Properties of the z-Transform *Ying Sun*

ZT: $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

IZT: $x[n]=\frac{1}{2\pi j}\oint X(z)z^{n-1}dz$ where $z=re^{j\omega}$, $\left|\sum_{-\infty}^{\infty} x[n]z^{-n}\right|<\infty$

For the following, let $X(z)$ be the ZT of $x[n]$, with region of convergence (ROC): R_X;
Let $X_1(z)$ be the ZT of $x_1[n]$, with ROC: R_1; and
Let $X_2(z)$ be the ZT of $x_2[n]$, with ROC: R_2.

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1. Linearity

$a\,x_1[n]+b\,x_2[n] \quad \Leftarrow z \Rightarrow \quad a\,X_1(z)+b\,X_2(z)$ ROC: at least $R_1\cap R_2$

2. Time shift

$x[n-k] \quad \Leftarrow z \Rightarrow \quad z^{-k}X(z)$ ROC: R_X except at $z=0$

3. Frequency shift

$e^{j\omega_0 n}x[n] \quad \Leftarrow z \Rightarrow \quad X(e^{-j\omega_0}z)$ ROC: R_X

4. Amplitude scaling

<u>complex</u> $z_0^n x[n] \quad \Leftarrow z \Rightarrow \quad X(\frac{z}{z_0})$ ROC: $z_0 R_X$

<u>real</u> $a^n x[n] \quad \Leftarrow z \Rightarrow \quad X(\frac{z}{a})$ ROC: $|a|R_X$

5. Time scaling

$w[n]=\begin{cases} x[r], & n=rk \\ 0, & n\neq rk \end{cases} \quad \Leftarrow z \Rightarrow \quad X(z^k)$ ROC: $R_X^{\frac{1}{k}}$

6. Time reversal

$x[-n] \quad \Leftarrow z \Rightarrow \quad X(z^{-1})$ ROC: R_X^{-1}

7. Convolution

$x_1[n] \otimes x_2[n] \quad \Leftarrow z \Rightarrow \quad X_1(z)X_2(z)$ ROC: at least $R_1 \cap R_2$

8. Differentiation in z-domain

$n\ x[n] \quad \Leftarrow z \Rightarrow \quad -z\frac{d\,X(z)}{dz}$ ROC: R_X except at $z=0$

9. Summation in time-domain

$\sum_{k=-\infty}^{n} x[k] \quad \Leftarrow z \Rightarrow \quad \frac{1}{1-z^{-1}}X(z)$ ROC: at least $R_X \cap \{|z|>1\}$

10. Initial value theorem

If $x[n] = 0$, for $n < 0 \quad \Rightarrow \quad x[0] = \lim_{z\to\infty} X(z)$