}\right]_{R_i = R_S} = \frac{V_S^2}{4R_S}$.

Figure on the right shows how the power transfer changes as a function of R_i . The maximum power transfer is $V_S^2/(4R_S)$, which occurs at $R_i = R_S$. For $R_i < R_S$, the system favors a higher current transfer at the sacrifice of the power transfer. For $R_i > R_S$, the system favors a higher voltage transfer at the sacrifice of the power transfer.



Example:

If we prefer a higher voltage transfer and by letting the power transfer down to 1/2 of the maximum power transfer, what should R_i be?

$$P = V_S^2 \frac{R_i}{(R_i + R_S)^2} = \frac{1}{2}\left(\frac{V_S^2}{4R_S}\right) = \frac{V_S^2}{8R_S}$$

Solving for R_i , we have

$$8R_S R_i = R_i^2 + 2R_S R_i + R_S^2 \; ,$$

$$R_i^2 - 6R_S R_i + R_S^2 = 0 \; ,$$

$$R_i = \frac{6 \pm \sqrt{6^2 - 4}}{2} R_S = (3 \pm \sqrt{8}) R_S = 0.172 R_S , \quad 5.828 R_S$$

Side note:

Quadratic equation: $ax^2 + bx + c = 0$

Roots: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

To favor voltage transfer we choose $R_i = 5.828 R_S$.

The voltage transfer is $V_i = V_S \frac{R_i}{R_i + R_S} = \frac{5.828 R_S}{5.828 R_S + R_S} V_S = 0.854 V_S$