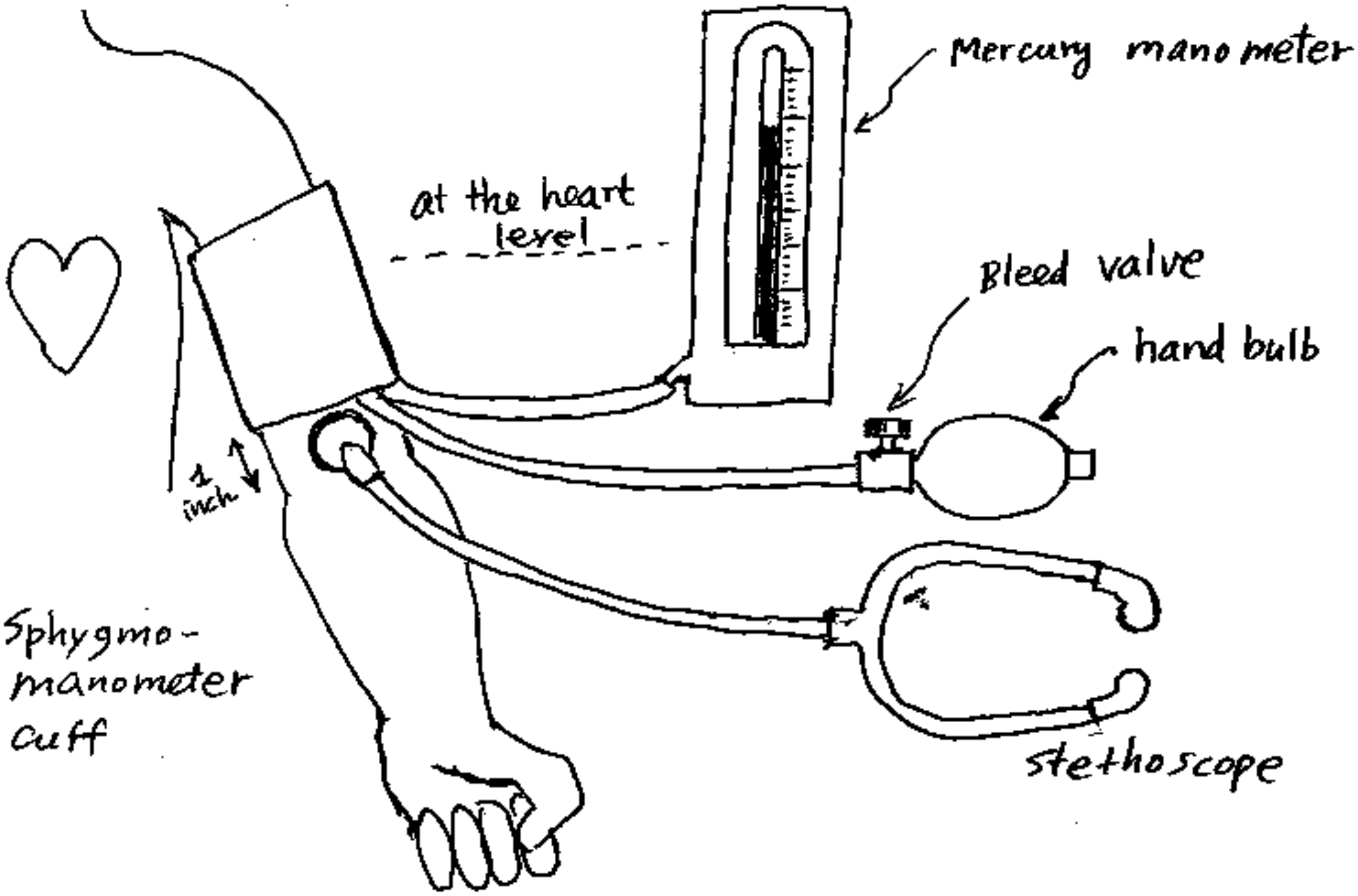
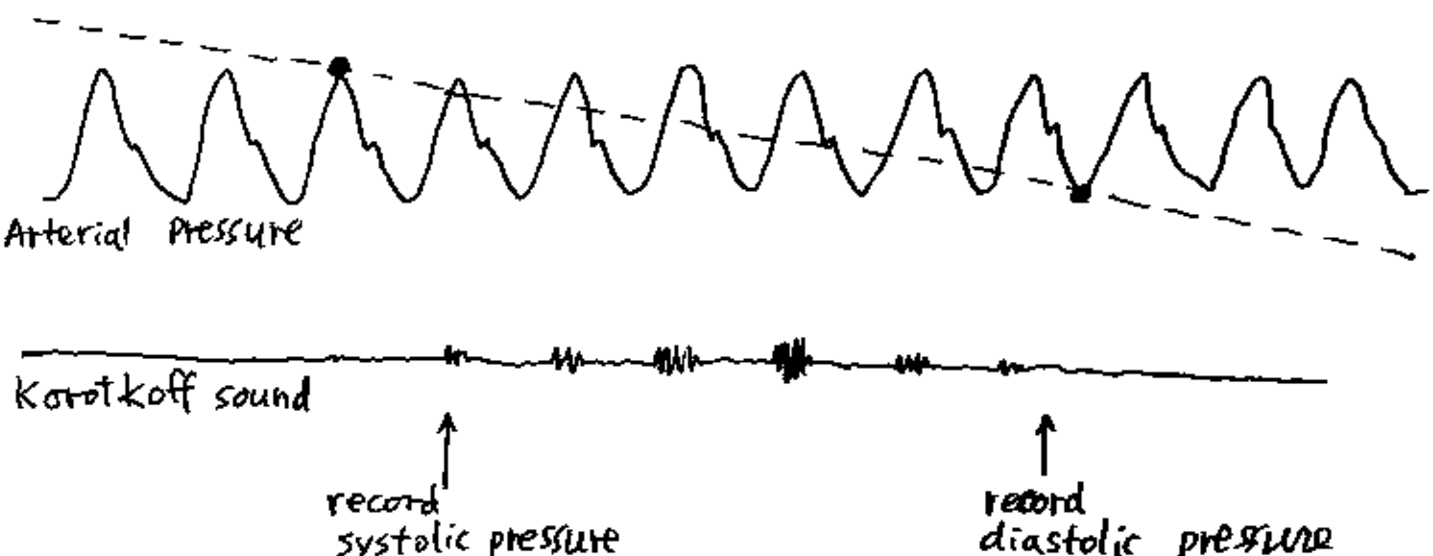
Indirect Measurement of Blood Pressure (see P.317 text)

- Palpatory method – to measure systolic pressure only
- Auscultatory method – to measure both systolic and diastolic pressures

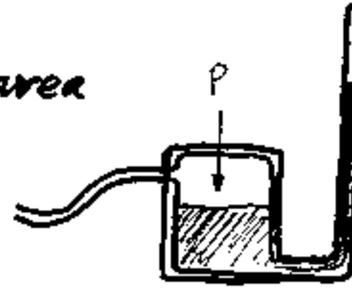




Blood Pressure Guidelines: $\left(\begin{array}{lll} \text{systolic} & \sim 120\ \text{mmHg} & < 140\ \text{mmHg} \\ \text{diastolic} & \sim 80\ \text{mmHg} & < 90\ \text{mmHg} \end{array} \right.$

Pressure Measurement

Invasive – Catheter & Pressure Transducer 1



- Unit $F = ma$; pressure = force / area

For cgs system : pressure : $dyne/cm^2$

$dyne = g \cdot cm/s^2$

$1\ mmHg = (13.6\ g/cm^3)(980\ cm/s^2)(\frac{1}{10}\ cm)$

density of mercury acceleration due to gravity at sea level

$\Rightarrow$

$$\boxed{1\ mmHg = 1333\ dyne/cm^2}$$

$$\boxed{\begin{aligned} 1\ atm &= 1.01 \times 10^5\ Pa\ (Pascal) = 1.01\ bar = 760\ mmHg \\ &= 760\ torr = 14.7\ psi \end{aligned}}$$

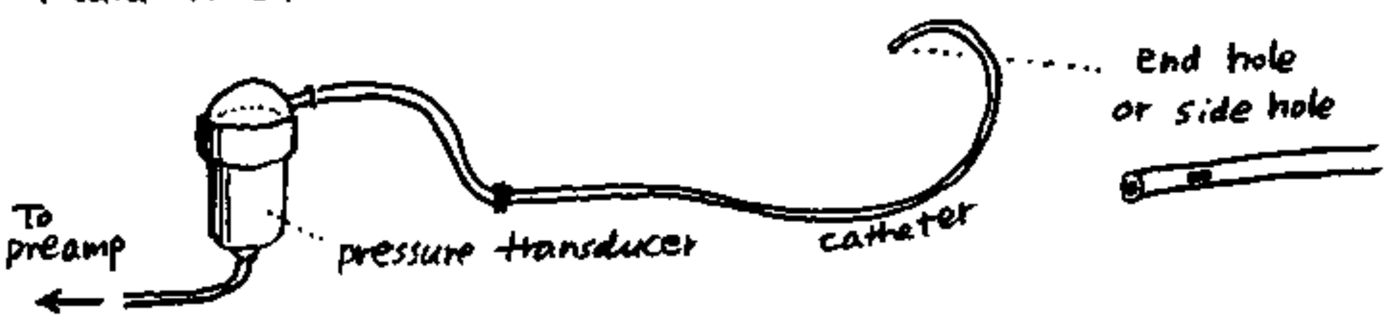
$$\boxed{1\ psi = 51.7\ mmHg}$$

human venous pressure ≈ 5 mmHg ≈ 1/10 psi
human arterial pressure ≈ 80-120 mmHg ≈ 2 psi
household water pressure ≈ 60 psi ≈ 3000 mmHg
car tire pressure ≈ 30 psi ≈ 1500 mmHg

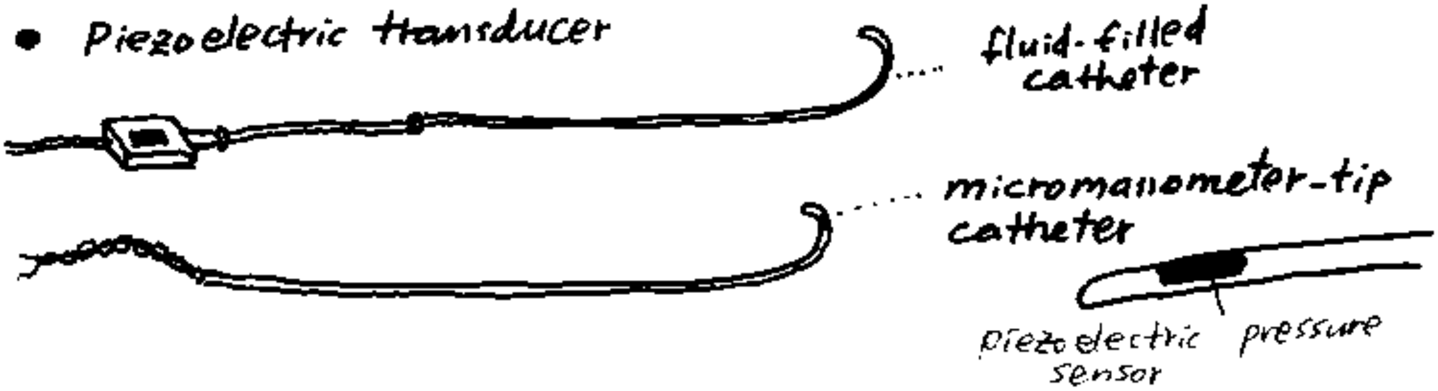
on top of atm

Direct Pressure Measurement (p.289)

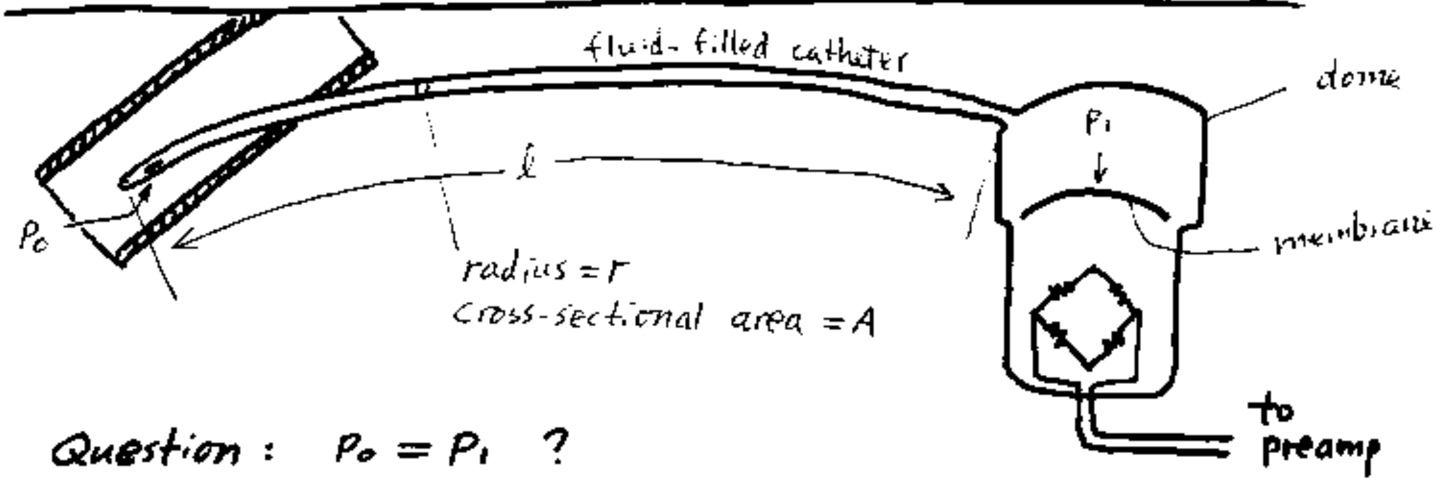
- Fluid-filled catheter and conventional pressure transducer



- Piezoelectric transducer



Conventional Fluid-filled catheter/transducer system



Question: $P_0 = P_1$?

Newton's 2nd law of motions: $\Sigma F = ma$

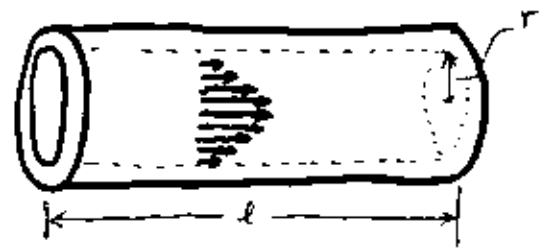
- Forces $= \pi r^2 (P_0 - P_1)$ — viscous force $= m\frac{dv}{dt}$

  v: velocity of fluid flows through the catheter (cm/s)

- Viscous force ?

  Assume laminar flow through circular pipe

  Poiseuille's Law: $\frac{\Delta P}{Q} = \frac{8\mu\ell}{\pi r^4}$



  ΔP: pressure gradient (mmHg)

  Q: volume flow rate (cm³/s)

  μ: viscosity (poise, dyne.s/cm²)

  usually 45%

  water or saline: $\mu = 0.01$ poise
  blood: $\mu = 0.025$ poise, depending on hematocrit

  Viscous force $= \Delta P(\pi r^2) = 8\mu\ell Q/r^2$; $Q = vA$

- $\pi r^2 (P_0 - P_1) - 8\mu\ell Q/r^2 = m\frac{dv}{dt}$

  $v = Q/A$, $dv/dt = (\frac{1}{A})\, dQ/dt = (1/\pi r^2)\, dQ/dt$

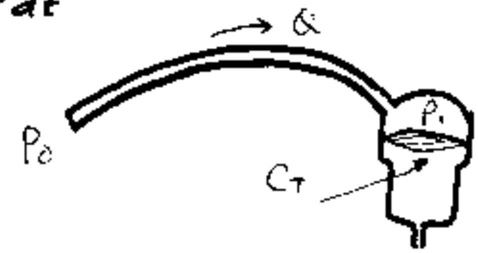
  $m = \pi r^2 \ell \rho$. ρ: density of the fluid

Reynolds number: a dimensionless parameter to indicate tendency of turbulent flows

$Re = (2Q\rho)/(\pi\mu r)$

$\begin{cases} Re < 200, & \text{laminar flows} \\ Re > 2000, & \text{turbulent flows} \end{cases}$

- $\pi r^2 (P_0 - P_1) - 8\mu \ell Q / r^2 = \ell \rho \, dQ/dt$
- The compliance (capacitance) in the system is usually dominated by the transducer compliance (the membrane in the dome)



$$C_T = \frac{dV}{dP_1} = \frac{1}{E}, \quad V\text{: volume } (cm^3),$$

factors specified

E: modulus of elasticity for the transducer

(capacitance = 1/elastance)

$$Q = dV/dt = C_T \frac{dP_1}{dt}$$

- $$\underbrace{\pi r^2 (P_0 - P_1) - \frac{8\mu\ell}{r^2}\left(C_T \frac{dP_1}{dt}\right) = \ell\rho\left(C_T \frac{d^2P_1}{dt^2}\right)}$$

2nd-order differential equation

after rearranging terms:

$$P_0 = \left(\frac{\ell\rho}{\pi r^2}\right) C_T \frac{d^2P_1}{dt^2} + \left(\frac{8\mu\ell}{\pi r^4}\right) C_T \frac{dP_1}{dt} + P_1$$

- $$\boxed{P_0 = L C_T \frac{d^2P_1}{dt} + R C_T \frac{dP_1}{dt} + P_1}$$

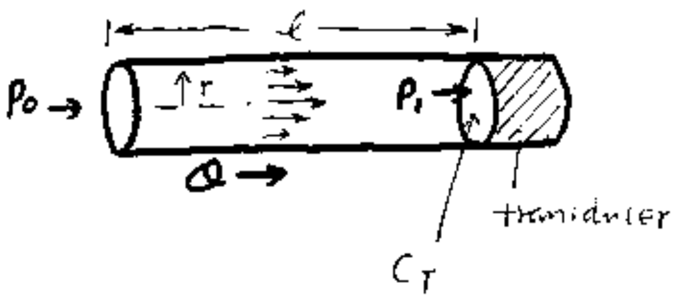
conversion factor from cgs pressure unit to mmHg

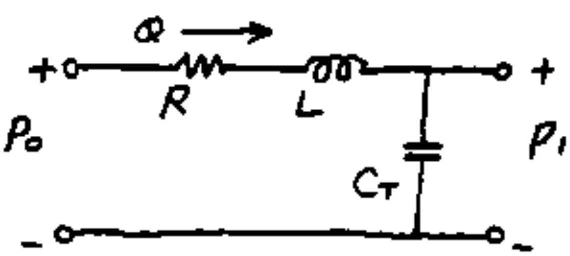
where $L = \frac{\ell\rho}{\pi r^2}$, inductance (inertance), $\div 1333$

$C_T = \frac{1}{E}$, capacitance (compliance), $\times 1333$

$R = \frac{8\mu\ell}{\pi r^4}$, resistance (viscous damping), $\div 1333$

- Electrical analog for the fluid mechanic system



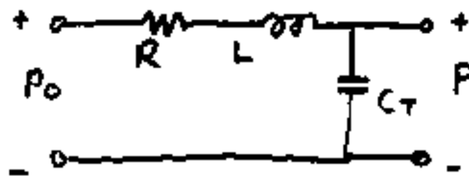


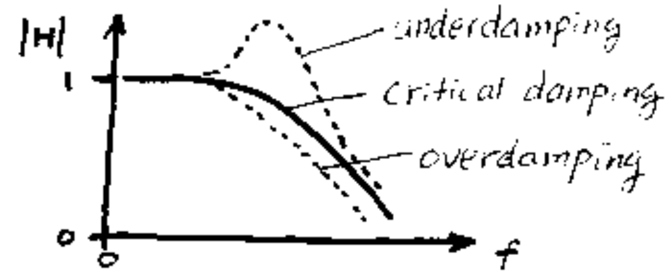
$$Q = C_T \, dP_1/dt$$

$$P_0 = RQ + L\,dQ/dt + P_1$$

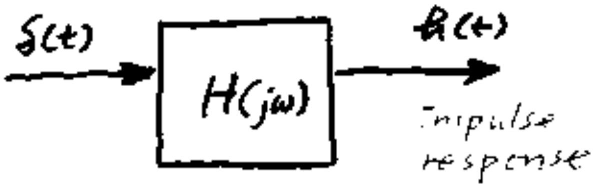
$$= RC_T \, dP_1/dt + LC_T \, d^2P_1/dt^2 + P_1$$

- 2nd-order system, low-pass filter

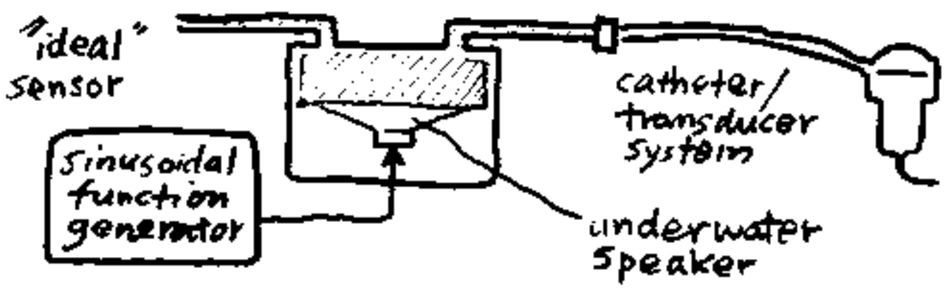




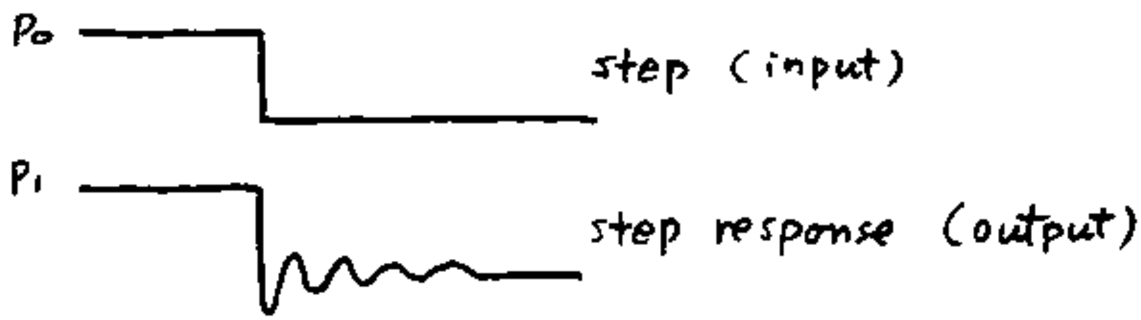
frequency response



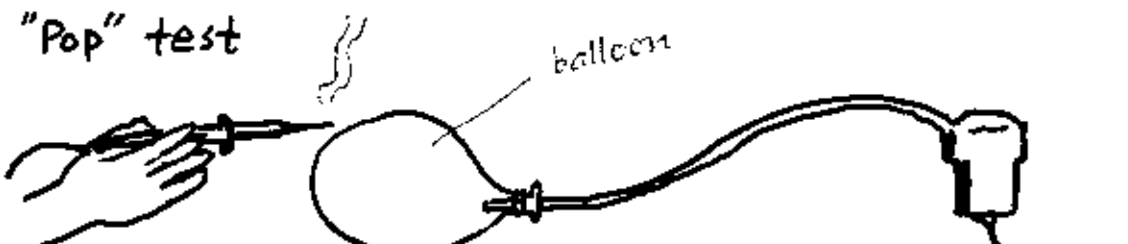
- Sinusoidal pressure-generator test system (p.304)

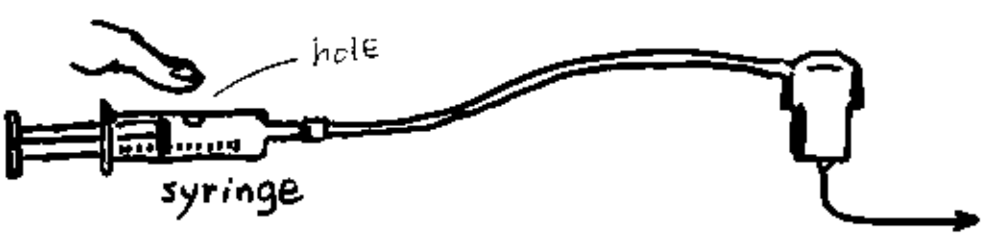


- Transient step response (p.303)



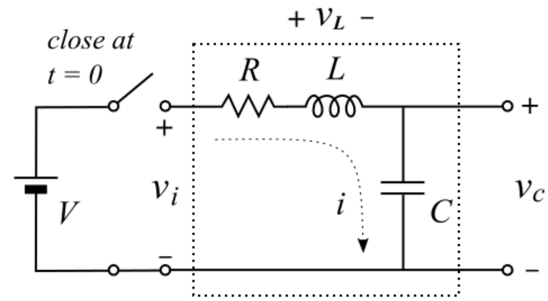
"Pop" test





Circuit Analysis Using Laplace Transform and Fourier Transform: RLC Low-Pass Filter

Ying Sun



The schematic on the right shows a 2nd-order RLC circuit. A constant voltage (*V*) is applied to the input of the circuit by closing the switch at $t = 0$. The output is the voltage across the capacitor *(C)*. The circuit can be represented as a linear time-invariant (LTI) system. The input is v_i . If a constant voltage is applied at $t = 0$, it is a step input. We further normalize the input voltage $V = 1$ such that it's unit step function. Thus, the input is the unit step function *u(t),* and the output is the step response *s(t)*. The LTI system can be completely characterized by its impulse response *h(t)*. The step response is the convolution between the input step function and the impulse response: $s(t) = u(t) \otimes h(t)$.

Circuit analysis using Laplace transform

The circuit analysis can be done by use of the Kirchhoff's voltage law and the properties of capacitor and inductor:

$$i = C\frac{d\,v_c}{dt} \text{ , and } v_L = L\frac{d\,i}{dt} \text{ --- (1)}$$

$$v_i = Ri+v_L+v_c = R\,i+L\frac{d\,i}{dt}+v_c \text{ -- (2)}$$

By substituting (1) into (2), we have:

$$v_i = LC\frac{d^2 v_c}{d\,t^2}+RC\frac{d\,v_c}{dt}+v_c \text{ -- (3)}$$

Although a closed form solution can be obtained by solving the above 2nd-order differential equation, we will take the frequency-domain approach. Taking LT on both side, we have:

$$V_i(s) = LC\,s^2\,V_c(s)+RC\,s\,V_c(s)+V_c(s) \text{ -- (4)}$$

The transfer function is given by:

$$H(s) = \frac{V_c(s)}{V_i(s)} = \frac{1}{LC\,s^2+RC\,s+1} \text{ -- (5)}$$

We now use a different set of parameters:

$$H(s) = \frac{\omega_n^2}{s^2+2\alpha\,s+\omega_n^2} \text{ --- (6)}$$

where $\omega_n = \dfrac{1}{\sqrt{LC}}$, natural frequency

$\alpha = \dfrac{R}{2L}$, damping factor

We further define damped frequency ω_d:

$$\omega_d = \sqrt{\omega_n^2 - \alpha^2}\ ; \quad \text{or} \quad \omega_n^2 = \omega_d^2 + \alpha^2\ ; \quad \text{or} \quad \omega_n = \sqrt{\omega_d^2 + \alpha^2}$$

To obtain the impulse response, the transfer function is further extended to:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\alpha s + \omega_n^2} = \frac{\omega_n^2}{(s+\alpha+\sqrt{\alpha^2-\omega_n^2})(s+\alpha-\sqrt{\alpha^2-\omega_n^2})} = \frac{\omega_n^2}{(s+\alpha+j\omega_d)(s+\alpha-j\omega_d)} =$$

$$\frac{\omega_d^2+\alpha^2}{s^2+2\alpha s+\alpha^2+\omega_d^2} = (\omega_d + \frac{\alpha^2}{\omega_d})(\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}) \quad \text{-- (7)}$$

The system has two poles at: $-\alpha + j\omega_d$ and $-\alpha - j\omega_d$.

Taking the ILT, the impulse response is:

$$h(t) = (\omega_d + \frac{\alpha^2}{\omega_d}) e^{-\alpha t} \sin \omega_d t \quad \text{-- (8)}$$

Next, we want to get a closed form solution for the step response. This will be accomplished by extending *H(s)* to *H(s)/s*, or *S(s)*, which is the LT of the step response.

$$S(s) = \frac{H(s)}{s} = (\omega_d + \frac{\alpha^2}{\omega_d})(\frac{1}{s})(\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}) = \frac{a}{s} + \frac{bs+c}{(s+\alpha)^2+\omega_d^2}$$

$$\omega_d^2 + \alpha^2 = a(s+\alpha)^2 + a\omega_d^2 + bs^2 + cs = (a+b)s^2 + (2a\alpha + c)s + a(\omega_d^2+\alpha^2)$$

We have $a+b=0$, $2a\alpha+c=0$, and $\omega_d^2+\alpha^2 = a(\omega_d^2+\alpha^2) \Rightarrow a = 1, b = -1$, and $c = -2\alpha$.

$$S(s) = \frac{1}{s} - \frac{s+2\alpha}{(s+\alpha)^2+\omega_d^2} = \frac{1}{s} - \frac{s+\alpha}{(s+\alpha)^2+\omega_d^2} - (\frac{\alpha}{\omega}) \frac{\omega_d}{(s+\alpha)^2+\omega_d^2} \quad \text{------------------------- (9)}$$

Using the LT Table, we obtain the step response *s(t)*:

$$s(t) = 1 - e^{-\alpha t}[\cos \omega_d t + (\frac{\alpha}{\omega_d}) \sin \omega_d t] \quad \text{-- (10)}$$

Next, we want to combine cosine and sine into one term with a phase angle. We further define the damping ratio η, $\eta = \frac{\alpha}{\omega_n}$.

$$\frac{\alpha}{\omega_d} = \frac{\alpha}{\sqrt{\omega_n^2 - \alpha^2}} = \frac{1}{\sqrt{\omega_n^2/\alpha^2 - 1}} = \frac{1}{\sqrt{1/\eta^2 - 1}} = \frac{\eta}{\sqrt{1-\eta^2}} \quad \text{--------------------------------- (11)}$$

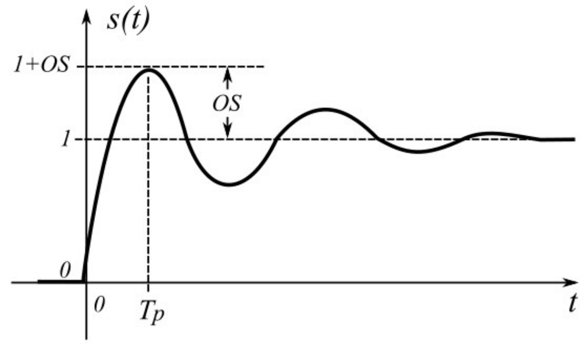
$$s(t) = 1 - e^{-\alpha t}[\cos \omega_d t + (\frac{\eta}{\sqrt{1-\eta^2}}) \sin \omega_d t] = 1 - \frac{e^{-\alpha t}}{\sqrt{1-\eta^2}}[\sqrt{1-\eta^2} \cos \omega_d t + \eta \sin \omega_d t] =$$

$$1 - \frac{e^{-\alpha t}}{\sqrt{1-\eta^2}}[\sin \phi \cos \omega_d t + \cos \phi \sin \omega_d t] = 1 - \frac{e^{-\alpha t}}{\sqrt{1-\eta^2}} \sin(\omega_d t + \phi) \quad \text{--------------------- (12)}$$

where $\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{\sqrt{1-\eta^2}}{\eta}$, $\phi = \tan^{-1} \frac{\sqrt{1-\eta^2}}{\eta}$.

From the previous circuit course, such as ELE 212, you might have learned what the damping ratio means: a) underdamping $(\eta < 1)$; b) critical damping $(\eta = 1)$; and c) overdamping $(\eta > 1)$.

Assume the underdamping situation, *s(t)* is shown on the right. The 2nd-order system requires two parameters to define, such as the damped frequency ω_d and the damping factor α . These two parameters can be obtained from the *s(t)* curve by making two measurements. A prominent feature point is the first peak after the onset. We measure the time to peak T_p and the amount of overshoot (OS). This point occurs when the derivative of the curve is 0.



$$\frac{d\,s(t)}{dt} = 0$$

$$\frac{d\,s(t)}{dt} = h(t) = (\omega_d + \frac{\alpha^2}{\omega_d})e^{-\alpha t}\sin\omega_d t = 0 \text{ . --- (13)}$$

The peaks and valleys occur when $\omega_d t = 0, \pi, 2\pi, 3\,\pi, 4\pi, \ldots$. The first peak occurs when $\omega_d t = \pi$.

Thus, $T_p = \frac{\pi}{\omega_d}$, or $\omega_d = \frac{\pi}{T_p}$ --- (14)

At the first peak T_p ,

$$s(T_p) = 1 - \frac{e^{-\alpha T_p}}{\sqrt{1-\eta^2}}\sin(\omega_d T_p + \tan^{-1}\frac{\sqrt{1-\eta^2}}{\eta}) = 1 - \frac{e^{-\alpha T_p}}{\sqrt{1-\eta^2}}\sin(\pi + \tan^{-1}\frac{\sqrt{1-\eta^2}}{\eta}) =$$

$$1 + \frac{e^{-\alpha T_p}}{\sqrt{1-\eta^2}}\sin(\tan^{-1}\frac{\sqrt{1-\eta^2}}{\eta}) = 1 + \frac{e^{-\alpha T_p}}{\sqrt{1-\eta^2}}\sqrt{1-\eta^2} = 1 + e^{-\alpha T_p} = 1 + OS \text{ . ------------ (15)}$$

Thus, $e^{-\alpha T_p} = OS$, or $\alpha = -\frac{\ln OS}{T_p}$. -- (16)

Frequency response

Let's assign values to the circuit components so that we can plot the frequency response as an example. Let's assume:

$$R = 40\,K\Omega \text{ ; } L = 1 \text{ } mH \text{ ; } C = 0.1\,\mu F \text{ --- (17)}$$

Based on these values, we have:

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\,mH \times 0.1\,\mu F}} = 100000 \text{ radians/s; and}$$

$$\alpha = \frac{R}{2\mathrm{L}} = \frac{40\,K\Omega}{2 \times 1\,mH} = 20000$$

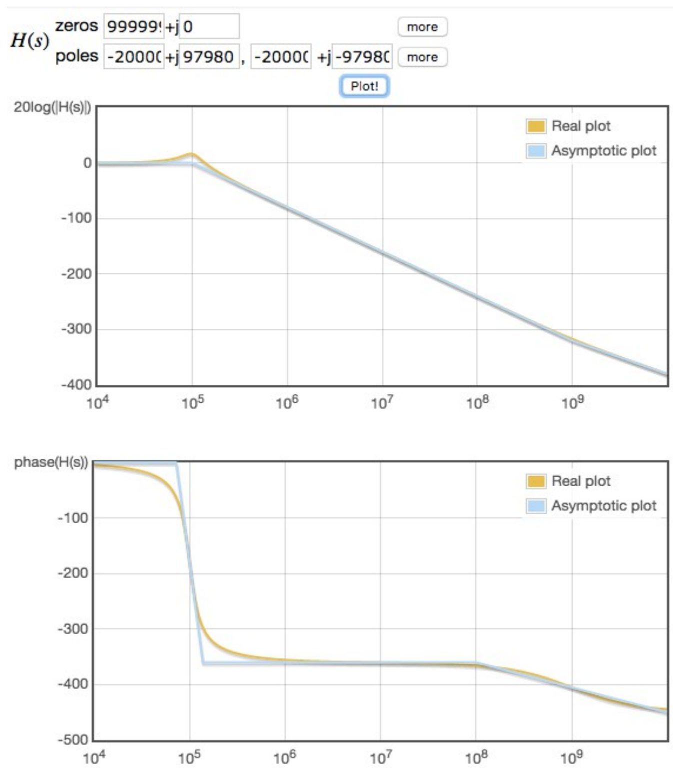
$$\omega_d = \sqrt{\omega_n^2 - \alpha^2} = \sqrt{100000^2 - 20000^2} = 97980 \text{ radians/s.}$$

$$\eta = \frac{\alpha}{\omega_n} = \frac{20000}{100000} = 0.2 \text{ ; } \phi = \tan^{-1}\frac{\sqrt{1-\eta^2}}{\eta} = \tan^{-1}\frac{\sqrt{1-0.2}}{0.2} = 1.35 \text{ radians.}$$

The transfer function is $H(s) = (\omega_d + \frac{\alpha^2}{\omega_d})(\frac{\omega_d}{(s+\alpha)^2+\omega_d^2})$. The system has the poles at $-\alpha \pm j\omega_d$, or $-20000 \pm 97980\, j$. The zeros are at infinity.

Bode plot

Enter the poles of $-20000 \pm 97980\, j$ at the online Bode plot generator <http://http://www.onmyphd.com/?p=bode.plot>. We obtain the following:



Step response

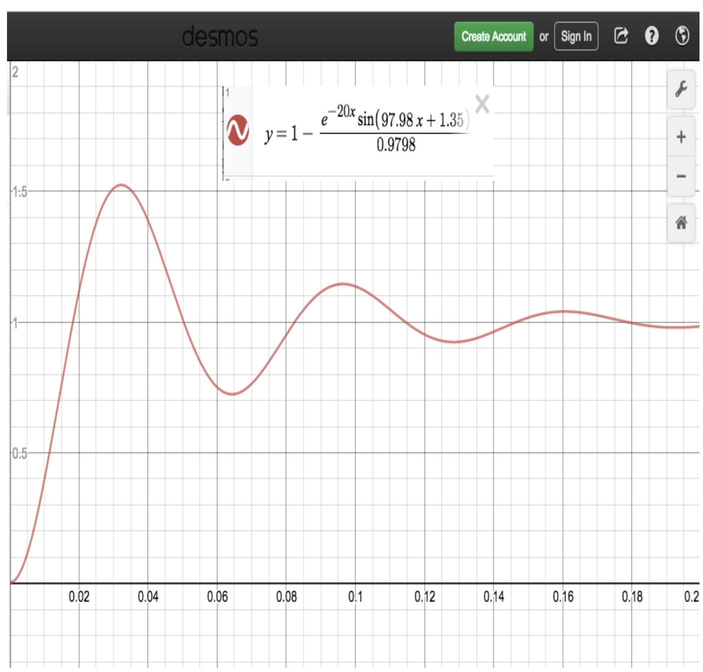
$$s(t) = 1 - \frac{e^{-\alpha t}}{\sqrt{1-\eta^2}} \sin(\omega_d t + \phi) =$$

$$1 - \frac{e^{-20000t}}{0.9798} \sin(97980\, t + 1.35) =$$

Due to a range issue with the graphing calculator, we change the time unit from s to ms:

$$s(t) = 1 - \frac{e^{-20\, t}}{0.9798} \sin(97.98\, t + 1.35)$$

From the graph, we see that the peak occurs roughly at: $T_p = 0.0323$ ms, and OS = 0.52.



Using the formula derived previously:

$$\omega_d = \frac{\pi}{T_p} = \frac{3.14159}{0.0000323} = 97263 ,$$

as compare to $\omega_d = 97980$.

$$\alpha = -\frac{\ln OS}{T_p} = -\frac{\ln 0.52}{0.0000323} = 20245 ,$$

as compare to $\alpha = 20000$.