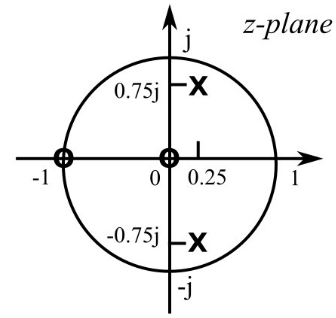
**ELE314 Linear Systems and Signals** Exam #3 Summer 2017 Name: Solution

Open book/notes (10 questions, 10 points each)

1. ( **D** ) The pole-zero plot of a digital filter shows two poles at 0.25 ± 0.75 *j* and two zeros at 0 and -1. What is its *H(z)*? (A) $\dfrac{z^2+1}{z^2+0.5z+0.625}$, (B) $\dfrac{z^2-0.5z+0.625}{z^2-z}$, (C) $\dfrac{z^2+z}{z^2+0.5z+0.625}$, (D) $\dfrac{z^2+z}{z^2-0.5z+0.625}$, (E) none of the above.




$$H(z) = \frac{z(z+1)}{(z-0.25+0.75j)(z-0.25-0.75j)} = \frac{z^2+z}{(z-0.25)^2+0.75^2} =$$

$$\frac{z^2+z}{z^2-0.5z+0.0625+0.5625} = \frac{z^2+z}{z^2-0.5z+0.625}.$$

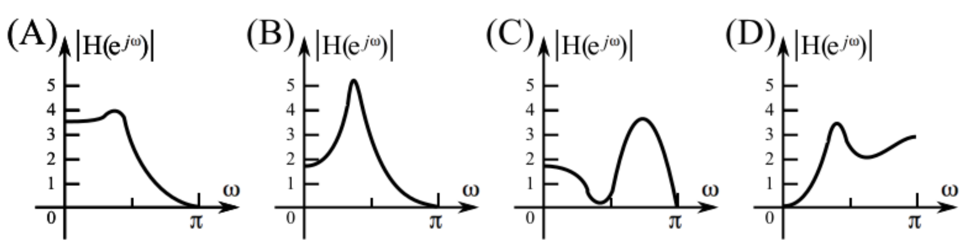
2. ( **D** ) For the above problem, what is the filter equation?
   (A) $y[n]=x[n]+x[n-2]-0.5y[n-1]-0.625\,y[n-2]$,
   (B) $y[n]=x[n]-0.5\,x[n-1]+0.625x[n-2]+y[n-1]$,
   (C) $y[n]=x[n]+x[n-1]-0.5y[n-1]-0.625\,y[n-2]$,
   (D) $y[n]=x[n]+x[n-1]+0.5y[n-1]-0.625\,y[n-2]$, (E) none of the above.

$$H(z)=\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-(0.5z^{-1}-0.625z^{-2})} \Rightarrow Y(z)[1-(0.5z^{-1}-0.625z^{-2})] = X(z)(1+z^{-1})$$

$$\Rightarrow Y(z) = X(z)(1+z^{-1})+Y(z)(0.5z^{-1}-0.625z^{-2})$$

$$\Rightarrow y[n]=x[n]+x[n-1]+0.5y[n-1]-0.625\,y[n-2]$$

3. ( **B** ) For the above problem, what does its $\left|H(e^{i\omega})\right|$ look like?

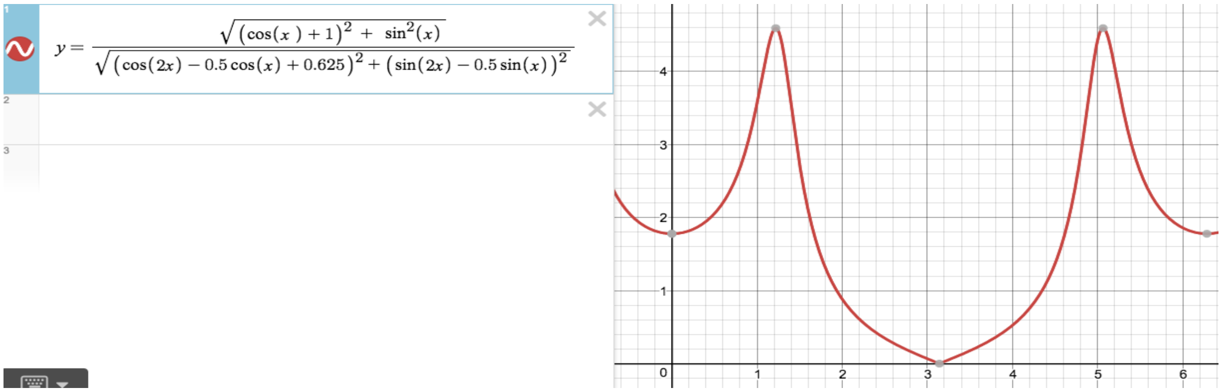



(E) none of the above.

$$H(e^{j\omega}) = \frac{e^{j\omega}(e^{j\omega}+1)}{e^{2j\omega}-0.5e^{j\omega}+0.625} = \frac{e^{j\omega}(\cos\omega+j\sin\omega+1)}{\cos 2\omega+j\sin 2\omega-0.5\cos\omega-0.5\,j\sin\omega+0.625} =$$

$$\frac{e^{j\omega}(\cos\omega+1+j\sin\omega)}{(\cos 2\omega-0.5\cos\omega+0.625)+j(\sin 2\omega-0.5\sin\omega)} =$$

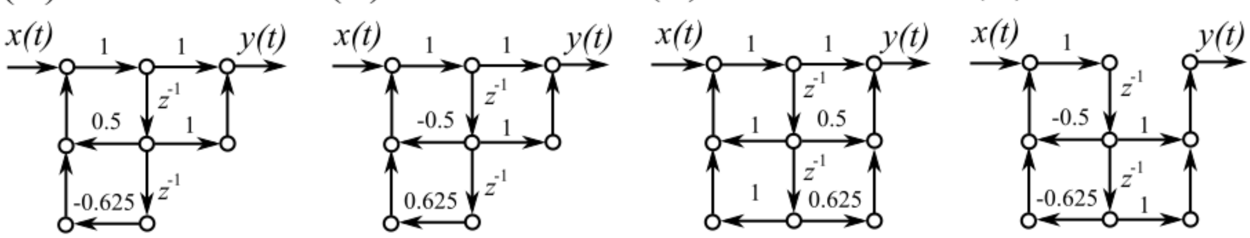
$$\left|H(e^{j\omega})\right| = \frac{\sqrt{(\cos\omega+1)^2+\sin^2\omega}}{\sqrt{(\cos 2\omega-0.5\cos\omega+0.625)^2+(\sin 2\omega-0.5\sin\omega)^2}}$$




The system is substantially underdamped as indicated by the prominent peak, which is related to the two poles close to the unit circle.

4. ( **A** ) For the above problem, what is its Direct Form 2 realization?

(A) (B) (C) (D)




(E) none of the above.

$$H(z) = \frac{1+z^{1}}{1-(0.5z^{1}-0.625z^{2})} \Rightarrow \boxed{\begin{array}{ll} a_1=0.5 & b_0=1 \\ a_2=-0.625 & b_1=1 \end{array}}$$

5. ( **B** ) If $H(z)=\dfrac{\frac{\sqrt{2}}{2}z^{-1}}{1-\sqrt{(2)}\,z^{-1}+z^{-2}}$, what is $h[n]$? (Hint: Use the ZT Table.)

(A) $h[n]=\sin(\frac{\pi n}{2})$, (B) $h[n]=\sin(\frac{\pi n}{4})$, (C) $h[n]=\cos(\frac{\pi n}{2})$,

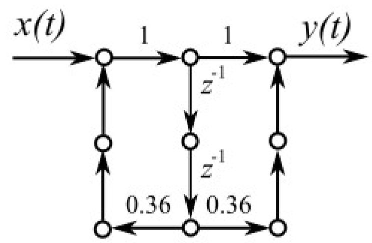
(D) $h[n]=\cos(\frac{\pi n}{4})$, (E) none of the above.

From the ZT Table: $\sin\omega n \quad \Leftarrow z \Rightarrow \quad H(z)=\dfrac{z^{-1}\sin\omega}{1-2z^{-1}\cos\omega+z^{-2}}$

Let $\omega=\frac{\pi}{4}$ . $\Rightarrow$ $\cos\frac{\pi}{4}=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$. Thus, $h[n]=\sin(\frac{\pi n}{4})$.

6. ( **A** ) The Direct form 2 realization of a filter is shown below. What is its filter equation?

(A) $y[n]=x[n]+0.36x[n-2]+0.36\,y[n-2]$,
(B) $y[n]=x[n]-0.36\,x[n-2]-0.36\,y[n-2]$,
(C) $y[n]=x[n]+0.36x[n-1]+0.36\,y[n-1]$,
(D) $y[n]=x[n]-0.36x[n-1]-0.36\,y[n-1]$,
(E) none of the above.



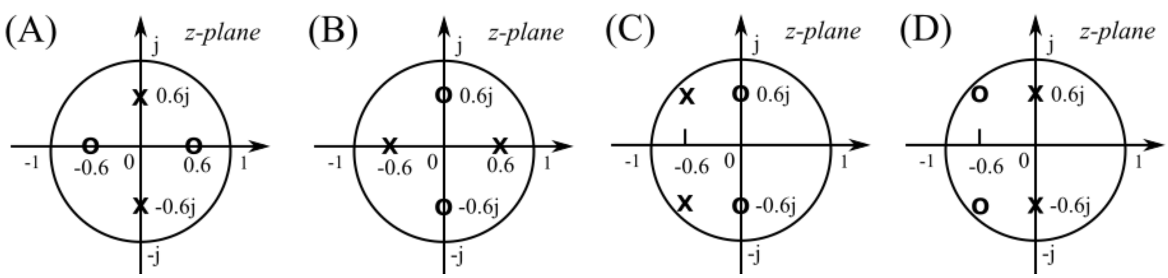

| | |
|---|---|
| $a_1=0$ | $b_0=1$ |
| $a_2=0.36$ | $b_1=0$ |
| | $b_2=0.36$ |

$$H(z)=\frac{Y(s)}{X(s)} = \frac{1+0.36\,z^{-2}}{1-0.36z^{-2}} \quad \Rightarrow$$

$$Y(s)(1-0.36z^{-2}) = X(s)(1+0.36\,z^{-2}) \quad \Rightarrow$$

$$Y(s) = X(s)(1+0.36\,z^{-2})+Y(z)0.36z^{-2} \quad \Rightarrow \quad y[n]=x[n]+0.36x[n-2]+0.36\,y[n-2]$$

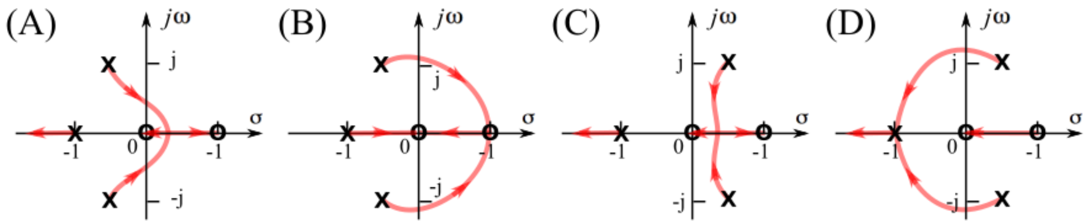
7. ( **B** ) For the above problem, what is the pole-zero plot?




(E) none of the above.

Zeros at: $s = \pm 0.6\,j$ ; Poles at: $s = \pm 0.6$ .

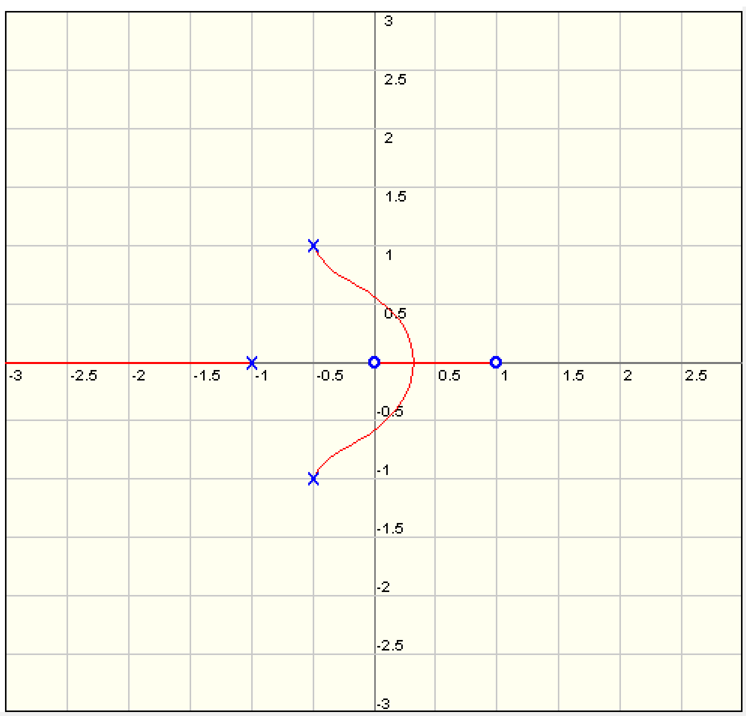
8. ( **A** ) The loop gain of a feedback control system is $G(s)H(s)=\dfrac{s(s-1)}{(s^2+s+1.25)(s+1)}$. What is its root locus for negative feedback?




(E) none of the above.

$$G(s)H(s)=\frac{s(s-1)}{((s+0.5)^2+1)(s+1)} = \frac{s(s-1)}{(s+0.5+j)(s+0.5-j)(s+1)}$$

Zeros at: 0, -1; Poles at: $-0.5\pm j$, -1.




| Poles Real | Poles Imag. | Zeros Real | Zeros Imag. |
|---|---|---|---|
| - 0.5 | +1 | 0 | 0 |
| - 0.5 | - 1 | +1 | 0 |
| - 1 | 0 | ...... | ...... |

| Gain Points Nr. | Real | Imag. | RL.Gain |
|---|---|---|---|

9. ( **C** ) A state-space representation is developed for the 2nd-order differential equation $\ddot{y}(t)+2\dot{y}+5y = 2x$ by choosing the state variables: $s_1=y$; $s_2=\dot{y}$. The state equation in matrix form is $\underline{\dot{s}}=A\underline{s}+\underline{b}x$. What is the plant matrix $A$? (A) $\begin{bmatrix} 0 & 1 \\ -2 & -5 \end{bmatrix}$, (B) $\begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix}$, (C) $\begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix}$, (D) $\begin{bmatrix} 0 & 1 \\ 5 & 2 \end{bmatrix}$, (E) none of the above.

10. ( **A** ) For the above problem, what is the input vector $\underline{b}$? (A) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$, (B) $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$, (C) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, (D) $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$, (E) none of the above.

$$\ddot{y}(t) = -2\dot{y} - 5y + 2x$$

$$\begin{matrix} s_1=y \\ s_2=\dot{y} \end{matrix} \Rightarrow \begin{matrix} \dot{s}_1 = \dot{y} = s_2 \\ \dot{s}_2 = \ddot{y} = -5y-2\dot{y}+2x = -5s_1-2s_2+2x \end{matrix}$$

Thus, the state-space representation is given by

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} x$$

The system equation is $\underline{\dot{s}} = A\underline{s} + \underline{b}x$, where

the plant matrix $A = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix}$, and the input vector $\underline{b} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.