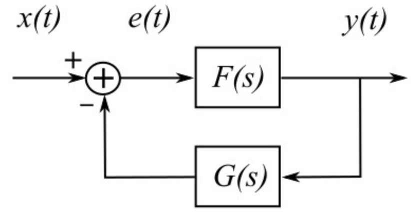
ELE314 Linear Systems and Signals Exam #2 Summer 2017 Name: **Solution**

Open book/notes (10 questions, 10 points each)

1. (**A**) The figure on the right shows a negative feedback configuration of two continuous-time LTI systems with the individual transfer functions: $F(s)=\frac{1}{s+1}$, and $G(s)=\frac{1}{s+1}$.

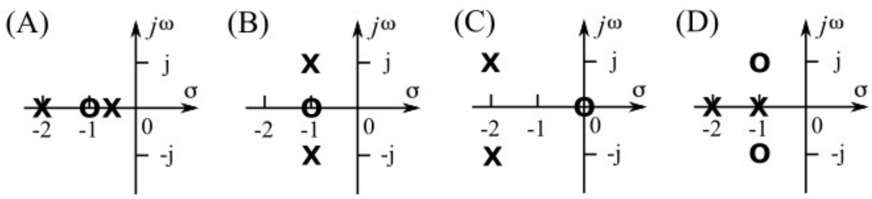


What is the overall transfer function *H(s)*? (A) $\frac{s+1}{s^2+2s+2}$,

(B) $\frac{s}{s^2+2s+1}$, (C) $\frac{1}{s^2+2s+2}$, (D) $\frac{s+1}{s^2+2s+1}$, (E) none of the above.

$$H = \frac{F}{1+GF} = \frac{1/(s+1)}{1+\frac{1}{s+1}\frac{1}{s+1}} = \frac{s+1}{(s+1)^2+1} = \frac{s+1}{s^2+2s+2}$$

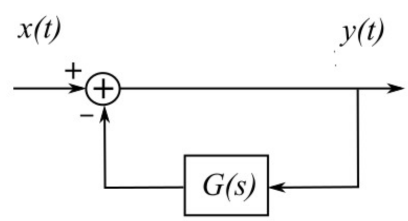
2. (**B**) For the above problem, what is the pole-zero plot of the overall system?



(E) none of the above.

$$H = \frac{s+1}{(s+1)^2+1} = \frac{s+1}{(s+1+j)(s+1-j)}$$ Zero: -1; poles: $-1\pm j$

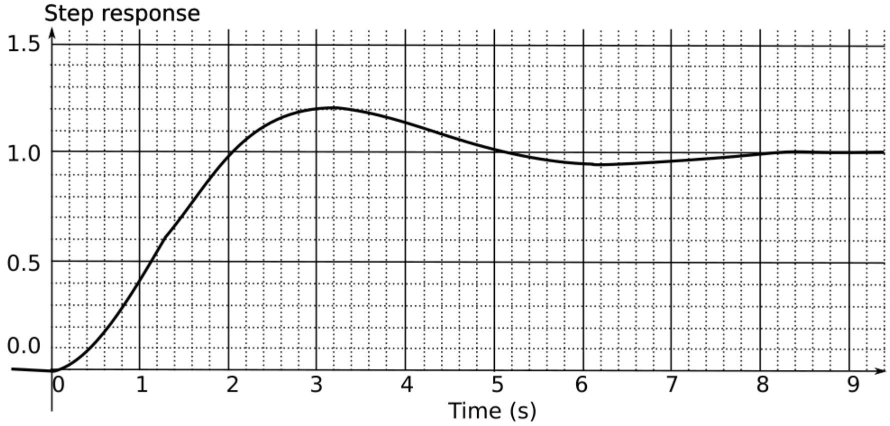
3. (**C**) The figure on the right shows a negative feedback system with *F(s)* = 1. If we want the overall transfer function *H(s)* to be $\frac{s+1}{s+2}$, what should *G(s)* be? (A) $\frac{s}{s+1}$, (B) $\frac{s}{s+2}$,

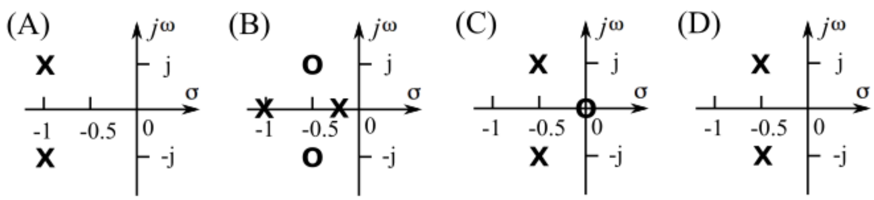


(C) $\frac{1}{s+1}$, (D) $\frac{1}{s+2}$, (E) none of the above.

$$H = \frac{F}{1+GF} = \frac{1}{1+G} = \frac{s+1}{s+2} \Rightarrow G = \frac{s+2}{s+1}-1 = \frac{1}{s+1}$$

4. (**D**) The step response of a 2nd-order continuous-time LTI system is shown below. Which of the following is its pole-zero plot?





(E) none of the above.

From the graph, $T_p = 3.14\,s$; $OS = 0.21$

$$\omega_d = \frac{\pi}{T_p} = 1\,; \quad \alpha = -\frac{\ln 0.21}{T_p} = \frac{1.56}{3.14} = 0.5$$

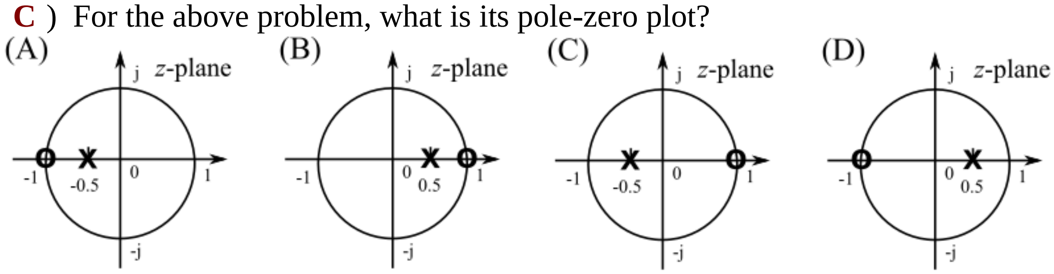
poles at: $-\alpha \pm j\omega_d = -0.5 \pm j$; zeros at ∞.

5. (**A**) The filter equation for a digital filter is given by $y[n] = \frac{1}{4}x[n] - \frac{1}{4}x[n-1] - \frac{1}{2}y[n-1]$.

What is its transfer function *H(z)*? (A) $\frac{z-1}{4z+2}$, (B) $\frac{z+1}{4z-2}$, (C) $\frac{z-2}{4z+1}$, (D) $\frac{z+4}{2z+1}$, (E) none of the above.

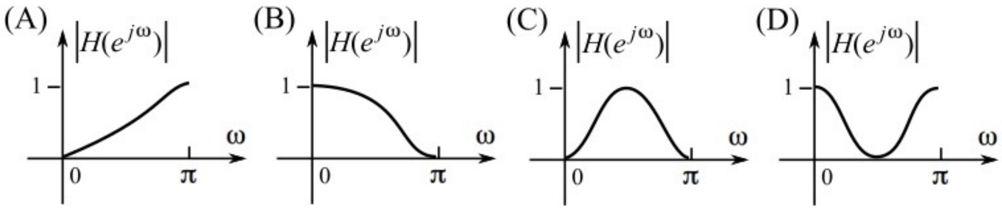
$$Y(1+z^{-1}) = \frac{1}{4}X(1-z^{-1})\,; \quad H = \frac{1}{4}\frac{1-z^{-1}}{1+\frac{1}{2}z^{-1}} = \frac{z-1}{4z+2}$$ ⇒ pole at -1/2; zero at 1.

6. (**C**) For the above problem, what is its pole-zero plot?



(E) none of the above.

7. (**A**) For the above problem, what does its $|H(e^{i\omega})|$ look like?



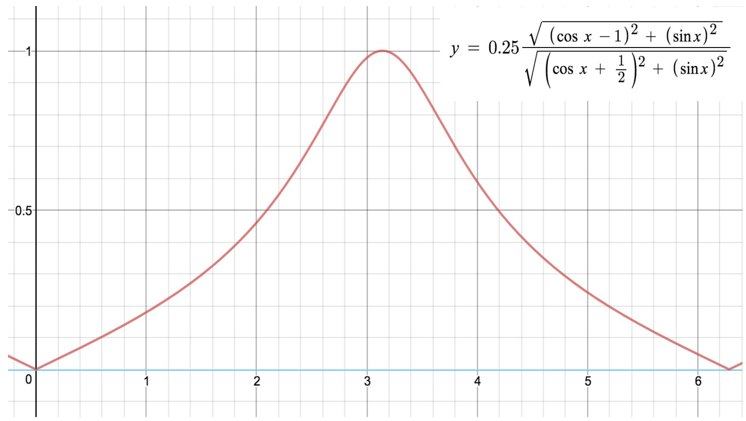
(E) none of the above.

$H(e^{j\omega}) = \frac{1}{4}\frac{e^{-j\omega}-1}{e^{j\omega}+1/2} =$

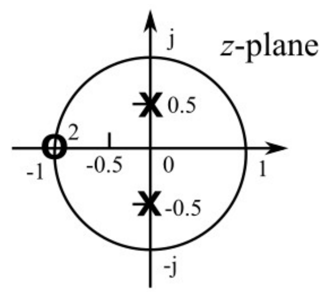
$\frac{1}{4}\frac{(\cos\omega-1)+j\sin\omega}{(\cos\omega+1/2)+j\sin\omega} \Rightarrow$

$|H(e^{j\omega})| =$

$\frac{1}{4}\frac{\sqrt{(\cos\omega-1)^2+\sin^2\omega}}{\sqrt{(\cos\omega+1/2)^2+\sin^2\omega}}$



8. (**D**) The pole-zero plot of a digital filter on the right shows two poles at ±0.5*j* and a double zero at -1. What is its *H(z)*? (A) $\frac{z^2+1}{z^2+0.5}$, (B) $\frac{z^2-2z+1}{z^2-0.25}$, (C) $\frac{z^2+0.25}{z^2-2z+1}$, (D) $\frac{z^2+2z+1}{z^2+0.25}$, (E) none of the above.



Zeros at -1, -1; poles at +0.5j, -0.5j.

$$H(z) = \frac{(z+1)^2}{(z+0.5j)(z-0.5j)} = \frac{z^2+2z+1}{z^2+0.25} = \frac{1+2z^{-1}+z^{-2}}{1+0.25z^{-2}} = \frac{Y(z)}{X(z)}$$

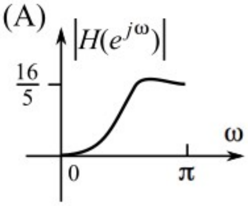
$$Y(z)(1+0.25z^{-2}) = X(z)(1+2z^{-1}+z^{-2}) \Rightarrow$$

$$y[n]=x[n]+2x[n-1]+x[n-2]-0.25y[n-2]$$

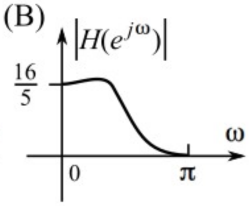
9. (**D**) For the above problem, what is the filter equation?
(A) $y[n]=x[n]+x[n-2]+0.25y[n-2]$,
(B) $y[n]=x[n]-2x[n-1]+x[n-2]+0.25y[n-2]$,
(C) $y[n]=x[n]+0.25x[n-2]+y[n-1]-y[n-2]$,
(D) $y[n]=x[n]+2x[n-1]+x[n-2]-0.25y[n-2]$, (E) none of the above.

10. (**B**) For the above problem, what does its $|H(e^{i\omega})|$ look like?

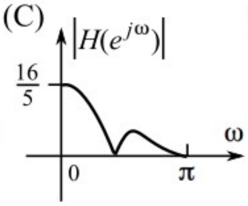
(A)



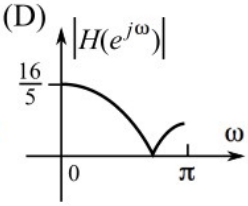
(B)



(C)



(D)



(E) none of the above.

$$H(e^{j\omega}) = \frac{(\cos\omega + j\sin\omega + 1)^2}{(\cos\omega + j\sin\omega)^2 + 0.25} = \frac{[(\cos\omega+1)^2 - \sin^2\omega] + 2j(\cos\omega+1)\sin\omega}{(\cos^2\omega - \sin^2\omega + 0.25) + 2j\cos\omega\sin\omega} \Rightarrow$$

$$|H(e^{j\omega})| = \frac{\sqrt{[(\cos\omega+1)^2 - \sin^2\omega]^2 + 4[(\cos\omega+1)\sin\omega]^2}}{\sqrt{(\cos^2\omega - \sin^2\omega + 0.25)^2 + 4(\cos\omega\sin\omega)^2}}$$