

# Linear Transforms Between the Time Domain and the Frequency Domain

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	Continuous in Time Domain	Discrete in Time Domain
Periodic signal	<p style="text-align: center;"><b>Fourier Series (FS)</b></p> <p>FS: <math display="block">c_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt</math></p> <p>IFS: <math display="block">x(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t}</math></p>	<p style="text-align: center;"><b>Discrete Fourier Series (DFS)</b></p> <p>DFS: <math display="block">X(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}</math></p> <p>IDFS: <math display="block">x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}</math></p>
Fourier Transform	<p style="text-align: center;"><b>Fourier Transform (continuous case)</b></p> <p>FT: <math display="block">X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt</math></p> <p>IFT: <math display="block">x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega</math></p>	<p style="text-align: center;"><b>Fourier Transform (discrete case)</b></p> <p>FT: <math display="block">X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}</math></p> <p>IFT: <math display="block">x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega</math></p> <hr/> <p style="text-align: center;"><b>Discrete Fourier Transform (DFT)</b></p> <p>DFT: <math display="block">X(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}</math></p> <p>IDFT: <math display="block">x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}</math></p>
Laplace or z-Transform	<p style="text-align: center;"><b>Laplace Transform (LT)</b></p> <p>LT: <math display="block">X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt</math></p> <p>ILT: <math display="block">x(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} X(s) e^{st} ds</math></p> <p>where</p> <p style="text-align: center;"><math>s = \sigma + j\omega, \quad \left  \int_{-\infty}^{\infty} x(t) e^{-st} dt \right  &lt; \infty</math></p>	<p style="text-align: center;"><b>Z Transform (ZT)</b></p> <p>ZT: <math display="block">X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}</math></p> <p>IZT: <math display="block">x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz</math></p> <p>where</p> <p style="text-align: center;"><math>z = r e^{j\omega}, \quad \left  \sum_{-\infty}^{\infty} x[n] z^{-n} \right  &lt; \infty</math></p>