ELE 338 Electronics I

I. Review of Linear Circuit Theory

Tools for linear circuit analysis?

1. KVL $\left| \sum_{mesh} V_i = 0 \right|$

2. KCL $\left| \sum_{Node} I_i = 0 \right|$

3. Device eq. (e.g. Ohm's Law $V_R = R \cdot I_R$)

Device — V-I Characteristics (time domain)

$R = \frac{V_R}{I_R}$ $\quad V_R = R \cdot I_R$ $\quad I_R = \frac{1}{R} V_R$

$L = \frac{\Phi_L}{I_L}$ - Flux $\quad V_L = \frac{d\Phi_L}{dt} = L \cdot \frac{dI_L}{dt}$ or $I_L = \frac{1}{L} \int V_L \, dt$

$C = \frac{Q_C}{V_C}$ - charge $\quad I_C = \frac{dQ_C}{dt} = C \cdot \frac{dV_C}{dt}$ or $V_C = \frac{1}{C} \int I_C \, dt$

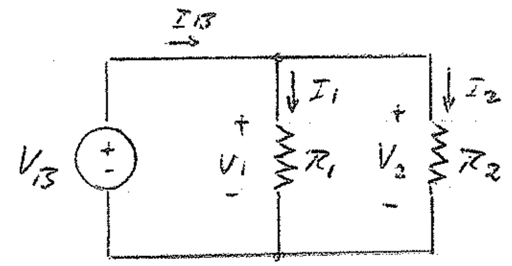
Laplace domain

$V_L(s) = L \cdot I_L(s) \cdot s$ or $I_L(s) = \frac{1}{sL} V_L(s)$

$I_C(s) = C \cdot V_C(s) \cdot s$ or $V_C(s) = \frac{1}{sC} I_C(s)$

<u>Note:</u> for sinusoidal signals $s \rightarrow j\omega$

Example 1:



Determine the 2 voltages V_1 and V_2 and the 3 currents I_B, I_1 and I_2 as a function of V_B, R_1 and R_2

Solution:

Step 1: 5 unknowns $\therefore$ 5 eq. needed

KVL 1: $V_B = V_1$

KVL 2: $V_B = V_2$

KCL: $I_B = I_1 + I_2$

Ohm's law 1: $V_1 = R_1 \cdot I_1$

Ohm's law 2: $V_2 = R_2 \cdot I_2$

Step 2: Solve for unknowns

$V_1 = V_B$

$V_2 = V_B$

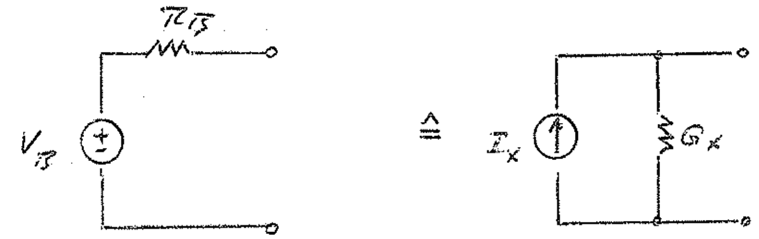
$I_1 = V_B / R_1$

$I_2 = V_B / R_2$

$I_B = V_B/R_1 + V_B/R_2$

Note: Before you can write down any equations, you clearly have to define your variables (e.g. currents and voltages).

Example 2:



Determine I_x and G_x so that the 2 circuits show identical terminal characteristics.

Solution:

We need 2 equations to determine the 2 unknowns I_x and G_x. To find them, we look at 2 specific load conditions of the 2 circuits and determine I_x and G_x so that the resulting output voltages and currents become identical.

Condition 1: $R_L \to \infty$ (open loop)

$$V_{o1} = V_B \qquad V_{o2} = I_x / G_x$$

$$\text{since} \quad V_{o1} \equiv V_{o2} \quad \therefore \left| V_B = I_x / G_x \right| \qquad (1)$$

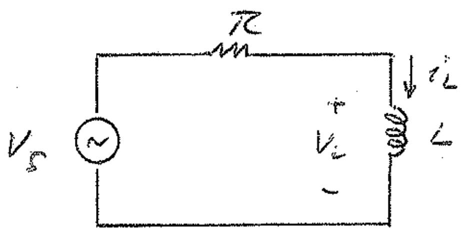
Condition 2: $R_L \to 0$ (short circuit)

$$I_{S1} = V_B / R_B \qquad I_{S2} = I_x$$

$$\text{since} \quad I_{S1} = I_{S2} \quad \therefore \left| I_x = V_B / R_B \right| \qquad (2)$$

$$\therefore \quad \left\| \begin{array}{l} I_x = V_B / R_B \\ G_x = 1 / R_B \end{array} \right\|$$

Example 3:



Determine i_L and V_L if V_S represents a sinusoidal voltage with a voltage swing of $\hat{V}_S$. $V_S(t) = \hat{V}_S \sin(\omega t)$

Solution: (work in Laplace domain since input is sinusoidal)

KVL: $V_S = i_L \cdot R + i_L \cdot s \cdot L$

Dev. eq; $V_L = i_L \cdot s L$

$$\left| \begin{array}{l} i_L = \dfrac{V_S}{R + sL} \\[2ex] V_L = V_S \dfrac{sL}{R + sL} \end{array} \right| \qquad \begin{array}{l} i_L(j\omega) = \dfrac{V_S(j\omega)}{R + j\omega L} \\[2ex] V_L(j\omega) = V_S(j\omega) \dfrac{j\omega L}{R + j\omega L} \end{array}$$

Def. Voltage transfer function

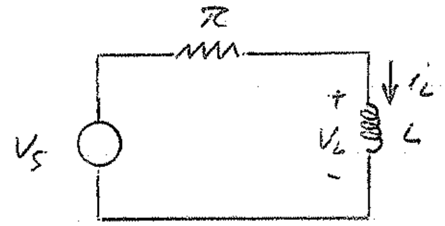
$$\left| T = \frac{V_L}{V_S} \right|$$

$$\therefore \quad \left| T(j\omega) = \frac{V_L(j\omega)}{V_S(j\omega)} = \frac{j\omega L}{R + j\omega L} \right| \quad \text{komplex function}$$

Bode plot: Magnitude $|T(j\omega)| = \left| \dfrac{j\omega L}{R + j\omega L} \right|$

Phase $\varphi_{T(j\omega)} = \varphi_{Num} - \varphi_{Den} = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L}{R}\right)$

Example 4:



Determine i_L and V_L if V_S represents a step function that changes from 0 to V_{S0} at time $t=0$.

Solution:

A) Time domain equations

KVL: $V_{S0} = i_L(t)\cdot R + L \cdot di_L(t)/dt$ $\quad t \geq 0$

Dev. eq.: $V_L(t) = L \cdot di_L(t)/dt$ $\quad t \geq 0$

Init. cond. $V_L(t=0) = V_{S0}$ $\quad$ Steady state: $V_L(t \to \infty) = 0$

$i_L(t=0) = 0$ $\quad$ $i_L(t \to \infty) = V_{S0}/R$

Solution of 1st order diff. eq.

$$\left| i_L(t) = A + B e^{-t/\tau} \right| \text{ and } \left| V_L(t) = -\frac{BL}{\tau} e^{-t/\tau} \right| \quad t \geq 0$$

$i_L(t=0) = 0 \quad \therefore B = -A$ $\quad$ KVL: $\therefore \frac{A \cdot L}{\tau} = V_{S0}$

$i_L(t \to \infty) = V_{S0}/R \quad \therefore A = V_{S0}/R$

$$\therefore \left\| \begin{array}{l} i_L(t) = \frac{V_{S0}}{R}\left(1 - e^{-t/\tau}\right) \\ V_L(t) = \frac{V_{S0} L}{R\tau} e^{-t/\tau} \end{array} \right\| \quad \tau = \frac{L}{R} = L \cdot G \quad t \geq 0$$

B) Laplace domain equations

$$\begin{aligned} \text{KVL:} \quad & V_{so} \cdot \frac{1}{s} = i_L(s) R + sL\, i_L(s) \qquad (1) \\ \text{Dev. eq.:} \quad & V_L(s) = sL\, i_L(s) \qquad (2) \end{aligned}$$

$\frac{1}{s}$ is unit step in Laplace domain

From (1) $\therefore$

$$i_L(s) = \frac{V_{so}}{L} \frac{1}{s(s + R/L)} = \frac{V_{so}}{L} \frac{L}{R} \left[\frac{1}{s} - \frac{1}{s + R/L} \right]$$

$\therefore$

$$i_L(s) = \frac{V_{so}}{R} \left[\frac{1}{s} - \frac{1}{s + R/L} \right]$$

$$V_L(s) = V_{so} \frac{1}{s + R/L}$$

$\mathcal{L}^{-1}$

$$i_L(t) = \frac{V_{so}}{R} u(t) \left[1 - e^{-\frac{t \cdot R}{L}} \right]$$

$$V_L(t) = V_{so}\, u(t)\, e^{-\frac{t \cdot R}{L}}$$

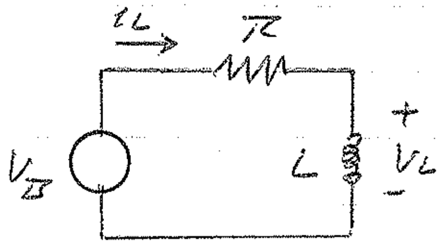
where $u(t)$ is the unit step function

or

$$i_L(t) = \begin{cases} V_{so}/R \left[1 - e^{-\frac{t \cdot R}{L}} \right] & \text{for } t \geq 0 \\ 0 & \text{else} \end{cases}$$

$$V_L(t) = \begin{cases} V_{so}\, e^{-\frac{t \cdot R}{L}} & \text{for } t \geq 0 \\ 0 & \text{else} \end{cases}$$

Step Response of RL Circuit



where

$V_B(t) = V_{B0} \cdot u(t)$

$u(t)$: unit step function

2 unknowns $\therefore$ 2 equations needed

KVL: $V_B(t) = i_L(t)\,R + V_L(t)$ (1) where $V_B(t) = V_{B0} \cdot u(t)$

Dev.: $V_L(t) = L \cdot \dfrac{di}{dt}$ (2)

$\downarrow \mathcal{L}$

$V_B(s) = i_L(s)\,R + V_L(s)$ (3) where $V_B(s) = V_{B0} \cdot \frac{1}{s}$

$V_L(s) = L \cdot s\; i_L(s)$ (4)

Solving for i_L and V_L yields

$$i_L(s) = V_B(s)\,\frac{1}{R + sL} = V_{B0} \cdot \frac{1}{s}\,\frac{1/L}{(R/L + s)} \quad (5)$$

$$V_L(s) = V_B(s)\,\frac{sL}{R + sL} = V_{B0}\,\frac{1}{s}\,\frac{s}{(R/L + s)} \quad (6)$$

(5) and (6) can be rewritten as

$$i_L(s) = \frac{V_{BO}}{R}\left[\frac{1}{s} - \frac{1}{(R/L + s)}\right] \qquad (7)$$

$$V_L(s) = V_{BO}\,\frac{1}{(R/L + s)} \qquad (8)$$

$$\mathcal{L}^{-1}$$

$$i_L(t) = \frac{V_{BO}}{R}\,u(t)\left[1 - e^{-t\frac{R}{L}}\right] \qquad (9)$$

$$V_L(t) = V_{BO}\,u(t)\,e^{-t\frac{R}{L}} \qquad (10)$$

<u>Graphical Solution</u> (RL Step Response)