ELE 314 Linear Systems and Signals – HomeWork #5 (5%) *Ying Sun*

Name: ______________________________ Due date: ____________________

ZT Properties, Signal Flow Graph, Root Locus Analysis, and State Space Analysis

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ZT properties

Geometric Series:

$$a_0 \sum_{n=0}^{N-1} r^n = a_0 \frac{1-r^N}{1-r}$$

for $r \neq 0$

Example 1: Show that

$$x[n]=u[n] \quad \Leftarrow z \Rightarrow \quad X(z)=\frac{1}{1-z^{-1}}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} u[n] z^{-n} = \sum_{n=0}^{\infty} z^{-n}$$

This is a geometric series with a ratio of z^{-1}, which converges for $|z|>1$.

$$X(z) = \frac{1-z^{-\infty}}{1-z^{-1}} = \frac{1}{1-z^{-1}}$$

Example 2: Prove the *Differentiation in z-domain* property (#8):

$$n \; x[n] \quad \Leftarrow z \Rightarrow \quad -z\frac{dX(z)}{dz} \qquad \text{ROC: } R_X \text{ except at } z=0$$

Take the derivative of *X(z)* with respect to *z*:

$$\frac{dX(z)}{dz} = \frac{d}{dz}\sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} x[n] {-n} z^{-n-1} = -z^{-1}\sum_{n=-\infty}^{\infty} n\, x[n] z^{-n}$$

$$-z\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} n\, x[n] z^{-n}$$

Thus, $\quad n \; x[n] \quad \Leftarrow z \Rightarrow \quad -z\frac{dX(z)}{dz}$

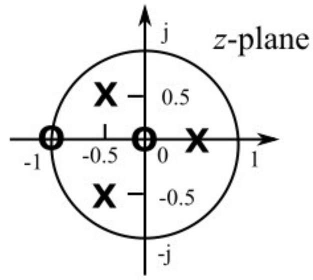
Homework 5a: **(1%)** Prove the *Summation in time-domain* property (#8):

$$\sum_{k=-\infty}^{n} x[k] \quad \Leftarrow z \Rightarrow \quad \frac{1}{1-z^{-1}} X(z) \qquad \text{ROC: at least } R_X \cap \{|z|>1\}$$

(Hint: Use the result from Example 1. Arrange the time-domain expression as the convolution between $x[n]$ and $u[n]$.)

Signal Flow Graph

Example 1: For the pole-zero plot shown on the right, find *H(z)* and realize the filter with Direct Form II.



$$H(z) = \frac{(z-0)(z+1)}{(z-0.5)(z+0.5+0.5j)(z+0.5-0.5j)} =$$

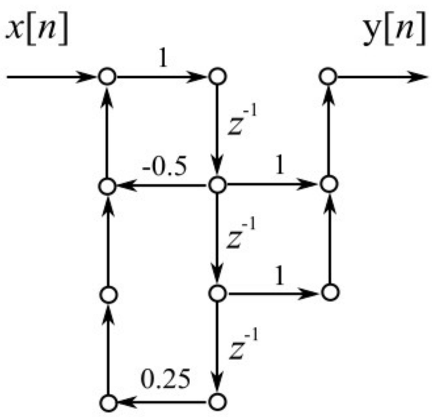
$$\frac{z^2+z}{(z-0.5)(z^2+z+0.5)} = \frac{z^2+z}{z^3+0.5z^2-0.25} = \frac{z^{-1}+z^{-2}}{1+0.5z^{-1}-0.25z^{-3}} = \frac{z^{-1}+z^{-2}}{1-(-0.5z^{-1}+0.25z^{-3})},$$

which fits the form of

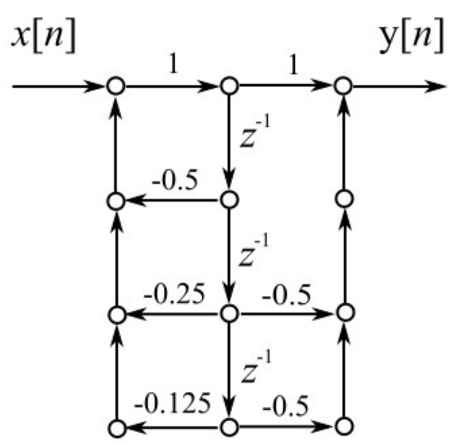
$$H(z)=\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}}, \text{ with}$$

$a_1=-0.5$	$b_0=0$
$a_2=0$	$b_1=1$
$a_3=0.25$	$b_2=1$

Thus, the Direct Form 2 realization is as shown.



Homework 5b: **(1%)** What is the filter equation for the Direct Form 2 realization shown on the right?



Root Locus Analysis

Homework 5c: **(1%)** Plot the root locus for $1+K\,G(s)H(s)=0$, where

the loop gain is given by $G(s)H(s)=\frac{s+1}{s^2+s+1.25}$.

You can use RootLocs, which is a freeware downloadable from <<http://www.coppice.myzen.co.uk/RootLocs_Site/RootLocs.html>.

State Space Analysis

Homework 5d: **(2%)** An LTI system is characterized by a second-order differential equation:

$$\frac{d^2 y(t)}{dt^2}+2\frac{d\,y(t)}{dt}+y(t) = x(t)$$

(i) Find the transfer function *H(s)* by use of the traditional method.

(ii) Derive the state-space representation.

(iii) Find *H(s)* by use of the state-space method.