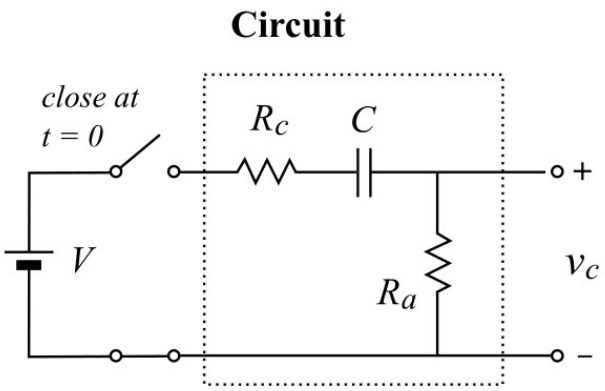
ELE 314 Linear Systems and Signals – HomeWork #3 (6%) *Ying Sun*

Name: _____ Due date: _____

Circuit Analysis Using Laplace Transform and Fourier Transform: High-Pass Filter

The schematic on the right shows a first-order high-pass filter. A constant voltage (V) is applied to the input of the circuit by closing the switch at $t = 0$. The output is the voltage across the resistor (R_a). The circuit can be represented as a linear time-invariant (LTI) system. We first normalize the input voltage: $V = 1$. Thus, the input is the unit step function $u(t)$, and the output is the step response $s(t)$. The LTI system can be completely characterized by its impulse response $h(t)$. The step response is the convolution between the input step function and the impulse response: $s(t) = u(t) \otimes h(t)$.



Following the steps of the example for the low-pass circuit. Complete the following.

Homework 3a: **Circuit analysis using differential equations (1%)**

Obtain the first-order differential equation by evaluating the Kirchhoff's current law at the output node. Solve the differential equation and plot the output (step response) as a function of time.

Homework 3b: **Transfer function (1%)**

Derive the transform function using the Laplace transforms of the circuit components.

Homework 3c: **Impulse response (1%)**

Take the inverse Laplace transform of the transfer function to obtain the impulse response. Plot the impulse response as a function of time.

Homework 3d: **Fourier transform (1%)**

Obtain the Fourier transform $H(j\omega)$ by substituting $s = j\omega$ in the transfer function. Obtain the expression of the magnitude and phase of $H(j\omega)$.

Homework 3e: **Frequency response (1%)**

Assign values to the circuit components: $R_a = R_c = 10\,K\,\Omega$; $C = 0.1\,\mu F$. Use the online graphing calculator at <www.desmos.com/calculator> to plot the magnitude and phase of the frequency response.

Homework 3f: **Bode plot (1%)**

Fine the locations of the pole and the zero of the system. Use the online Bode plot generator at <http://http://www.onmyphd.com/?p=bode.plot> to generate the Bode plot for the magnitude and the phase.