MOSFET Equations

Threshold Voltage V_t ($V_{SB}=0$)

$$V_{t0} = V_{FB} + 2\varphi_F + \frac{Q_{depl.0}}{C_{ox}} \qquad\qquad C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

where V_{FB}: Flat-Band Voltage; $\varphi_F = V_T \ln\left(\frac{N_{sub}}{n_i}\right)$ Fermi Potential

$$Q_{depl_0} = q N_{sub} \cdot w_{depl} = \sqrt{2\varepsilon_s q N_{sub} 2\varphi_F}$$

$V_{SB} \neq 0$ Body Effect

$$V_t = V_{FB} + 2\varphi_F + \frac{Q_{depl.0}}{C_{ox}} + \frac{Q_{depl.} - Q_{depl.0}}{C_{ox}}$$

$$= V_{t0} + \gamma\left[\sqrt{2\varphi_F + V_{SB}} - \sqrt{2\varphi_F}\right]$$

where $\gamma = \frac{\sqrt{2\varepsilon_s q N_{sub}}}{C_{ox}}$ Body Effect increases with $\sqrt{N_{sub}} \cdot \frac{t_{ox}}{\varepsilon_{ox}}$

Notes: Since V_{FB} is material and process dependent and φ_F varies weakly with the substrate carrier concentration, threshold tuning is achieved by adjusting $\frac{Q_{depl}}{C_{ox}}$

Channel Length Modulation

If $V_{DS} > (V_{GS} - V_t)$, the drain side of the conducting channel is pinched-off. The effective channel length is then given by

$$L_{eff} = L - W_d(V_{DS})$$

where

$$W_d(V_{DS}) \cong \sqrt{\frac{2\varepsilon_s(\Phi_0 + V_{DS})}{q N_{sub}}}$$

Φ_0: Built-in potential

$\Phi_0 \approx 1V$

$$\frac{dI_D}{dV_{DS}} = \frac{dI_D}{dL_{eff}} \frac{dL_{eff}}{dV_{DS}} = -\frac{I_{D0}}{L} \frac{dL_{eff}}{dV_{DS}}$$

$$\frac{dL_{eff}}{dV_{DS}} = -\frac{dW_d}{dV_{DS}} = -\frac{1}{2}\sqrt{\frac{2\varepsilon_s}{q N_{sub}(\Phi_0 + V_{DS})}}$$

$$\therefore \quad \frac{dI_D}{dV_{DS}} = I_{D0}\frac{1}{L}\sqrt{\frac{\varepsilon_s}{2q N_{sub}(\Phi_0 + V_{DS})}}$$

$$I_D = I_{D0}(1 + \lambda V_{DS})$$

$$\frac{dI_D}{dV_{DS}} = I_{D0} \cdot \lambda$$

$$\therefore \quad \lambda = \frac{1}{L}\sqrt{\frac{\varepsilon_s}{2q N_{sub}(\Phi_0 + V_{DS})}}$$

$\varepsilon_s \cong 10^{-10} \frac{As}{Vm}$ $\quad \Phi_0 + V_{DS} = 1.5V$

$N_{sub} \cong 10^{23} m^{-3}$ $\quad L = 500nm$

$\therefore \lambda \cong 0.09 \, V^{-1}$

Current-Voltage Relationship

Triode Region $V_{GS} > V_t$; $V_{DS} < V_{GS} - V_t$

$$\left\| I_D = \mu C_{ox} \frac{W}{L} \left[(V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \right\|$$

μ: Carrier Mobility

$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$ Area cap. gate to channel

$\frac{W}{L}$ width to length ratio of channel

Saturation or Active Region $V_{GS} > V_t$

$V_{DS} \geq V_{GS} - V_t$

$$\left\| I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} \left[V_{GS} - V_t \right]^2 \left[1 + \lambda V_{DS} \right] \right\|$$

λ: Channel Length Modulation Factor

Note: $\lambda \propto \frac{1}{L}$ Designer can modify slope of I-V characteristic by adjusting L