Signal Flow Graphs of Digital Filters *Ying Sun*

Realization of Digital Filters

The LTI systems in the discrete time can be generalized according to the following:

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k] .$$

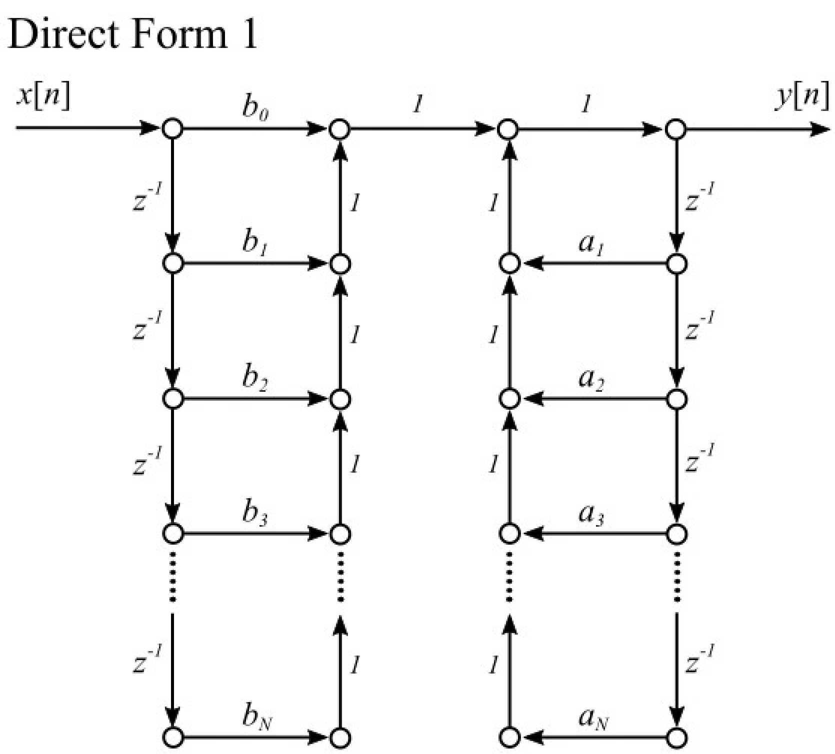
Without loosing generality, we can let $N = \mathrm{M}$ and assigning 0 to the extra coefficients a_k or b_k. Taking ZT of the filter equation, we have

$Y(z) = Y(z)\sum_{k=1}^{N} a_k z^{-k} + X(z)\sum_{k=0}^{N} b_k z^{-k}$. The transfer function is given by.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}} .$$

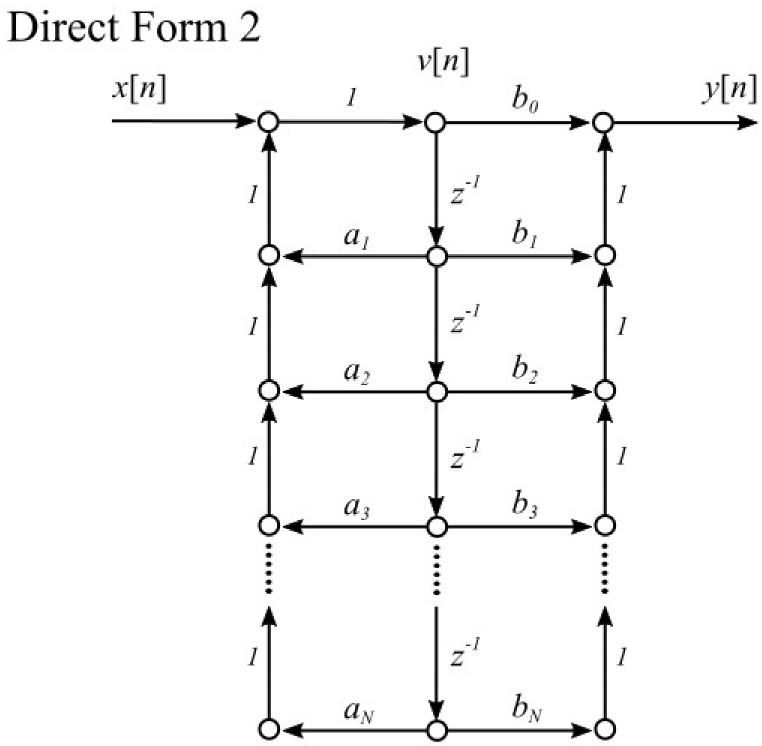
<u>Direct Form 1</u>

An realization of this system is represented by the following signal flow diagram called Direct Form 1. The validity of Direct Form 1 is obvious by comparing the filter equation and the signal flow graph.



Direct Form 2

Another realization of this system, called Direct Form 2, is more efficient by using the minimum number of delays (z^{-1}).



The validity of Direct Form 2 is not immediately obvious. Thus, we will prove that the Direct Form 2 does result in the same transfer function. Let $v[n]$ be the intermediate signal as shown above.

$$v_n = x[n]+\sum_{k=1}^{N} a_k v[n-k], \quad \text{and} \quad y_n = b_0 v[n]+\sum_{k=1}^{N} b_k v[n-k].$$

By taking the ZT of the above, we have

$$V(z) = X(z)+V(z)\sum_{k=1}^{N} a_k z^{-k}, \quad \text{and} \quad Y(z) = b_0 V(z)+V(z)\sum_{k=1}^{N} b_k z^{-k};$$

$$V(z) = \frac{X(z)}{1-\sum_{k=1}^{N} a_k z^{-k}}, \quad \text{and} \quad Y(z) = V(z)\sum_{k=0}^{N} b_k z^{-k}.$$

Substituting $V(z)$ into $Y(z)$, we have $Y(z) = X(z)\dfrac{\sum_{k=0}^{N} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}}$. Thus, $H(z) = \dfrac{\sum_{k=0}^{N} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}}$.

Variations of Direct Form 2

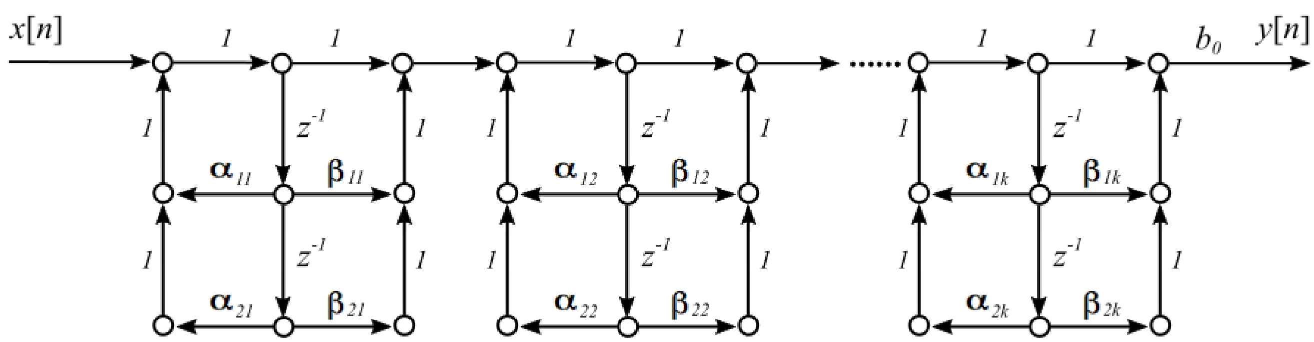
The Direct Form 2 realization shown above is based on the rational form of *H(z)*. There are two variations of Direction Form 2 based on the factored form and the partial-fraction form of *H(z)*.

<u>Cascade Direct Form 2</u>

The transfer function can be arranged into the factored form, resulting in Cascade Direct Form 2 realization as shown:

$$H(z)=\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}} = b_0 \prod_{k=1}^{\lfloor \frac{N+1}{2} \rfloor} \frac{1+\beta_{1k} z^{-1}+\beta_{2k} z^{-2}}{1-\alpha_{1k} z^{-1}-\alpha_{2k} z^{-2}} .$$

Cascade Direct Form 2



Parallel Direct Form 2

The transfer function can be arranged into the partial-fraction form, resulting in Parallel Direct Form 2 realization as shown:

$$H(z)=\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1-\sum_{k=1}^{N} a_k z^{-k}} = \sum_j \gamma_j z^{-j} + \sum_k \frac{\delta_{0k}+\delta_{1k} z^{-1}}{1-\epsilon_{1k} z^{-1}-\epsilon_{2K} z^{-2}} .$$

Parallel Direct Form 2