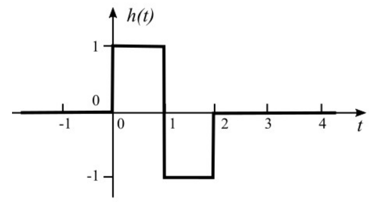
ELE314 Linear Systems and Signals Exam #1a Summer 2017 Name: **SOLUTION**

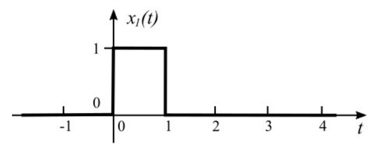
Open book/notes (10 questions, 10 points each)

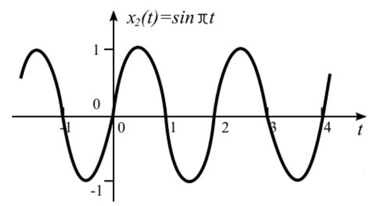
1. (**C**) Which of the following is a nonlinear time-varying system? (A) $y(t)=\sqrt{x(t)}$, (B) $y(t)=\sin[x(t)]$, (C) $y(t)=\sin[x(t)]+t$, (D) $y(t)=\sin[x(t)]+\cos[x(t)]$, (E) none of the above.

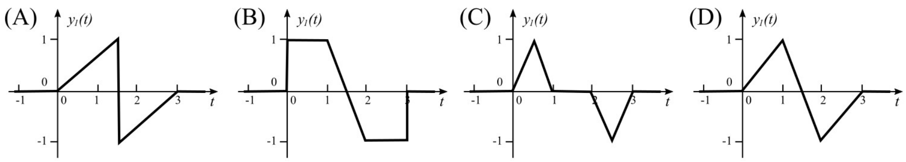
   (A), (B) and (D) are nonlinear time-invariant systems.

2. (**D**) The impulse response of a LTI system *h(t)* is shown on the right. The input $x_1(t)$ is a square pulse, which is 1 for $0\le t\le 1$ and 0, otherwise. The output $y_1(t)$ = ?





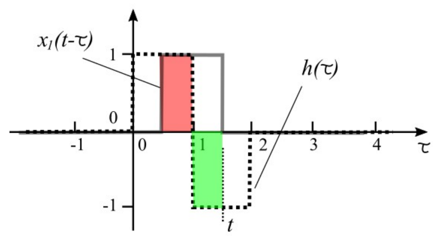




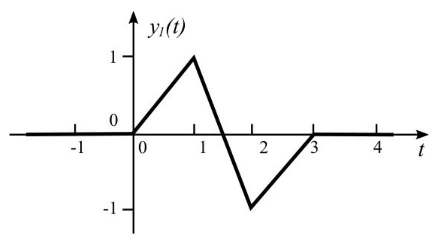
(E) none of the above.

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau =$$

$$\begin{cases} 0, & 0\le t,\ t>3 \\ \int_0^t d\tau, & 0<t\le 1 \\ \int_{t-1}^{1} d\tau - \int_1^t d\tau, & 1<t\le 2 \\ -\int_{t-1}^{2} d\tau, & 2<t\le 3 \end{cases} =$$



$$\begin{cases} 0, & 0\le t,\ t>3 \\ t, & 0<t\le 1 \\ 3-2\mathrm{t}, & 1<t\le 2 \\ t-3, & 2<t\le 3 \end{cases} =$$

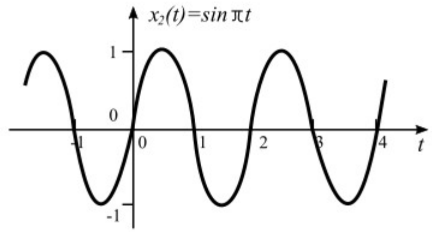


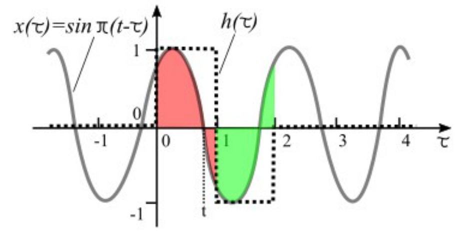
3. (**C**) For the above problem, what is the Laplace transform of the square pulse $x_1(t)$?

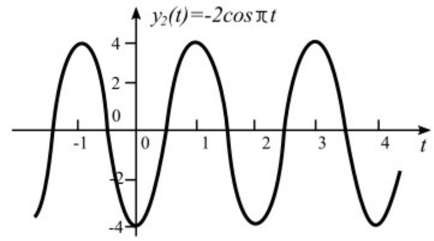
   (A) $s(1+e^{-s})$, (B), $s(1-e^{-s})$ (C) $\frac{1-e^{-s}}{s}$, (D) $\frac{1+e^{-s}}{s}$, (E) none of the above.

$$H(s) = \int_0^{\infty} x_1(t)e^{-st}dt = \int_0^1 e^{-st}dt = \left[\frac{-e^{-st}}{s}\right]_0^1 = \frac{-e^{-st}}{s}+\frac{1}{s} = \frac{1-e^{-st}}{s}$$

4. (**E**) With the same *h(t)* of the above problem, the input is now is a sine wave $x_2(t)=\sin \pi t$. The output $y_2(t)$ = ?
(A) $-2\cos \pi t$, (B) $-\cos \pi t+\sin \pi t$,
(C) $\cos 2\pi t$, (D) $2\sin \pi t$, (E) none of the above.







$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau =$$

$$\int_0^1 \sin \pi(t-\tau)d\tau - \int_1^2 \sin \pi(t-\tau)d\tau =$$

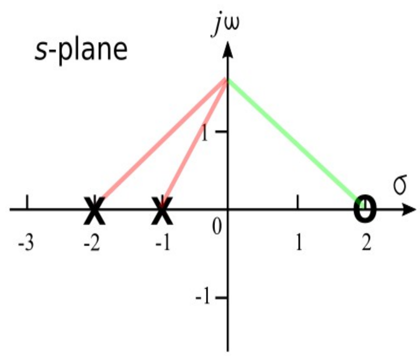
$$[\cos \pi(t-\tau)]_0^1 - [\cos \pi(t-\tau)]_1^2 =$$

$$\cos \pi(t-1) - \cos \pi t - \cos \pi(t-2) + \cos \pi(t-1) =$$

$$-\cos \pi t - \cos \pi t - \cos \pi t - \cos \pi t =$$

$$-4\cos \pi t$$

5. (**B**) The pole-zero plot of a LTI system is shown on the right. Assume that the magnitude of the DC gain is unity. What is the transfer function *H(s)*? (A) $\dfrac{s+2}{(s-1)(s-2)}$,
(B) $\dfrac{s-2}{(s+1)(s+2)}$, (C) $\dfrac{s(s+2)}{(s+1)(s+2)}$,
(D) $\dfrac{s-2}{(s-1)(s-2)}$, (E) none of the above.



Poles: -1, -2

Zeros: 2, ∞

$$H(s) = \frac{s-2}{(s-(-1))(s-(-2))} = \frac{s-2}{(s+1)(s+2)}$$

For the DC gain, set *s* = 0. We have *H(0)* = -1, which has a magnitude of unity.

6. (**A**) For the above problem, the red and green lines help to visualize the magnitude of the frequency response. What kind of filter is this? (A) low-pass, (B) high-pass, (C) band-pass, (D) band-stop, (E) none of the above.

The magnitude of the Fourier transfer is the product of the lengths of the vectors to the zeros over the product of the lengths of the vectors to the poles. Because there are two red lines and one green line, the magnitude decreases with increasing frequency. It is a low-pass filter.

7. (**D**) For the above problem, what is the impulse response *h(t)*? (A) $(3e^{-2t}+7e^{-t})u(t)$, (B) $(2e^{-2t}-5e^{-t})u(t)$, (C) $(e^{-2t}+3e^{-t})u(t)$, (D) $(4e^{-2t}-3e^{-t})u(t)$, (E) none of the above.

Do partial-fraction expansion: $\dfrac{s-2}{(s+1)(s+2)} = \dfrac{a}{s+1}+\dfrac{b}{s+2} = \dfrac{a(s+2)+b(s+1)}{(s+1)(s+2)}$

We have $s-2 = (a+b)s+(2a+b)$, or $a+b=1$ and $2a+b=-2$. Solving the simultaneous equations, we have $a=-3$ and $b=4$.

$H(s) = \dfrac{-3}{s+1}+\dfrac{4}{s+2}$. Take the ILT, we have $h(t) = (4e^{-2t}-3e^{-t})u(t)$

8. (**C**) The transfer function of a LTI system is $H(s)=\dfrac{2}{s^2+2s+5}$, what is its impulse response *h(t)*? (A) $(e^{-2t}\sin t)u(t)$, (B) $(e^{-2t}\cos t)u(t)$, (C) $(e^{-t}\sin 2t)u(t)$, (D) $(e^{-t}\cos 2t)u(t)$, (E) none of the above.

$H(s)=\dfrac{2}{s^2+2s+5}=\dfrac{2}{(s+1)^2+2^2}$. Using the LT table, we have $h(t)=(e^{-t}\sin 2t)u(t)$

9. (**D**) The factored form of *H(s)* is changed to the partial-fraction form according to: $\dfrac{s}{(s+3)(s+2)} = \dfrac{a}{s+3}+\dfrac{b}{s+2}$. a = ? (A) -2, (B) 2, (C) -3, (D) 3, (E) none of the above.

$\dfrac{s}{(s+3)(s+2)} = \dfrac{a}{s+3}+\dfrac{b}{s+2} = \dfrac{a(s+2)+b(s+3)}{(s+3)(s+2)}$. we have

$s = (a+b)s+(2a+3b)$, or $a+b=1$ and $2a+b=0$. Solving the simultaneous equations, we have $a=3$ and $b=-2$.

10. (**A**) For the above problem, b = ? (A) -2, (B) 2, (C) -3, (D) 3, (E) none of the above.