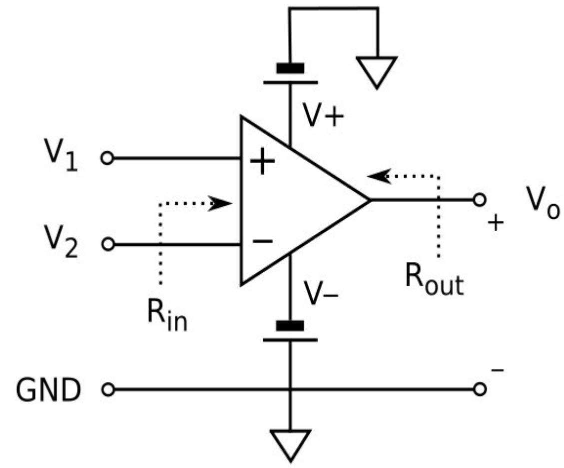
# Operational Amplifier

BME 360 Lecture Notes *Ying Sun*

## Characteristics of Op-Amp

An operational amplifier (op-amp) is an analog integrated circuit that consists of several stages of transistor amplification circuits, resulting in a high-gain DC-coupled electronic amplifier. As shown on the right, the op-amp typically has a differential input ($V_1$ and $V_2$) and a single-ended output ($V_o$). The properties that characterize an op-amp are:




Differential gain: $A_d = \dfrac{v_o}{v_1 - v_2}$

Common-mode gain: $A_c = \dfrac{v_o}{(v_1 + v_2)/2}$

Common-mode Rejection Ratio: $CMRR = \dfrac{A_d}{A_c}$

Input Impedance: $R_{\text{in}}$

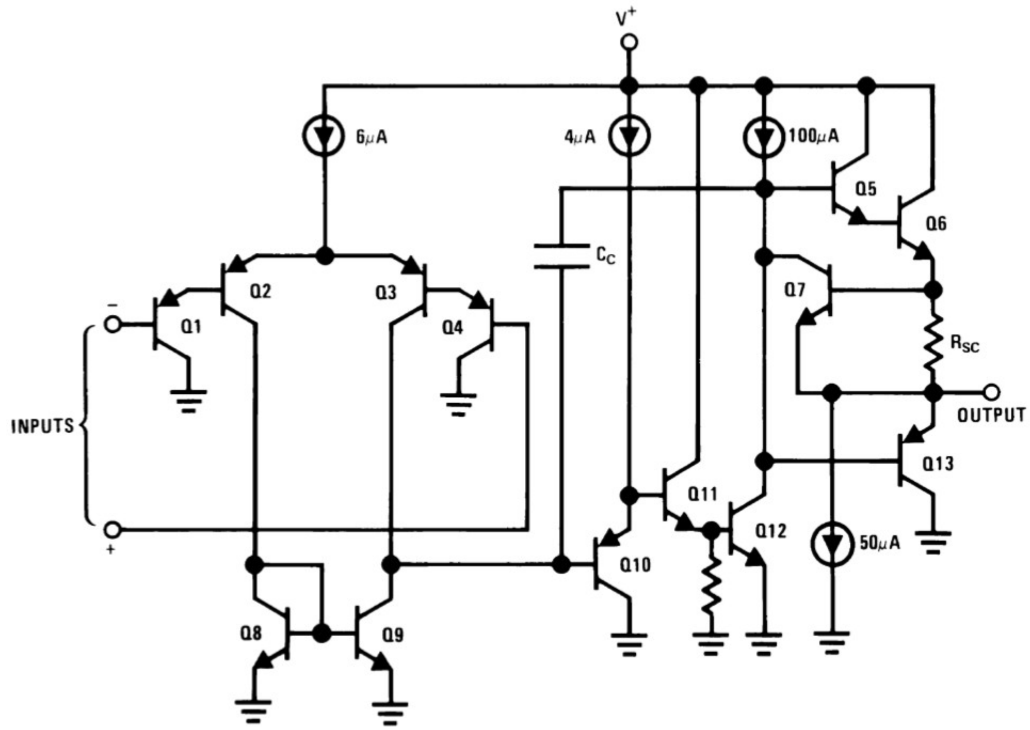
Output Impedance: $R_{\text{out}}$

Gain-bandwidth product: $GBWP = A_d \times \text{Bandwidth}$

| Parameter | Ideal op-amp | Low power quad op-amp LM324 | Ultra-low input current op-amp LMC6001 |
|---|---|---|---|
| Differential gain $A_d$ | ∞ | 100000 | 1000000 |
| Common-mode gain $A_c$ | 0 | 10 | 100 |
| Common-mode rejection ratio(CMRR) | ∞ | 10000 | 10000 |
| Input impedance $R_{\text{in}}$ | ∞ | 100 MΩ | 1 TΩ |
| Output impedance $R_{\text{out}}$ | 0 | 100 Ω | 100 Ω |
| Gain-bandwidth product (GBWP) | ∞ | 1 MHz | 1 MHz |
| Input (bias) current | 0 | 45 nA | 25 fA |
| Input offset voltage | 0 | 2 mV | 0.35 mV |
| Cost | N/A | $0.40 | $17.00 |

**Inside the Op-Amp**

The op-amp is a multi-stage transistor circuit. The schematic below shows one of the four op-amps in LM324, which is a quad op-amp integrated circuit. The input stage is a differential amplifier (Q1–Q4). A constant current source (Q8 and Q9) helps reduce the common-mode gain. The capacitor Cc provides a negative feedback for higher frequency signals, thereby setting the bandwidth of the amplifier. The output stage is a Darlington pair (Q5 and Q6), which provides a higher current output.




**Input offset voltage**

At the front stage, the four transistors should be matched, i.e. having the exact same characteristics. For example, their base-emitter voltage $V_{\text{BE}}$ should be the same. If not, the input voltages need to be slightly different in order to make the final output zero. This voltage is called "input offset voltage."

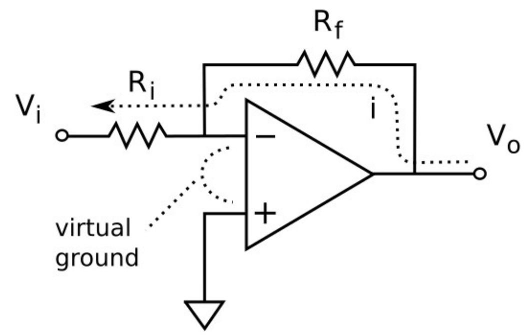
**Input (bias) current**

For the same reason above, the slight miss-match of the transistors can cause a small current flowing between the positive and negative input terminals. This current is called "input bias current."

**Negative Feedback and "Virtual Ground"**

For the ideal op-amp, the *open-loop gain* is infinity. We use negative feedback in the circuit design to result in a *closed-loop gain* of an appropriate value. For a realistic op-amp, the negative feedback also has the benefits of increasing $R_{\text{in}}$ , decreasing $R_{\text{out}}$ , and increasing bandwidth. For the following analyses of the op-amp circuits, we assume that the op-amp is ideal. The "virtual ground" concept significantly simplifies the analysis of an op-amp circuit. As shown in the circuit below, a negative feedback loop is established by connecting the feedback resistor $R_f$ from the output to the negative terminal of the input. The positive terminal is at ground. We claim that the negative terminal is at "virtual ground." The voltage at the negative terminal must be zero. Because the open-loop differential gain $A_d=\infty$ , any none-zero voltage sustained between the negative and the positive terminal would result in $V_o=\infty$ and the negative feedback would reduce the input voltage to zero. However, unlike the true ground that can drain any amount of current, the virtual ground cannot draw any current into it because the input impedance of the op-amp is **∞**. We will use "virtual ground" referring to the fact that the input terminals cannot sustain any differential voltage at the presence of a negative feedback.

**Inverter Amplifier**

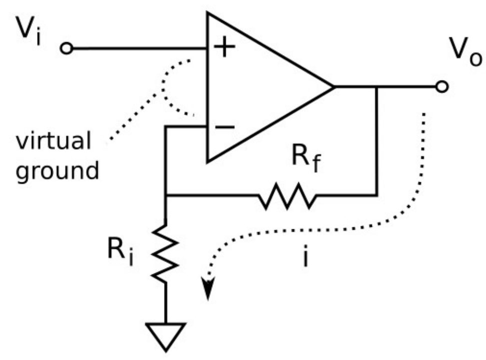



The inverter amplifier has an input resistor $R_i$ to the negative terminal, a feedback resistor $R_f$ from the output to the negative terminal, and the positive terminal to ground. Current $i$ flows from the output through $R_f$ . Because no current can flow into the op-amp ( $R_{\text{in}}=\infty$ ), $i$ continues to flow through $R_i$ . The negative terminal is at virtual ground with zero voltage.

$$i = \frac{V_o}{R_f}; \quad i = -\frac{V_i}{R_i} \quad \Rightarrow \quad \frac{V_o}{R_f} = -\frac{V_i}{R_i} \quad \Rightarrow \text{ The gain is: } A = \frac{V_o}{V_i} = -\frac{R_f}{R_i}.$$

The "–" sign in the gain means that the output is 180-degree out-of-phase with the input. Therefore, this circuit is called inverter amplifier.

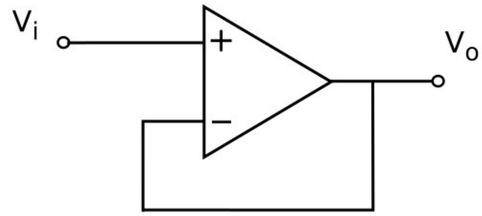
**Non-Inverting Amplifier**




The non-inverting amplifier has the input goes directly into the positive terminal. A feedback resistor $R_f$ is from the output to the negative input terminal and also to the ground through an input resistor $R_i$ . The *virtual ground* concept is still applied; the positive and negative input terminals cannot sustain any differential voltage due to the negative feedback. However, the voltage at the input terminals is not ground, but $V_i$ .

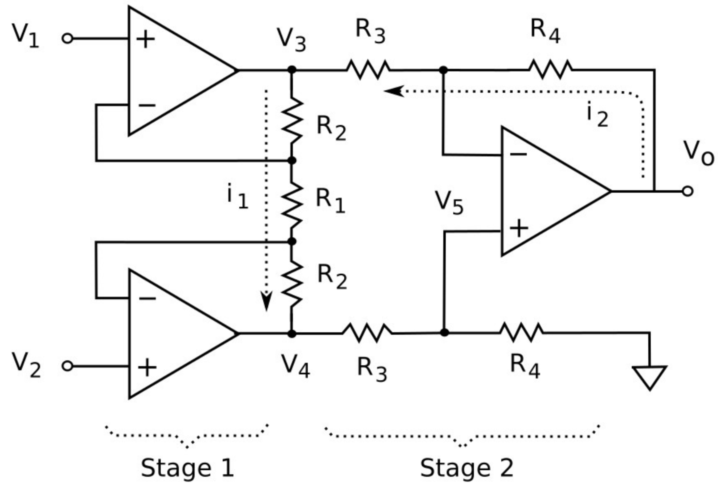
$$i = \frac{V_o}{(R_f+R_i)}; \quad i = \frac{V_i}{R_i} \quad \Rightarrow \quad \frac{V_o}{R_f+R_i} = \frac{V_i}{R_i} \quad \Rightarrow \text{ The gain is: } A = \frac{V_o}{V_i} = \frac{R_f+R_i}{R_i}.$$

**Voltage Follower**




The voltage follower is a special case of the non-inverting amplifier. Again, the *virtual ground* concept applies in the sense that the input terminals cannot sustain any differential voltage due to negative feedback. So $V_o = V_i$, and the gain is unity. With its high input impedance and low output impedance, the voltage follower serves as a "buffer" in the circuit design.

**Instrumentation Amplifier**




The instrumentation amplifier is often used at the front-end of an amplifier to interface with a biosensor that requires differential inputs such as the electrocardiogram (ECG). The first stage consists of two voltage followers, one for each of the input terminals. The second stage is an inverter amplifier that combines the differential inputs.

Applying the *virtual ground* concept, $V_1$ appears at the top of $R_1$, and $V_2$ appears at the bottom of $R_1$. $R_1$ is the input resistance of the overall amplifier. The current $i_1$ flows through the branch consisting of $R_2$, $R_1$, and $R_2$; $i_1$ does not split into the op-amps because of their infinite input impedance. The voltage gain of stage 1 ( $A_1$ ) is computed as follows.

$$i_1 = \frac{V_1 - V_2}{R_1}\ ;\ \ i_1 = \frac{V_3 - V_4}{2R_2 + R_1} \quad \Rightarrow \quad \frac{V_1 - V_2}{R_1} = \frac{V_3 - V_4}{2R_2 + R_1} \quad \Rightarrow$$

$$A_1 = \frac{V_3 - V_4}{V_1 - V_2} = \frac{2R_2 + R_1}{R_1}$$

For the second stage, current $i_2$ flows through $R_4$ and continues to flow through $R_3$.

$$i_2 = \frac{V_o - V_5}{R_4}\ ;\ \ i_2 = \frac{V_5 - V_3}{R_3} \quad \Rightarrow \quad \frac{V_o - V_5}{R_4} = \frac{V_5 - V_3}{R_3} \quad \Rightarrow \quad V_o = \frac{R_3 + R_4}{R_3} V_5 - \frac{R_4}{R_3} V_3$$

The voltage at the input terminals ( $V_5$ ) comes from $V_4$ through the voltage divider $R_3$ and $R_4$.

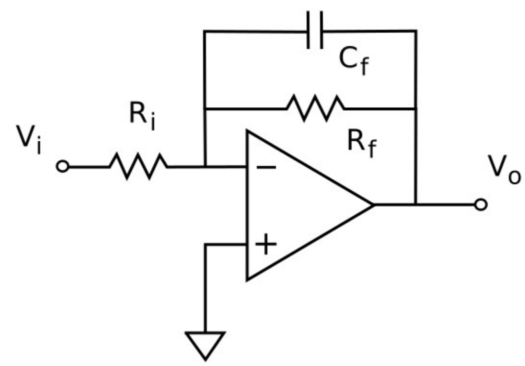
$V_5 = V_4 \dfrac{R_4}{R_3 + R_4}$ . Substituting this to the above equation, we have

$$V_o = \left(\frac{R_3 + R_4}{R_3}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_4 - \frac{R_4}{R_3} V_3 = -\frac{R_4}{R_3}(V_3 - V_4) \quad \Rightarrow$$

$A_2 = \dfrac{V_o}{V_3 - V_4} = -\dfrac{R_4}{R_3}$ . The overall voltage gain is: $A = A_1 A_2 = -\left(\dfrac{2R_2 + R_1}{R_1}\right)\left(\dfrac{R_4}{R_3}\right)$ .

**Low-Pass Filter**

Adding a capacitor $C_f$ in the negative feedback loop results in a low-pass filter. As frequency increases, more negative feedback goes through the capacitor and the gain becomes lower. We use the Laplace transform (LT) to represent the impedance ( $Z_f$ ) of the feedback loop.




$$Z_f = R_f // \frac{1}{sC_f} = \frac{R_f(1/sC_f)}{R_f+(1/sC_f)} = \frac{R_f}{1+sR_fC_f}$$

The transfer function $H(s)$ of the filter is the LT of the output voltage over the LT of the input voltage. Extending the result of the gain of the inverter amplifier, $H(s)$ is the negative of the feedback impedance over the input impedance, given by:

$$H(s) = \frac{V_o(s)}{V_i(s)} = -\frac{Z_f}{R_i} = -\frac{R_f}{R_i}\frac{1}{1+sR_fC_f}$$

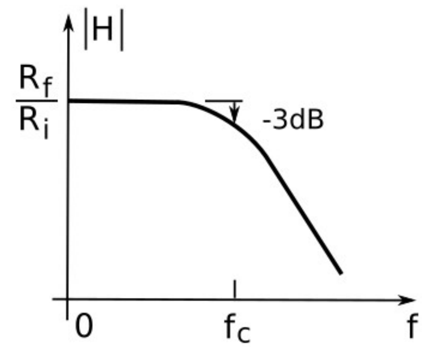
The Fourier transform (FT) is the LT evaluated on the $j\omega$ axis of the $s$-plane. By replacing $s$ by $j\omega$, we obtain $H(j\omega)$, which is the frequency response of the filter.

$$H(j\omega) = -\frac{R_f}{R_i}\frac{1}{1+j\omega R_fC_f}\text{, where the angular frequency } \omega = 2\pi f\text{ .}$$

At DC ($\omega = 0$), we have the gain = $-\ R_f/R_i$ . The cutoff frequency $f_c$ is defined at the point where the gain drops down to $1/\sqrt{2}$ (70.7%). This occurs at $2\pi f_c R_f C_f = 1$ .

$$H(j\omega_c) = -\frac{R_f}{R_i}\frac{1}{1+j2\pi f_c R_f C_f} = -\frac{R_f}{R_i}\frac{1}{1+j} \quad \Rightarrow$$

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}}\frac{R_f}{R_i}$$




The Bode plot is the plot of the frequency response on the log-log scales, as shown. On the Bode plot the "corner frequency" or the cutoff frequency is where the gain drops down to $1/\sqrt{2}$ . This point is often called the 3 dB point because:

$$20\log_{10}(1/\sqrt{2}) = 20\log_{10}0.707 = -3.01\,\text{dB} \simeq -3\,\text{dB}\text{ .}$$

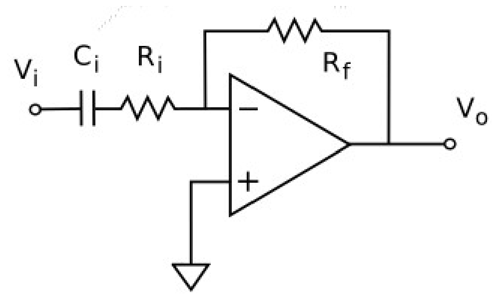
In summary, the cutoff frequency for the 1st-order low-pass filter is:

$$\boxed{f_c = \frac{1}{2\pi R_f C_f}}$$

## High-Pass Filter

Adding a capacitor $C_i$ in series with the input resistor $R_i$ results in a high-pass filter. This is because the capacitor impedes the low-frequency components in the signal. The input impedance is:

$$Z_i \;=\; R_i + \frac{1}{s\,C_i} \;=\; \frac{1+s\,R_i C_i}{s\,C_i} \,.$$




The transfer function is:

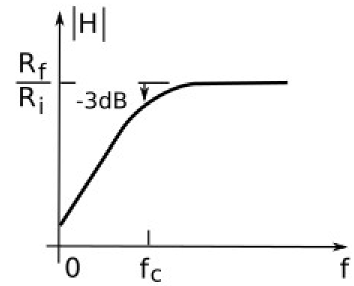
$$H(s) \;=\; \frac{V_o(s)}{V_i(s)} \;=\; -\frac{R_f}{Z_i} \;=\; -\frac{R_f}{R_i}\frac{s\,R_i C_i}{1+s\,R_i C_i}$$

The frequency response is:

$$H(j\,\omega) \;=\; -\frac{R_f}{R_i}\frac{j\,\omega\,R_i C_i}{1+j\,\omega\,R_i C_i}\,,\ \text{where}\ \ \omega \;=\; 2\pi f\ .$$

The magnitude of the frequency response is:

$$|H(j\,\omega)| \;=\; \frac{R_f}{R_i}\frac{\omega\,R_i C_i}{\sqrt{1+(\omega\,R_i C_i)^2}}\ .$$

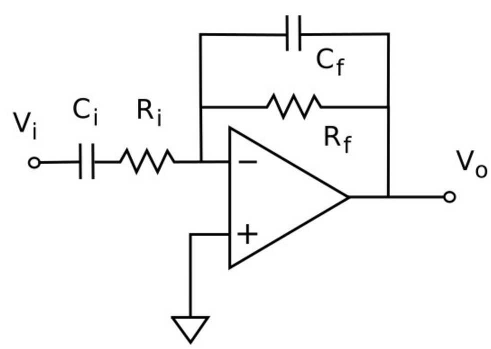



We see that

$$|H(j\,\omega)| \;=\; \begin{cases} 0, & \omega=0 \\ \dfrac{R_f}{R_i}, & \omega\rightarrow\infty \\ \dfrac{1}{\sqrt{2}}\dfrac{R_f}{R_i}, & \omega=\dfrac{1}{R_i C_i} \end{cases}$$

The Bode plot is shown on the right and the cutoff frequency is:

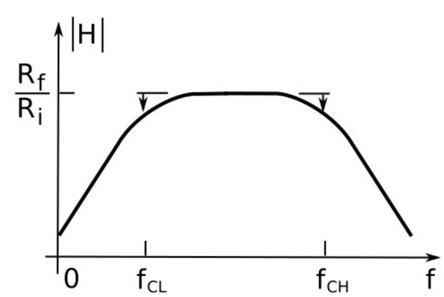
$$\boxed{f_c \;=\; \frac{1}{2\pi\,R_i C_i}}$$




## Band-Pass Filter

A band-pass filter is obtained by combining the low-pass filter and the high-pass filter as shown. The low and high cutoff frequencies are:

$$\boxed{\begin{aligned} f_{\mathrm{CL}} \;&=\; \frac{1}{2\pi\,R_i C_i} \\[2ex] f_{\mathrm{CH}} \;&=\; \frac{1}{2\pi\,R_f C_f} \end{aligned}}$$

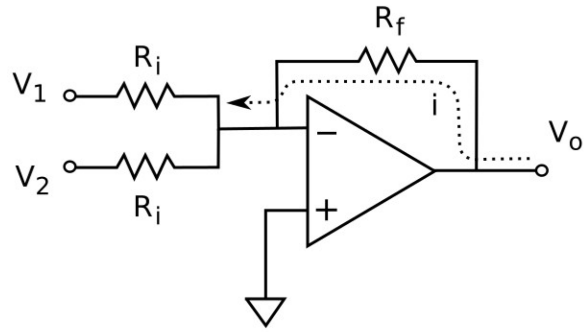


## Summer

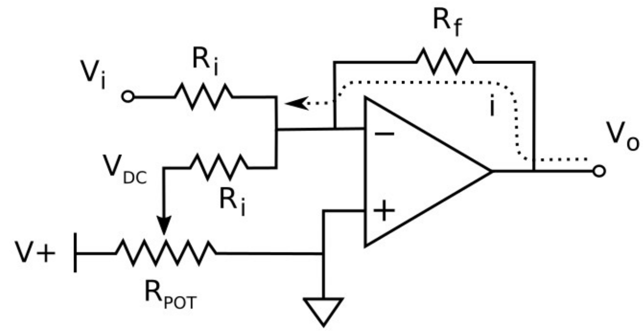
Adding two voltage signals together requires the connection of the two signals through resistors. The circuit analysis is as follows:

$$i = \frac{V_o}{R_f} = -\frac{V_1}{R_i} - \frac{V_2}{R_i} \quad \Rightarrow$$

$$V_o = -\frac{R_f}{R_i}(V_1 + V_2)$$



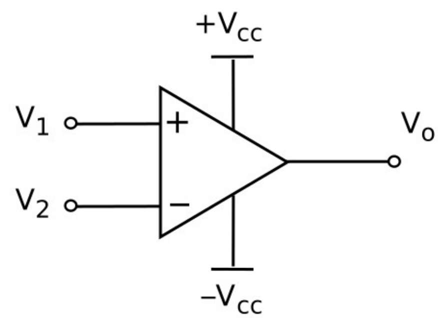

An application of the summer is to add a DC component to the signal. For example, the ECG is an AC signal, we need to add a DC component to it before sending it to the A/D converter of the processor that has an input range of 0-5V. The circuit on the right shows that a DC component $V_{DC}$ is added to the signal $V_i$. $V_{DC}$ is a positive DC offset that can be adjusted through the potentiometer $R_{POT}$.




## Comparator

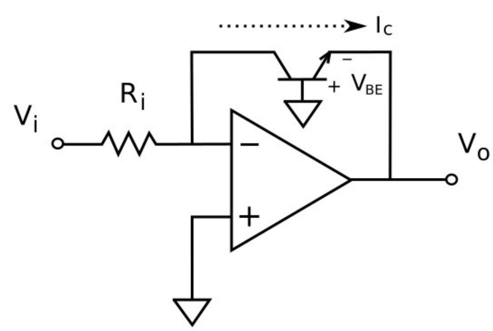
The comparator compares the two input signals $V_1$ and $V_2$ and outputs a binary signal, either positive or negative.

$$V_o = \begin{cases} +V_{CC}, & \text{if } V_1 > V_2 \\ -V_{CC}, & \text{if } V_1 < V_2 \end{cases}.$$




## Logarithmic Amplifier

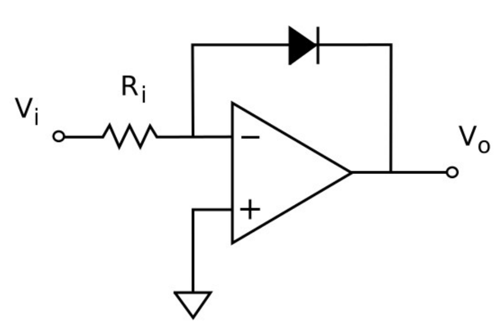
The logarithmic amplifier (or log amp) is useful to convert a voltage signal from a linear scale to a logarithmic scale. It is useful, for example, to display the sound level, which is expressed in dB. The base-emitter voltage ($V_{BE}$) of a transistor is proportional to the logarithm of the collector current ($I_C$). By putting a transistor in the feedback loop, a logarithmic relationship between the input and output can be obtained.



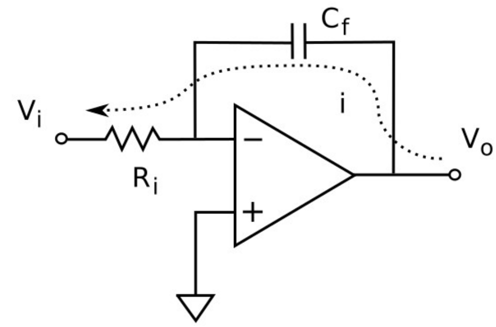

$$I_C = \frac{V_i}{R_i}; \quad V_{BE} \propto \log I_c; \quad V_o = -V_{BE} \quad \Rightarrow$$

$$V_o \propto \log V_i$$

A log amp can also be constructed by use of a diode, as shown.



**Integrator**

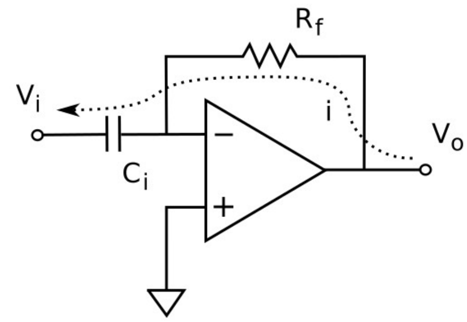



The integrator is a special case of the low-pass filter (by removing the feedback resistor).

$$i = -\frac{V_i}{R_i}; \quad V_o = \frac{1}{C_f}\int_{-\infty}^{t} i(\tau)\,d\tau \quad \Rightarrow$$

$$V_o(t) = -\frac{1}{R_i C_f}\int_{-\infty}^{t} V_i(\tau)\,d\tau$$

**Differentiator**




The differentiator is a special case of the high-pass filter (by removing the input resistor).

$$i = -C_i\frac{dV_i}{dt}; \quad i = \frac{V_o}{R_f} \quad \Rightarrow$$

$$V_o(t) = -R_f C_i \frac{dV_i(t)}{dt}$$

**Analog Computer**

The electronic analog computer is a device that uses reconfigurable electronic circuits to model and solve various computational problems. For example, an electronic circuit can be set up to characterize a specific differential equation; the solution of the differential equation is represented by a waveform in the circuit and can be observed on an oscilloscope. The analog computer was a very active field for many decades with its peak in the 1970's. Many electronic technologies including the op-amp have benefitted from the advancements in analog computers. However, since the 1980's, digital computers have significantly advanced in speed, accuracy, versatility, and numerical computational methods. Thus, almost all the tasks done by the traditional analog computers have been taken over by the digital computers. The integrator and the differentiator shown above are key components in an analog computer.