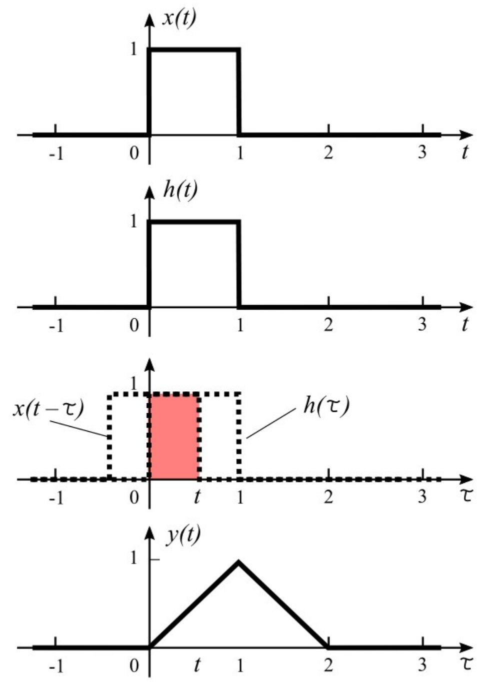
ELE 314 Linear Systems and Signals – HomeWork #2 Convolution (4%) *Ying Sun*

Name: ______________________________ Due date: ______________________



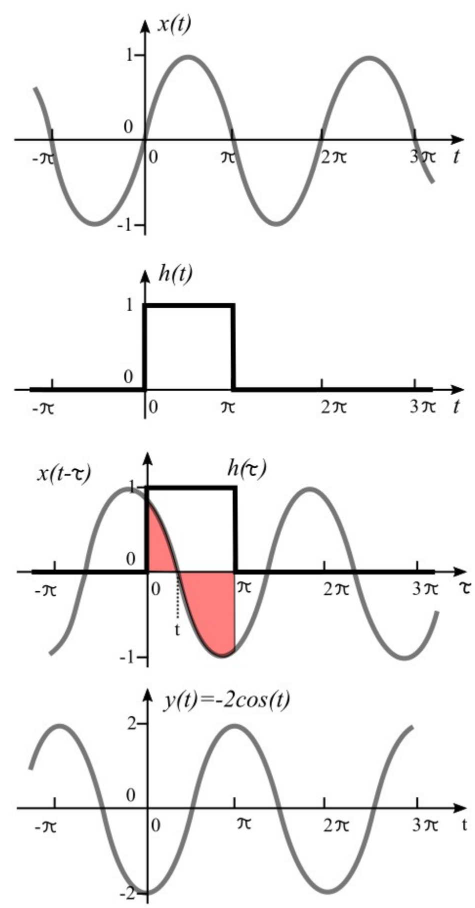
Example 1:

For a LTI system, the impulse response *h(t)* and input *x(t)* are $h(t)=x(t)=u(t)-u(t-1)$. Plot and obtain a mathematical expression for the output *y(t)*.

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau =$$

$$\begin{cases} 0, & t<0 \\ \int_0^t d\tau & 0\le t<1 \\ \int_{t-1}^1 d\tau & 1\le t<2 \\ 0, & t\ge 2 \end{cases} = \begin{cases} 0, & t<0 \\ [\tau]_0^t, & 0\le t<1 \\ [\tau]_{t-1}^1, & 1\le t<2 \\ 0, & t\ge 2 \end{cases}$$

$$y(t) = \begin{cases} 0, & t<0 \\ t, & 0\le t<1 \\ 2-t, & 1\le t<2 \\ 0, & t\ge 2 \end{cases}$$



Example 2:

For a LTI system, the impulse response is a square pulse between 0 and π $h(t)=u(t)-u(t-\pi)$, and the input is a sine wave $x(t)=\sin(t)$. Plot and obtain a mathematical expression for the output *y(t)*.

y(t) should also be a periodical signal with a period of 2 π.

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau =$$

$$\int_{-\infty}^{\infty} \sin(t-\tau)[u(\tau)-u(\tau-\pi)]d\tau =$$

$$\int_0^{\pi} \sin(t-\tau)d\tau = \left[-\cos(t-\tau)(-1)\right]_0^{\pi} =$$

$$\cos(t-\pi)-\cos t = -2\cos t$$

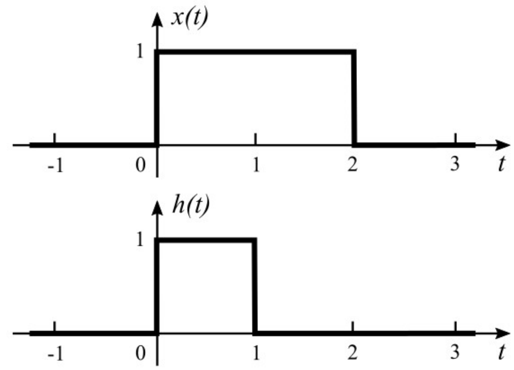
Note: from Trigonometry Table:

$$\cos(\alpha-\beta) = \cos\alpha \ \cos\beta + \sin\alpha \ \sin\beta$$

$$\cos(t-\pi) = \cos t \ \cos\pi + \sin t \ \sin\pi = -\cos t$$

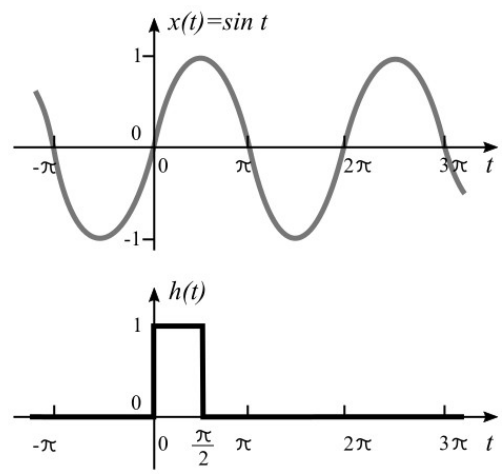
Homework 2a: **(1%)**

For a LTI system, the impulse response is a square pulse between 0 and 1 $h(t)=u(t)-u(t-1)$. The input *x(t)* a square pulse between 0 and 2 $x(t)=u(t)-u(t-2)$. Plot and obtain a mathematical expression for the output *y(t)*.



Homework 2b: **(1%)**

For a LTI system, the impulse response is a square pulse between 0 and π/2 $h(t)=u(t)-u(t-\pi/2)$, and the input is a sine wave $x(t)=\sin(t)$. Plot and obtain a mathematical expression for the output *y(t)*.



Homework 2c: **(2%)**

For a LTI system, the impulse response is a square pulse between 0 and 1 $h(t)=u(t)-u(t-1)$. The input *x(t)* a triangular pulse between 0 and 2, given below. Plot and obtain a mathematical expression for the output *y(t)*.

$$x(t)=\begin{cases} 0, & t<0 \\ t, & 0\le t<1 \\ 2-t, & 1\le t<2 \\ 0, & t\ge 2 \end{cases}$$