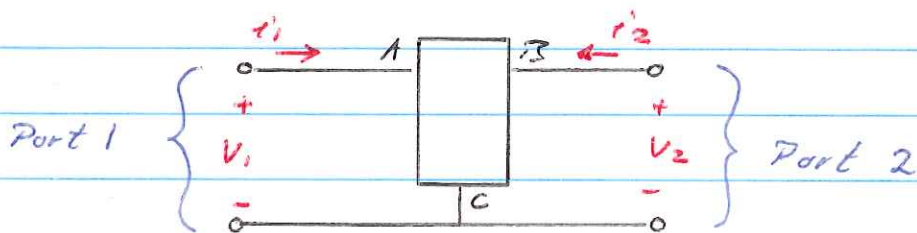


III Three-Terminal Devices

Three terminal devices such as the Bipolar Junction Transistor (BJT) or the Field-Effect Transistor (FET) form the foundation upon which modern electronics is built.

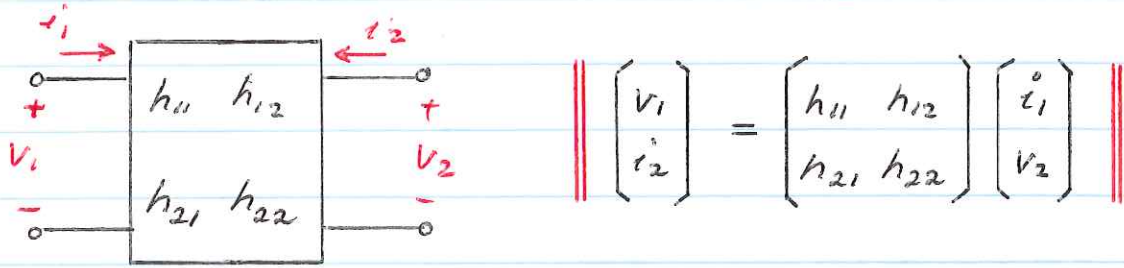
It is convenient to treat a three terminal device as a 2 port since this allows us to describe it by means of its input and output characteristics. In so doing, one terminal must be common to both the input and the output gate.



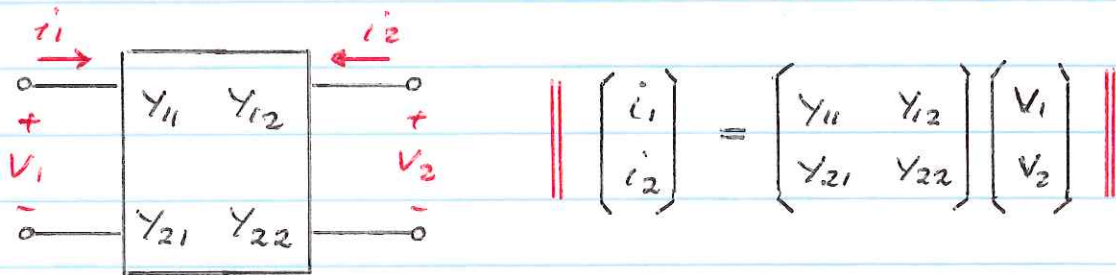
Since there are three choices for the common terminal, there exist three basic configurations how the three terminal device is utilized.

In order to formalize the description of the input and output $v-i$ characteristics, matrix notation is frequently used. The two most common matrices to describe transistors are the Hybrid matrix $[H]$ for BJTs and the Admittance matrix $[Y]$ for FETs.

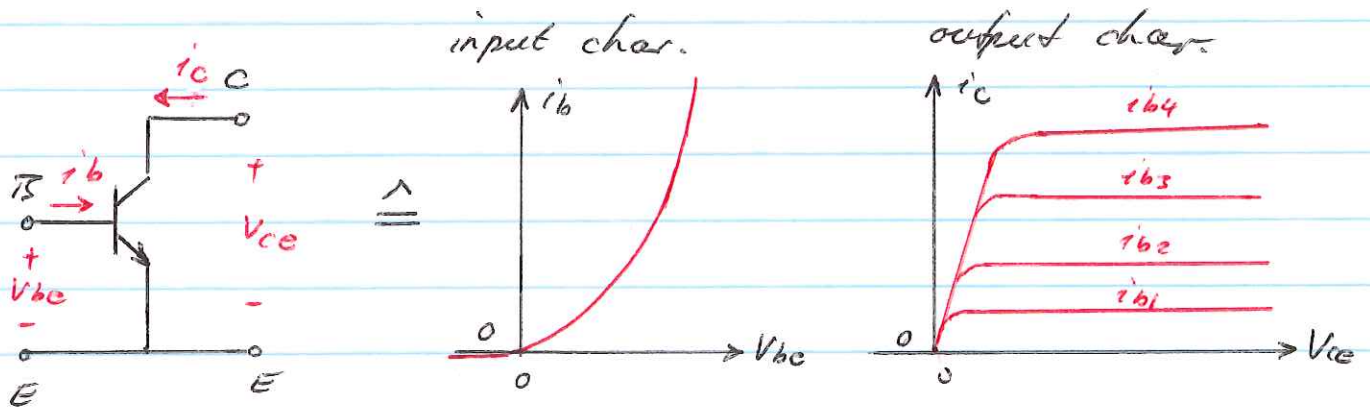
Def. Hybrid Matrix



Def Admittance Matrix



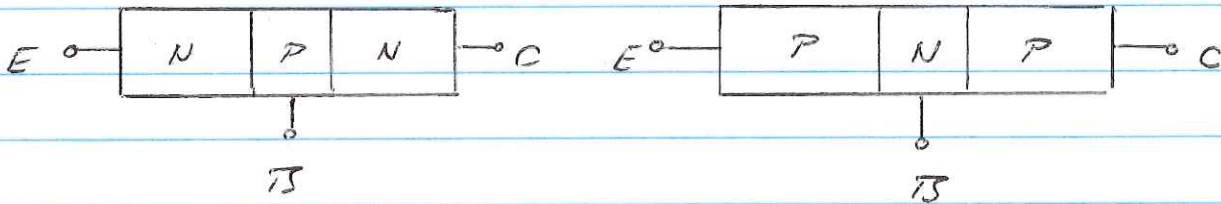
Alternatively, a two port can be described by its input and output o-i characteristics. For example, a BJT with the emitter as the common terminal possesses the following input and output characteristics:



5.1 The Bipolar Junction Transistor (BJT)

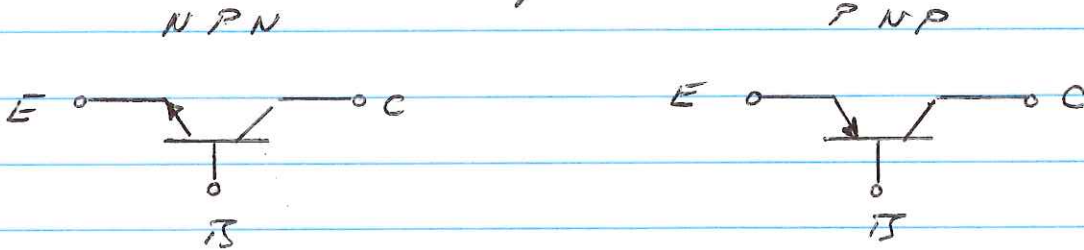
Current-Flow Mechanism

In essence, the BJT consists of two PN junctions placed back to back with a minimum distance between the resulting depletion layers. Consequently, there exist two possible implementations

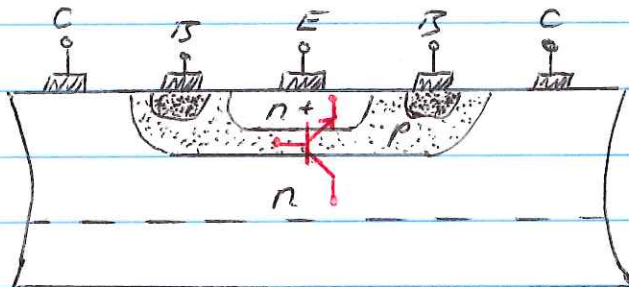


Note: Base region width w_B must be less than the diffusion length of the free carriers, i.e. $w_B < \sqrt{D \cdot \tau}$, where D = diff. const and τ = mean carrier lifetime

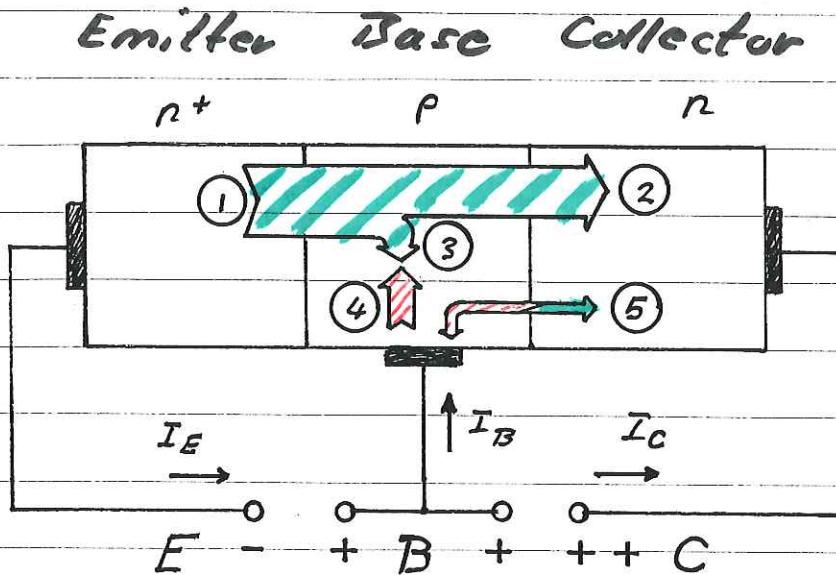
Symbol



Practical Implementation (Planar Process)



The n-p-n Transistor in the forward active state



- ① EL. injected from Emitter into Base
- ② EL. from Emitter that reach Collector (>90%)
- ③ EL. from Emitter that recombine in Base
- ④ Holes from Base that recombine in Base
- ⑤ Reverse saturation current of B-C junction

$$\begin{cases} I_E = I_C + I_B \\ I_C = \alpha I_E + I_{CBO} \\ I_B = (1-\alpha) I_E - I_{CBO} \end{cases}$$

Note: $I_{CBO} \ll I_C$
 $0.9 < \alpha < 0.999$

Current amplification: $I_C = \beta (I_B)$

we have: $I_B = (1 - \alpha_F) I_E - I_{CBO}$ (1)

$I_C = \alpha_F I_E + I_{CBO}$ (2)

from (1) $I_E = \frac{I_B}{1 - \alpha_F} + \frac{I_{CBO}}{1 - \alpha_F}$ (3)

(3) in (2) $\Rightarrow I_C = I_B \frac{\alpha_F}{1 - \alpha_F} + I_{CBO} (1 + \frac{\alpha_F}{1 - \alpha_F})$

Def: $\beta = \frac{dI_C}{dI_B} = \frac{\alpha}{1 - \alpha}$ small signal current amplification
 note: $\alpha = \beta (I_B, I_C)$

$\Rightarrow I_C = I_B \cdot \beta + I_{CBO} (1 + \beta) \approx I_B \cdot \beta$

e.g. $\alpha_F = 0.995$
 $I_{CBO} = 10 \text{ nA}$
 $I_B = 10 \mu\text{A}$

$\Rightarrow \underline{\beta_F = 199}$ $\underline{I_C = 1.99 \text{ mA} + 200 \text{ nA} = 1.9902 \text{ mA}}$

Notes: The transistor has basically 4 modes of operation:

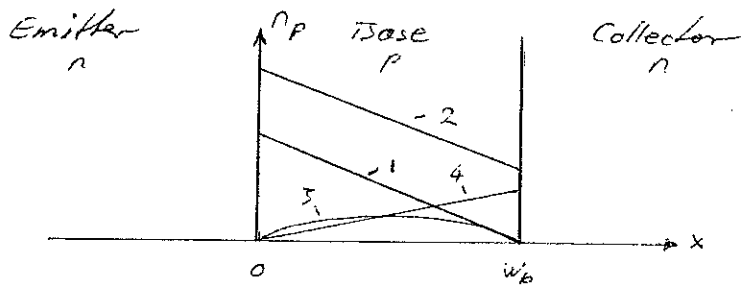
- 1) $V_{BE} > 0$, $V_{CB} > 0$ forward active mode
- 2) $V_{BE} > 0$, $V_{CB} < 0$ saturation mode
- 3) $V_{BE} < 0$, $V_{CB} > 0$ cut off mode
- 4) $V_{BE} < 0$, $V_{CB} < 0$ reverse active mode

saturation mode: large quantity of minority carriers in base.
 I_C can be large (depending on circuit config.)

cutoff mode: No minority carriers are injected into base
 $I_C \approx 0$

reverse-active mode: Minority carriers are injected into base from collector.
 The β in the reverse-active mode is much smaller than that of the forward-active mode due to asymmetries in layout and doping concentrations.

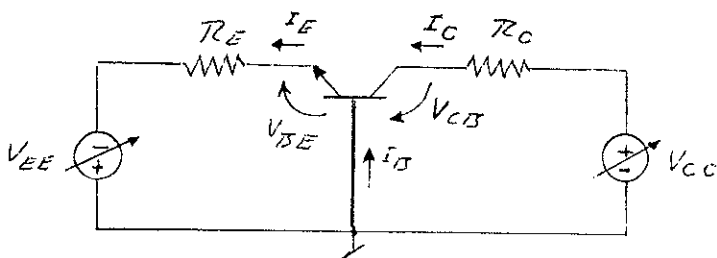
Minority carrier concentration in base:



- 1) forward-active mode
- 2) saturation mode
- 3) cutoff mode
- 4) reverse-active mode

2.2 Tr. current-voltage characteristics

Base-Emitter Junction
 (Common-Base Configuration)



Common-Base connection of npn transistor

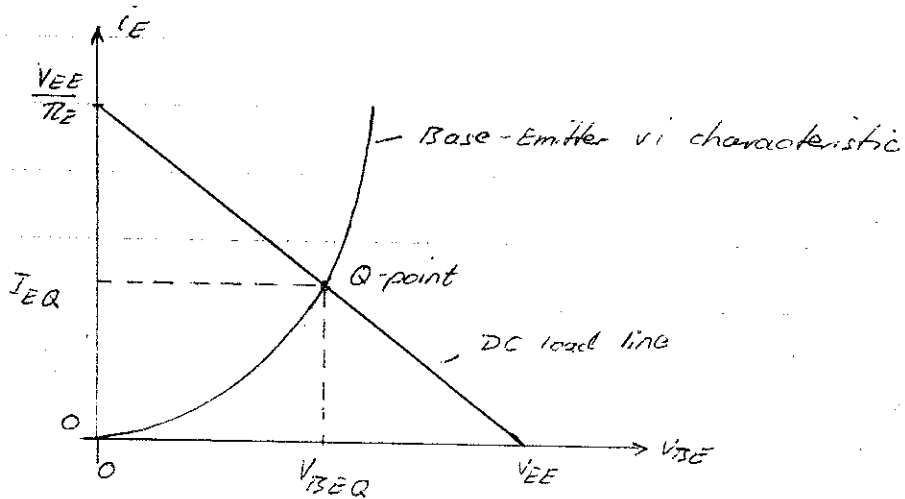
Base-Emitter Loop:

KVL: $V_{EE} = I_E \cdot R_E + V_{BE}$ (1)

$I_E = \frac{V_{EE} - V_{BE}}{R_E}$

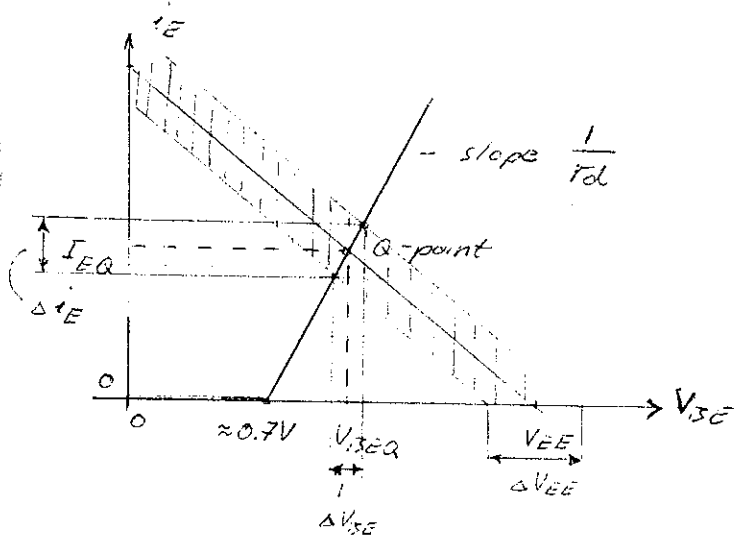
Load line

$I_E = I_{S_{BE}} \left[e^{\frac{V_{BE}}{V_T}} - 1 \right]$ (2) Diode equation



$I_{EQ} = \frac{V_{EE} - V_{BEQ}}{R_E}$

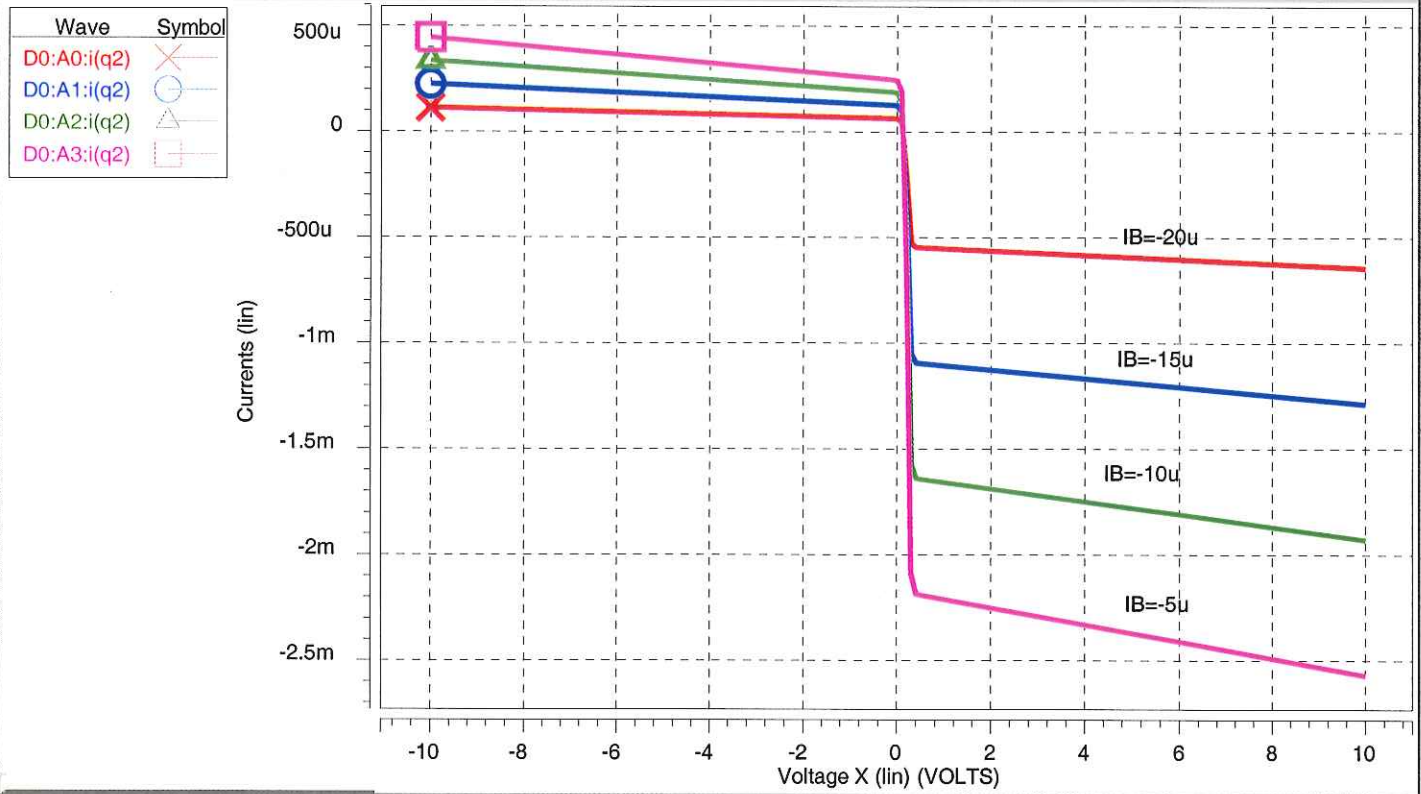
Linearized Base-Emitter circuit



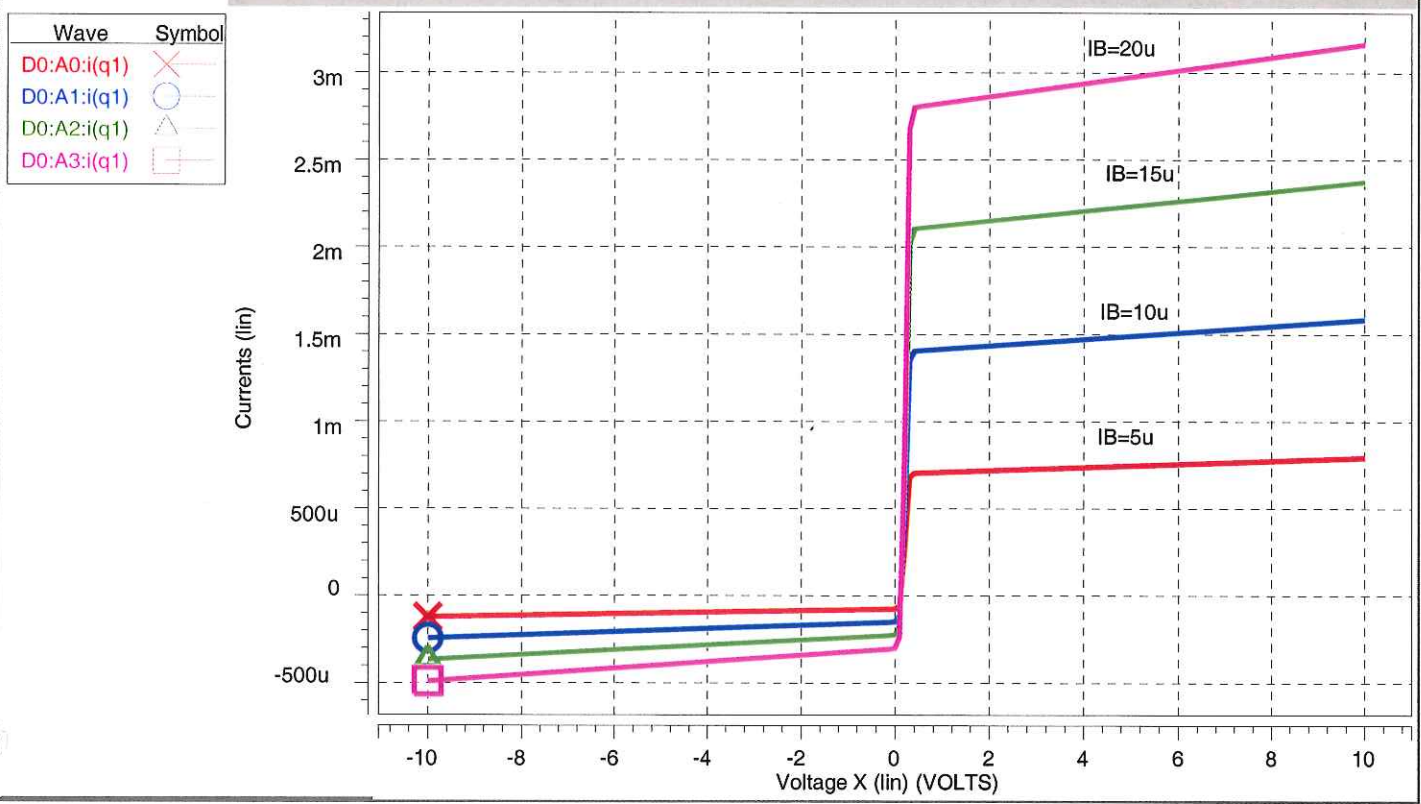
$r_d = \frac{V_T}{I_{EQ}}$

$$\bar{V} - 7\alpha$$

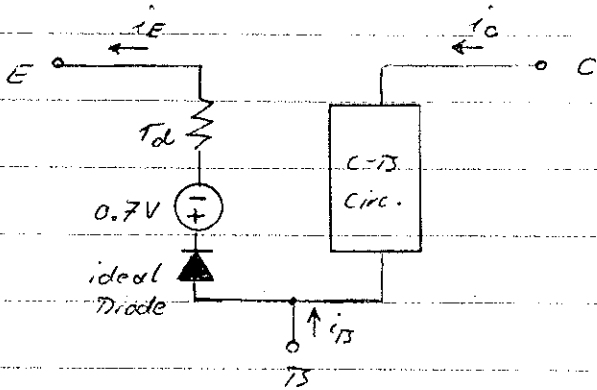
PNP Output Characteristics



NPN Output Characteristics



Linear equivalent circuit model



$$r_d = \frac{V_T}{I_E}$$

at $T = 300^\circ\text{K}$

$$r_d' = 25 \Omega/\text{mA}$$

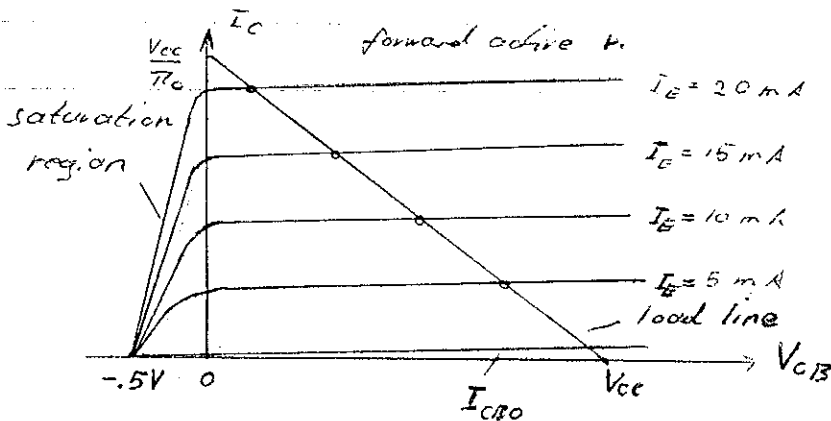
Collector - Base Junction

Collector - Base Loop:

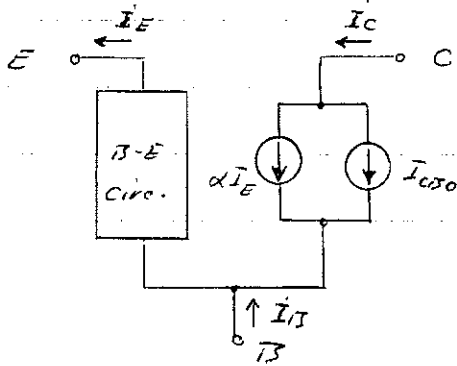
KVL: $V_{CC} = I_C R_C + V_{CB}$ \Rightarrow $I_C = \frac{V_{CC} - V_{CB}}{R_C}$ load line

A) $V_{CB} > 0 \Rightarrow$ forward active mode
 $\Rightarrow I_C = \alpha \cdot I_E + I_{CBO} \approx I_E$

B) $V_{CB} < 0$ CB and BE junction are forward biased
 \Rightarrow saturation region
 I_C decreases



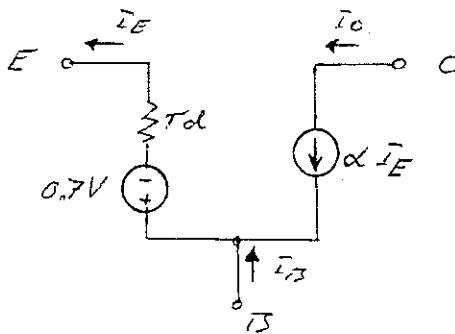
linear equivalent model for forward active region



Simplifications:

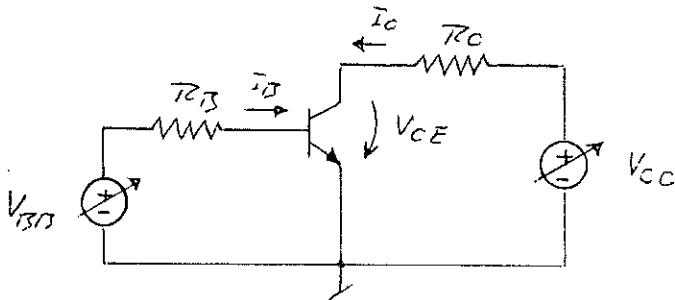
$$I_E \gg I_{CBO} \approx 0$$

simplified model for forward active mode:



$$r_d = \frac{V_T}{I_E} = \alpha \frac{V_T}{I_C} = \alpha \frac{1}{g_m}$$

Common Emitter configuration



C-E loop:

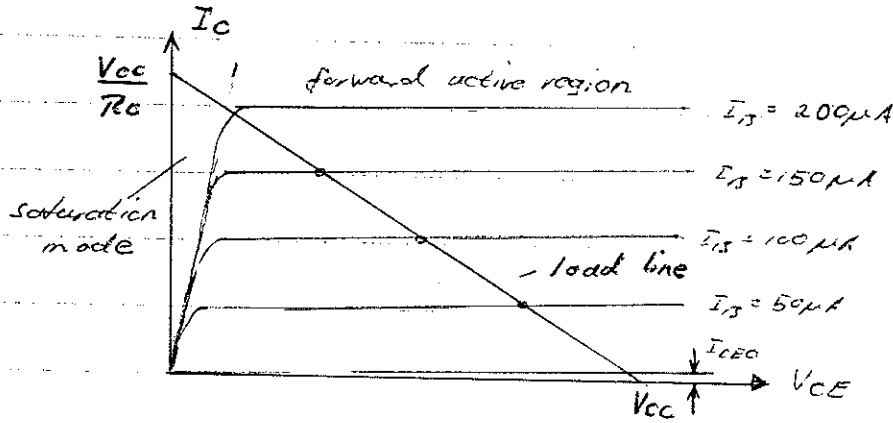
$$KVL: V_{CC} = I_C \cdot R_C + V_{CE} \Rightarrow \underline{I_C = \frac{V_{CC} - V_{CE}}{R_C}} \quad \text{load line}$$

A) $V_{CE} > V_{BE} \Rightarrow V_{CB} > 0 \Rightarrow$ forward active region

$$I_C = \alpha I_E + I_{CBO} = \beta (I_B + I_{CBO} \frac{1}{\alpha}) = \beta I_B + I_{CEO}$$

B) $V_{CE} < V_{BE} \Rightarrow V_{CB} < 0$ saturation mode

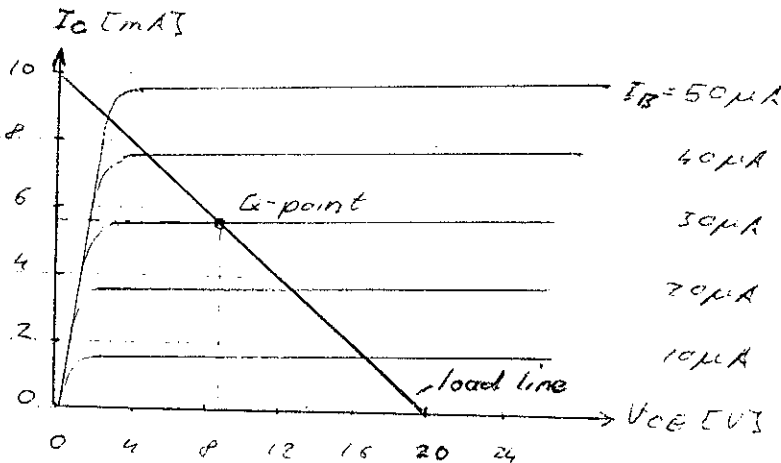
I_C decreases



Example Given: T_R in common emitter circuit configuration

$$V_{BE} = 4V, V_{CC} = 20V$$

$$R_B = 110k\Omega, R_E = 2k\Omega$$



load line

$$V_{CC} = I_C R_C + V_{CE}$$

$$\Rightarrow I_C = \frac{V_{CC} - V_{CE}}{R_C}$$

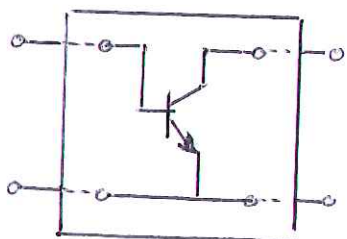
Estimate: I_B, I_C, V_{CE}

B-E Loop: KVL. $V_{BE} = I_B R_B + V_{BE}$

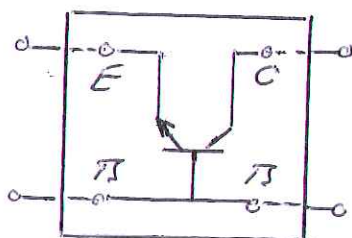
$$\Rightarrow I_B = \frac{V_{BE} - V_{BE}}{R_B} \approx \frac{4 - 0.7}{110} mA = \underline{\underline{30 \mu A}}$$

$$I_{CE} \approx 5.5 mA \quad V_{CEQ} \approx 9V$$

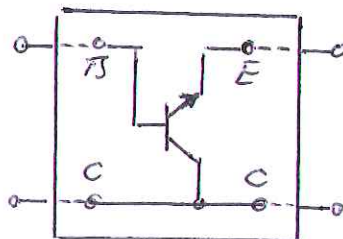
CE Configuration



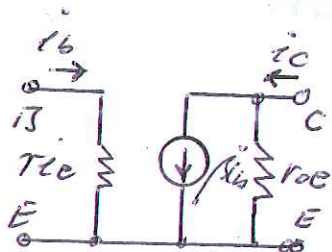
CS Configuration



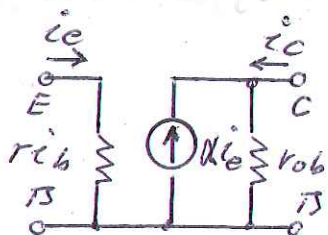
CC Configuration



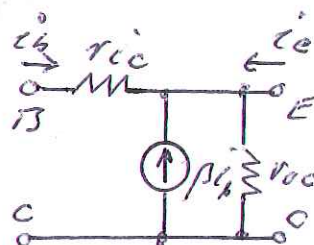
AC equivalent models



$$\left| \begin{array}{l} r_{iE} = \frac{\beta}{g_m} \\ r_{oE} = \frac{V_A}{I_{CQ}} \end{array} \right|$$



$$\left| \begin{array}{l} r_{iB} = \frac{\alpha}{g_m} \\ r_{oB} = (\beta + 1) \frac{V_A}{I_{CQ}} \end{array} \right|$$



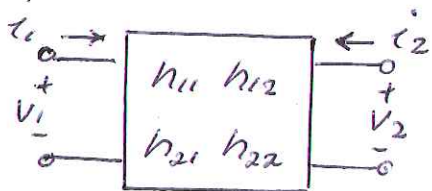
$$\left| \begin{array}{l} r_{iC} = \frac{\beta}{g_m} \\ r_{oC} = \frac{V_A}{I_{CQ}} \end{array} \right|$$

where $|g_m = \frac{I_{CQ}}{V_T}|$ $|\alpha = \frac{\beta}{1 + \beta}|$ $|\beta = \frac{i_C}{i_B}|$

Note: $\beta i_B = g_m V_{be}$ $\alpha i_e = g_m V_{be}$

Matrix Representation for 2-Ports

Hybrid Parameters (h-Parameters)



$$\begin{pmatrix} V_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ V_2 \end{pmatrix}$$

$$h_{11} = \frac{V_1}{i_1} \Big|_{V_2=0}$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{i_1=0}$$

$$h_{21} = \frac{i_2}{i_1} \Big|_{V_2=0}$$

$$h_{22} = \frac{i_2}{V_2} \Big|_{i_1=0}$$

Hybrid Parameters for BJTs

CE Configuration

$$\left| \begin{array}{ll} h_{ie} = \frac{\beta}{g_m} = \beta \frac{V_T}{I_{CQ}} & h_{re} \approx 0 \\ h_{fe} = \beta & h_{oe} = \frac{I_{CQ}}{V_A} \end{array} \right|$$

CB Configuration

$$\left| \begin{array}{ll} h_{ib} = \frac{\alpha}{g_m} = \alpha \frac{V_T}{I_{CQ}} & h_{rb} \approx 0 \\ h_{fb} = -\alpha & h_{ob} = \frac{I_{CQ}}{(\beta+1)V_A} \end{array} \right|$$

CC Configuration

$$\left| \begin{array}{ll} h_{ic} = \frac{\beta}{g_m} = \beta \frac{V_T}{I_{CQ}} & h_{rc} = 1 \\ h_{fc} = -(\beta+1) & h_{oc} = \frac{I_{CQ}}{V_A} \end{array} \right|$$

Numerical Example

$$\left| \begin{array}{ll} \beta = 100 & V_A = 100V \\ I_{CQ} = 1mA \end{array} \right|$$

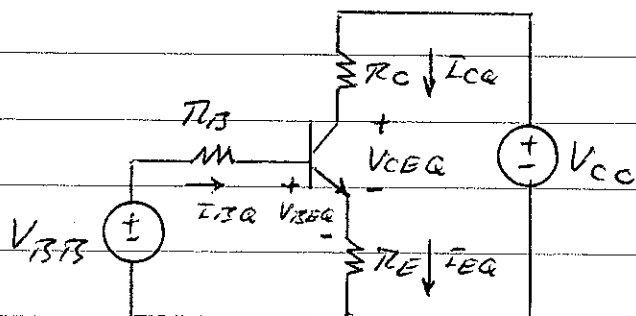
$$\left| \begin{array}{ll} h_{ie} = 2.5k\Omega & h_{re} = 0 \\ h_{fe} = 100 & h_{oe} = 10^{-5}S \end{array} \right|$$

$$\left| \begin{array}{ll} h_{ib} = 25\Omega & h_{rb} = 0 \\ h_{fb} = -0.99 & h_{ob} = 10^{-7}S \end{array} \right|$$

$$\left| \begin{array}{ll} h_{ic} = 2.5k\Omega & h_{rc} = 1 \\ h_{fc} = -101 & h_{oc} = 10^{-5}S \end{array} \right|$$

2.4 Biasing

Basic Configuration



Objective:

Describe the transistor Q-point, i.e. I_{CQ} and V_{CEQ} as a function of the two source voltages and the three resistors R_B , R_C and R_E .

Approach:

Apply KVL for both the input loops (V_{BIB} , V_{BEQ} , ...) and the output loops (V_{CC} , V_{CEQ} , ...) and use the known relationships between the currents I_{BQ} , I_{CQ} and I_{EQ} of the BJT.

Input loop:

$$\text{KVL: } V_{BIB} = I_{BQ} R_B + V_{BEQ} + I_{EQ} R_E$$

$$\text{Device: } I_{EQ} = (1 + \beta) I_{BQ}$$

$$I_{CQ} = \beta \cdot I_{BQ}$$

$$\therefore V_{BIB} = I_{BQ} R_B + V_{BEQ} + (1 + \beta) I_{BQ} R_E$$

$$\text{or } \left| I_{BQ} = \frac{V_{BIB} - V_{BEQ}}{R_B + (1 + \beta) R_E} \right|$$

$$I_{CQ} = \frac{V_{BIB} - V_{BEQ}}{R_B/\beta + (1 + \beta) R_E}$$

$$\left| I_{CQ} = \frac{V_{BIB} - V_{BEQ}}{R_B/\beta + R_E} \right|$$

Output Loop:

KVL: $V_{CC} = I_{CQ} R_C + V_{CEQ} + I_{EQ} R_E$

Device: $I_{EQ} = I_{CQ} (1 + \beta)$

$\therefore V_{CC} = I_{CQ} R_C + V_{CEQ} + I_{CQ} (1 + \beta) R_E$

or

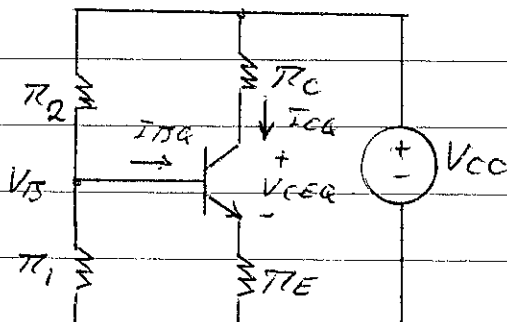
$V_{CEQ} = V_{CC} - I_{CQ} (R_C + (1 + \beta) R_E) \quad | \quad V_{CEQ} = V_{CC} - I_{CQ} (R_C + R_E)$

Q-point Equations:

$I_{CQ} = \frac{V_{B13} - V_{BEQ}}{R_B/\beta + (1 + \beta) R_E}$
$V_{CEQ} = V_{CC} - I_{CQ} (R_C + (1 + \beta) R_E)$

Practical Q-point Networks

A) Biasing with Voltage Divider



Question

What are the equivalent voltages of R_{B1} and V_{B1} ?

From basic configuration we deduce that

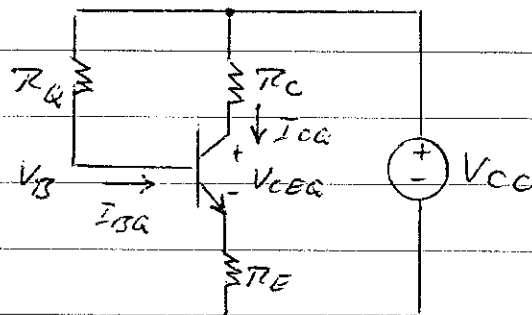
$$V_{B1B} = V_{B1} \quad I_{B1} = 0 \quad \beta_{B1} = \frac{V_{B1B} - V_{B1}}{I_{B1}}$$

$$\therefore \left\{ \begin{aligned} V_{B1B} &= V_{CC} \frac{\beta_1}{\beta_1 + \beta_2} \\ \beta_{B1} &= \frac{\beta_1 \cdot \beta_2}{\beta_1 + \beta_2} \end{aligned} \right\}$$

Thus

$$I_{CQ} = \frac{V_{CC} \frac{\beta_1}{\beta_1 + \beta_2} - V_{B1EQ}}{\frac{\beta_1 \cdot \beta_2}{(\beta_1 + \beta_2)\beta} + (1 + \frac{1}{\beta})\beta_2}$$

13) Biasing with Base Injection

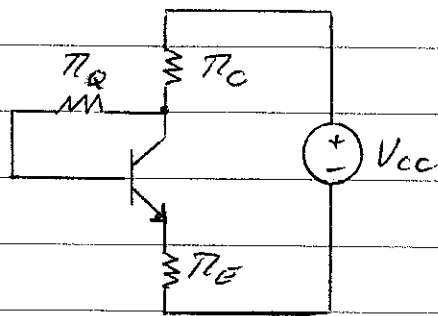


We deduce from the basic configuration that

$$\left\{ \begin{aligned} V_{B1B} = V_{B1} &= V_{CC} \quad \beta_{B1} = \frac{V_{B1B} - V_{B1}}{I_{B1}} = \beta_{B1} \\ I_{B1} &= 0 \end{aligned} \right\}$$

$$\left\{ I_{CQ} = \frac{V_{CC} - V_{B1EQ}}{\beta_1/\beta + (1 + \frac{1}{\beta})\beta_2} \right\}$$

c) Voltage Feedback Biasing



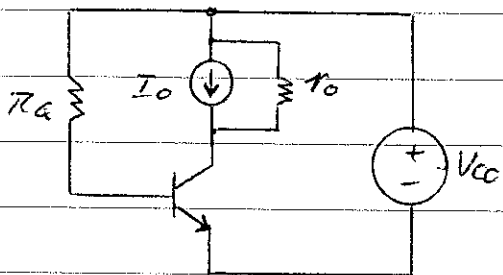
We deduce from basic configuration

$$\left\| \begin{aligned} V_{B13} = V_B &= V_{CC} \\ I_{B13} = 0 \end{aligned} \right. \quad R_{B13} = \frac{V_{B13} - V_B}{I_{B13}} = R_B + (1 + \beta) R_C \left\|$$

$$\left\| I_{CC} = \frac{V_{CC} - V_{BEQ}}{R_B / \beta + (1 + \beta)(R_C + R_E)} \right\|$$

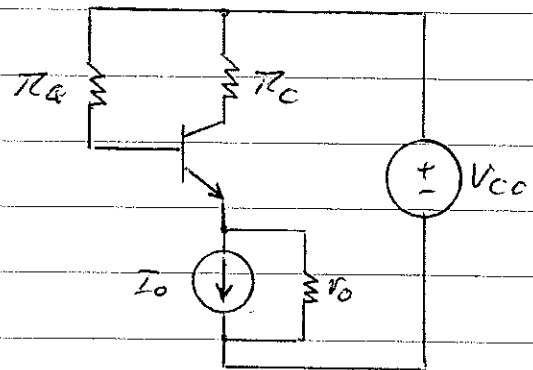
d) Current Source Biasing

Version A



$$\left\| I_{CC} = I_0 + \frac{V_{CC} - V_{CEQ}}{R_C} \right\|$$

Version B

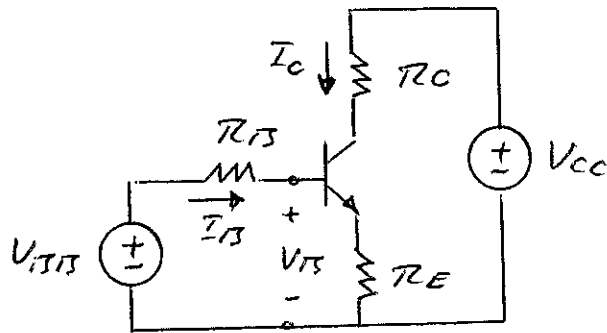


$$\left\| I_{CC} = (1 + \beta) \left[I_0 + \frac{V_{CC} - V_{CEQ} - I_{CC} R_C}{\beta} \right] \right\|$$

Note: R_B is not a free parameter. It has to be adjusted to provide the corresponding I_B .

Bipolar Transistor Biasing Summary

Basic Configuration



Def.:

$$V_{BIB} = V_{BE} \Big|_{I_B = 0}$$

$$R_B = \frac{V_{BIB} - V_{BE}}{I_B}$$

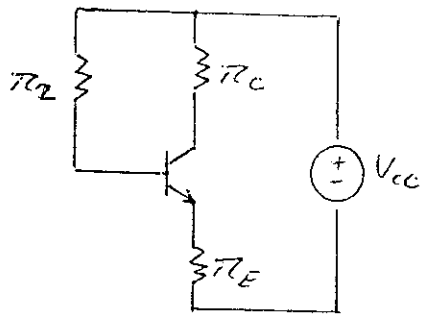
Q-Point Equations:

$$I_{CQ} = \frac{V_{BIB} - V_{BEQ}}{\frac{R_B}{\beta} + (1 + \frac{1}{\beta}) R_E}$$

$$V_{CEQ} = V_{CC} - I_{CQ} (R_C + [1 + \frac{1}{\beta}] R_E)$$

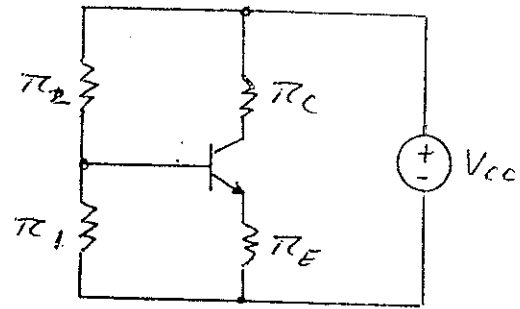
Silicon: $V_{BEQ} \cong 0.7V$

Base current Bias



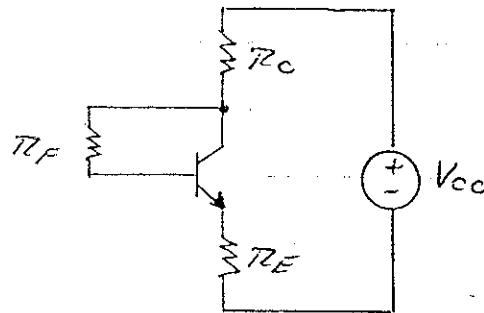
$$R_{B1} = R_2$$
$$V_{B1} = V_{CC}$$

Voltage divider Bias



$$R_{B1} = R_1 \parallel R_2$$
$$V_{B1} = V_{CC} \frac{R_2}{R_1 + R_2}$$

Voltage Feedback Bias



$$R_{B1} = R_F + (1 + \beta) R_C$$
$$V_{B1} = V_{CC}$$

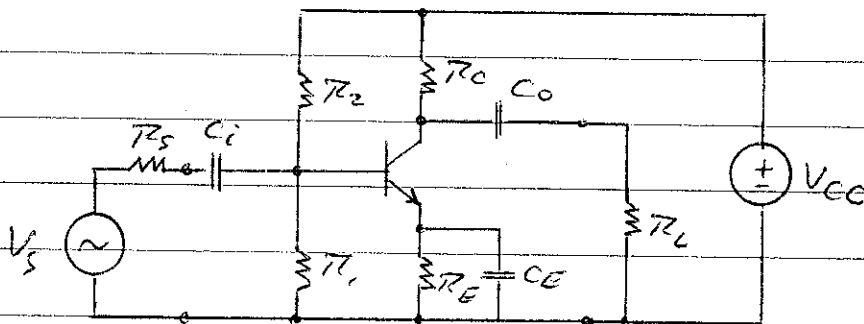
Examples

① Design a common emitter amplifier which realizes a voltage gain of -100. Pick an appropriate Q-point and determine the necessary values for the amplifiers input and output resistance.

$V_{cc} = 10V$, Transistor: $\beta_F = 150$, $V_A = 80V$, $r_s = 50\Omega$
 $r_L = 10k\Omega$

Approach:

1. Circuit Topology



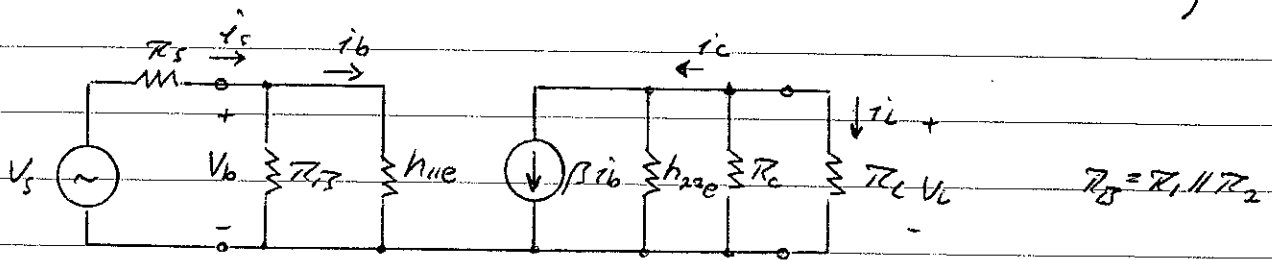
2. Q-point considerations

$I_{CQ} \approx \frac{V_{BIB} - V_{BEQ}}{R_B/\beta + R_E}$	where $V_{BIB} = V_{cc} \frac{R_2}{R_1 + R_2}$
$V_{CEQ} \approx V_{cc} - I_{CQ}(R_C + R_E)$	$R_{BIB} = R_1 \parallel R_2$

select $|I_{CQ} \approx 1mA|$ $\therefore |h_{ie} = \beta \frac{V_T}{I_{CQ}} \approx 4.5k\Omega|$
 $V_{CEQ} \approx V_{cc} - I_{CQ}(R_C + R_E)$ $|h_{22e} \approx \frac{I_{CQ}}{V_A} \approx 12.5\mu S|$

3. AC considerations

ac equivalent circuit (C_i, C_o and C_E act as ac shorts)



equations:

Input loop: $V_s = (i_b + \frac{V_b}{R_B})R_S + i_b h_{ie}$ (1)

Output loop: $V_L = - (R_L || R_C || \frac{1}{h_{oe}})$ (2)

Ohm's law: $V_b = i_b h_{ie}$ (3)

from (1) and (3)

$$V_s = i_b \left(R_S \left[1 + \frac{h_{ie}}{R_B} \right] + h_{ie} \right) \quad (4)$$

$$\therefore \frac{V_L}{V_s} = A_V = - \frac{R_L || R_C || \frac{1}{h_{oe}} \cdot \beta}{R_S \left[1 + \frac{h_{ie}}{R_B} \right] + h_{ie}}$$

or $A_V = - \frac{R_L || R_C || \frac{1}{h_{oe}}}{R_S \left[\frac{1}{\beta} + \frac{r_d}{R_B} \right] + r_d}$ (5)

If $r_d \ll R_B$

$$R_L || R_C || \frac{1}{h_{oe}} \cong A_V \cdot r_d \cong 3 k\Omega$$

$$\therefore R_C \cong 4.5 k\Omega$$

Select $R_{C_{nom}} = 4.7 k\Omega$
 $V_{CEQ} \cong 4.9 V$ (to maximize the output swing)

$\therefore R_E \cong 400 \Omega$
 $V_{BQ} \cong 1.1 V$

$$\bar{V} = 14e$$

select	$\pi_E = 470\Omega$	560Ω
	$\pi_1 = 10k\Omega$	4.7kΩ

\therefore	$\pi_2 \approx 68k\Omega$	33kΩ
--------------	---------------------------	------

	$\pi_B \approx 8.7k\Omega$	4.1kΩ
--	----------------------------	-------

4. Check Specifications

• Q-point:	$I_{CQ} \approx \frac{V_{CC} \frac{\pi_1}{\pi_1 + \pi_2} - V_{BEQ}}{\pi_B/\beta + \pi_E} \approx 1.1mA$	0.91mA
	$V_{CEQ} \approx V_{CC} - I_{CQ}(\pi_C + \pi_E) \approx 4.3V$	5.2V

• AC Gain

$$A_V \approx - \frac{\pi_C \parallel \pi_L \parallel \frac{1}{h_{FE}}}{\pi_E \left[\frac{1}{\beta} + \frac{r_d}{\pi_B} \right] + r_d} \approx -110$$

Adjust Gain by either

• Reducing π_C by 10%

• Reducing I_{CQ} by 10% (i.e. increase π_E by 10%
or reduce π_1 by approx 10%)

c.g. $\pi_C = 4.3k\Omega$	$I_{CQ} \approx 1.1mA$
	$V_{CEQ} \approx 4.7V$
	$A_V \approx -100$

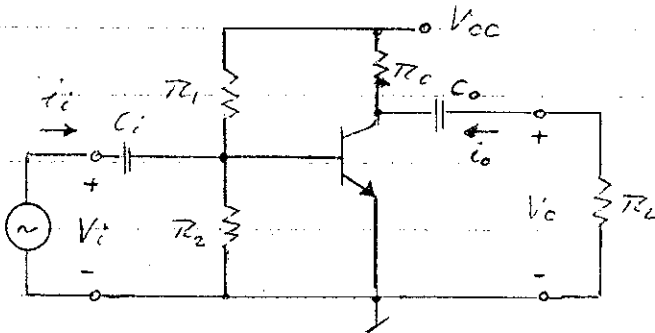
or

$\pi_E \approx 520\Omega$	$I_{CQ} \approx 1.0mA$
	$V_{CEQ} \approx 4.8V$
	$A_V \approx -100$

2.5 Common Emitter Amplifier

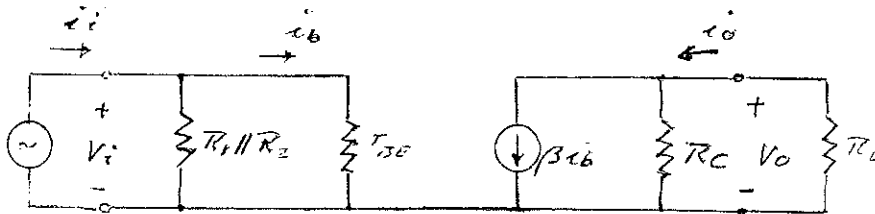
(small signal behavior)

Complete Amplifier Circuit with Biasing

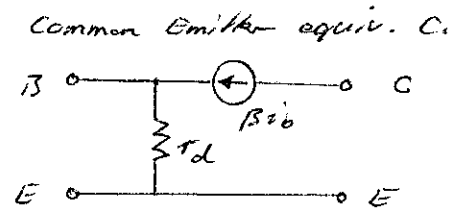


C_i, C_o Infinite Coupling Capacitors
They block DC currents while permitting AC signals to pass

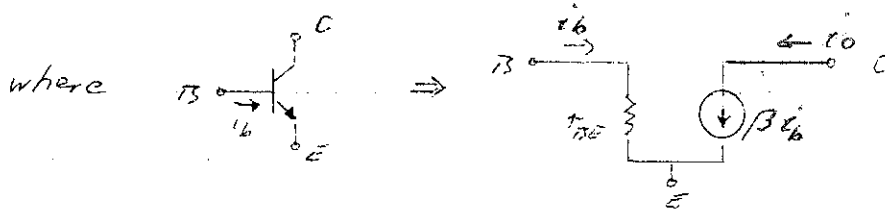
Small signal equivalent circuit



Recall:



$r_{BE} = (1 + \beta) r_d$



A) small-signal voltage gain

$$A_V = \frac{V_o}{V_i} = - \frac{\beta \cdot r_{BE} \parallel r_L}{r_{BE}}$$

input

output

$V_i = i_b r_{BE}$

$V_o = -i_o r_L$

$i_i = i_b + \frac{V_i}{r_{BE}}$

$i_o = \beta i_b + \frac{V_o}{r_L}$

e.g. $\beta = 100$ ($I_{CC} \approx 5mA$)

$r_{BE} = r_L = 1k\Omega$

\Rightarrow

$A_V = -100$

$r_{BE} = 500\Omega$

B) small-signal current gain

$$A_i = \frac{i_o}{i_i} = \beta \frac{R_C \parallel R_L}{R_L}$$

e.g. $\beta = 100$

$$R_C = R_L = 1k\Omega \Rightarrow \underline{\underline{A_i = 50}}$$

c) Input resistance (current source i_i at input)

$$R_{in} = \frac{V_i}{i_i} = R_1 \parallel R_2 \parallel r_{BE}$$

e.g. $R_1 = 120k\Omega$

$R_2 = 8.2k\Omega$

$r_{BE} = 500\Omega$

$$\Rightarrow \underline{\underline{R_{in} = 470\Omega}}$$

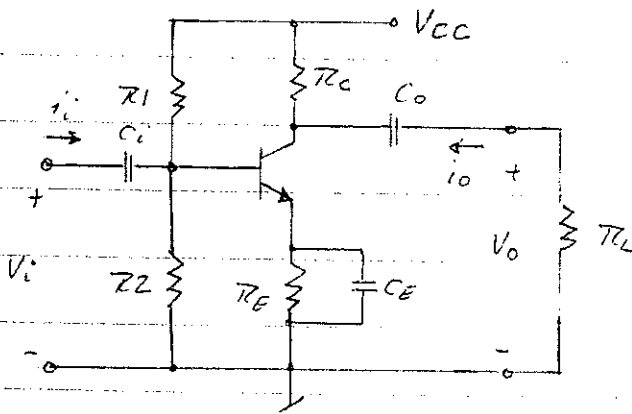
d) output resistance (current source i_o at output)

$$R_{out} = \frac{V_o}{i_o} = R_C$$

$R_C = 1k\Omega$

$$\Rightarrow \underline{\underline{R_o = 1k\Omega}}$$

Common Emitter Amplifier with current feedback



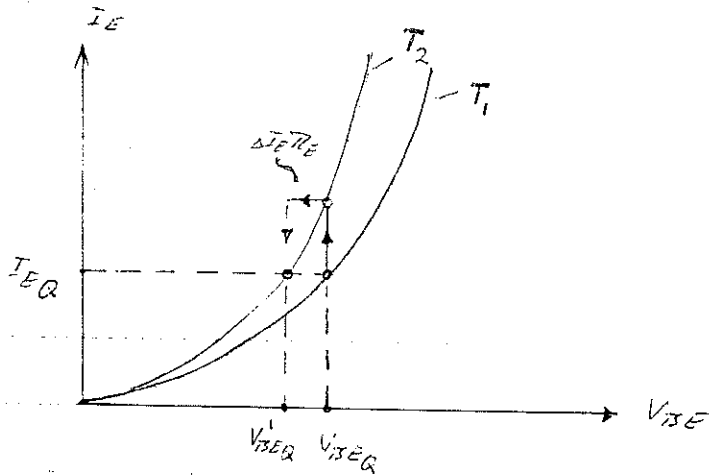
C_i, C_o Infinite Coupling capacitors

C_E Infinite Bypass capacitor

(short circuit for signal frequencies)

\Rightarrow AC performance of amplifier does not change.*

Input characteristics



$T_2 > T_1$

Since $I_{EQ} \cong \text{const} \Rightarrow V_{CEQ} \cong \text{const}$.

$\Rightarrow R_{FE}$ stabilizes Q-point

* notes without C_E ,
$$A_V = -\beta \frac{R_C \parallel R_L}{r_{BE} + (1+\beta)R_E} \cong -\frac{R_C \parallel R_L}{r_d + R_E}$$

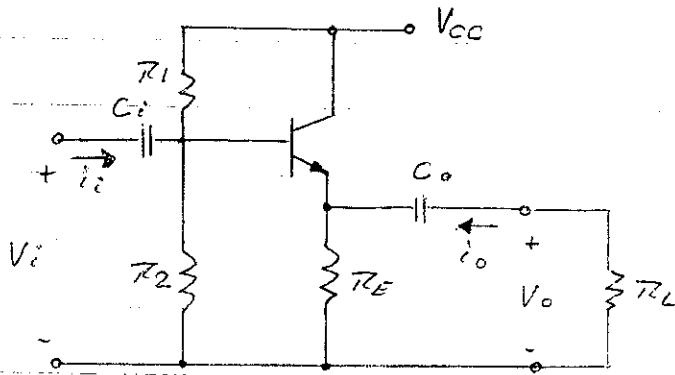
e.g. $R_C = R_L = 1k\Omega$

$\beta = 100, r_{BE} = 500\Omega$

$R_E = 100\Omega$

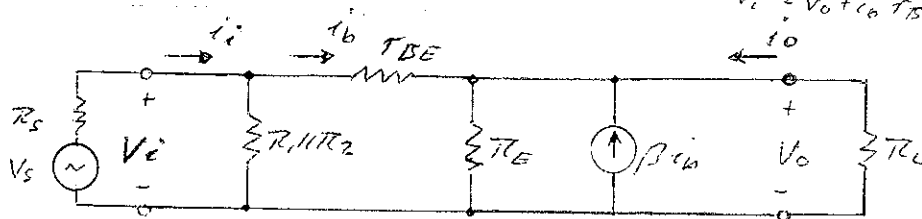
$\Rightarrow A_V \cong -5$

2.6 Common Collector Circuit (Emitter Follower)



C_i, C_o : Infinite coupling cap.

small signal equivalent circuit



input

$$V_s = V_i + i_i R_s$$

$$i_i = i_b + \frac{V_i}{R_1 || R_2}$$

$$V_i = V_o + i_o r_{BE}$$

output

$$V_o = -i_o R_L$$

$$i_o = -(1+\beta)i_b + \frac{V_o}{R_E}$$

$$r_{BE} = (1+\beta)r_{de}$$

A) small signal voltage gain

$$A_v = \frac{V_o}{V_i} = \frac{1}{1 + r_{de} / (R_E || R_L)}$$

e.g. $r_{de} = 5 \Omega$

$$R_E = R_L = 200 \Omega \quad 1k \Omega$$

$$\Rightarrow \underline{A_v = 0.95} \quad 0.99$$

B) small signal current gain

$$V_i = \frac{i_o}{i_i} = -(1+\beta) \frac{R_E}{(R_E + R_L)} \frac{1}{\left(1 + \frac{\beta R_E + r_{de}}{R_1 || R_2}\right)}$$

e.g. $R_E = R_L = 200 \Omega \quad 1k \Omega$

-8.3

$\beta = 100, r_{de} = 500 \Omega$

$\Rightarrow \underline{V_i = -25}$

$R_1 || R_2 = 20k \Omega$

c) Input resistance

$$\pi_i = \frac{V_i}{I_i} = (R_1 \parallel R_2) \parallel (r_{BE} + [1 + \beta] R_E \parallel R_L)$$

e.g. $R_E = R_L = 200 \Omega$

$\beta = 100$; $r_{BE} = 500$ $\Rightarrow \underline{\underline{\pi_i = 7.22 \text{ k}\Omega}}$

$R_1 \parallel R_2 = 20 \text{ k}\Omega$

d) Output resistance

$$\pi_o = \frac{V_o}{I_o} = R_E \parallel \frac{r_{BE} + R_1 \parallel R_2 \parallel R_S}{1 + \beta}$$

e.g. $R_E = 200 \Omega$; $R_S = 100 \Omega$

$\beta = 100$; $r_{BE} = 500$ $\Rightarrow \underline{\underline{\pi_o \approx 6 \Omega}}$

$R_1 \parallel R_2 = 20 \text{ k}\Omega$

This type of circuit is primarily used to provide a low source impedance (π_o is small) for driving subsequent loads. (π_i is rather great even though R_L is very small).

Class A CC Amplifier Power Efficiency

Total average Power:

$$\left| \overline{P}_{Tot} = 2V_{CC} \cdot (\overline{I}_{CC} + I_C \sin^2(\omega t)) = 2V_{CC} \cdot \overline{I}_{CC} \right|$$

where $\overline{I}_{CC} = \frac{V_{CC} - V_{BE}}{\pi R_E}$

$$\therefore \left\| \overline{P}_{Tot} = \frac{2V_{CC}(V_{CC} - V_{BE})}{\pi R_E} \right\|$$

Average load Power: (Max)

$$\left| \overline{P}_{L_{Max}} = V_{L_{Max}}^2 \frac{1}{\pi R_L} \sin^2(\omega t) = \frac{V_{L_{Max}}^2}{2\pi R_L} \right|$$

where $V_{L_{Max}} = \overline{I}_{CC} \frac{\pi R_E \pi R_L}{(\pi R_E + \pi R_L)} = \frac{(V_{CC} - V_{BE}) \pi R_L}{(\pi R_E + \pi R_L)}$

$$\therefore \left\| \overline{P}_{L_{Max}} = \frac{(V_{CC} - V_{BE})^2 \pi R_L}{2(\pi R_E + \pi R_L)^2} \right\|$$

Power Efficiency:

$$\left| \eta_{Max} = \frac{\overline{P}_{L_{Max}}}{\overline{P}_{Tot}} = \frac{(V_{CC} - V_{BE})^2 \pi R_L \cdot \pi R_E}{4(\pi R_E + \pi R_L)^2 V_{CC} (V_{CC} - V_{BE})} \right|$$

$$\therefore \left\| \eta_{Max} = \frac{(V_{CC} - V_{BE}) \pi R_L \pi R_E}{4 V_{CC} (\pi R_E + \pi R_L)^2} \right\|$$

For max Efficiency: $\pi R_E = \pi R_L$

$$\therefore \left| \eta_{Max} = \frac{(V_{CC} - V_{BE})}{16 V_{CC}} \right|$$

Class A/B CC Amplifier Power Efficiency

Total average Power:

$$\overline{P_{Tot\ max}} = V_{CC} \cdot \frac{V_{L\ max}}{T_L} \overline{\sin(\omega t)} = V_{CC} \frac{V_{L\ max}}{T_L} \frac{2}{\pi}$$

Half-wave only

where $V_{L\ max} \cong V_{CC} - V_{BE}$

$$\therefore \left| \overline{P_{Tot\ max}} = \frac{V_{CC}(V_{CC} - V_{BE})}{T_L} \frac{2}{\pi} \right|$$

Average load Power: (Max)

$$\overline{P_L} = V_{L\ max}^2 \frac{1}{T_L} \overline{\sin^2(\omega t)} = \frac{V_{L\ max}^2}{2T_L}$$

$$\left| \overline{P_{L\ max}} = \frac{(V_{CC} - V_{BE})^2}{2T_L} \right|$$

Power Efficiency:

$$\left| \eta_{\max} = \frac{\overline{P_{L\ max}}}{\overline{P_{Tot\ max}}} = \frac{(V_{CC} - V_{BE})^2 T_L \cdot \pi}{2 T_L V_{CC} (V_{CC} - V_{BE}) \cdot 2} \right|$$

$$\therefore \left| \eta_{\max} = \frac{(V_{CC} - V_{BE})}{V_{CC}} \frac{\pi}{4} \right|$$

$$\therefore \left| \frac{\eta_{\max\ A/B}}{\eta_{\max\ A}} = \frac{\pi}{4} \right|$$

Bipolar Transistor Circuits - Summary

Common Emitter	Common Base	Common Collector
$r_{cb} \cong \beta \frac{V_A}{I_C}$	$r_d \cong \frac{kT}{qI_C}$	
$[h_e] \cong \begin{bmatrix} \beta r_d & 0 \\ \beta & \frac{\beta}{r_{cb}} \end{bmatrix}$	$[h_b] \cong \begin{bmatrix} r_d & 0 \\ -\alpha & \frac{1}{r_{cb}} \end{bmatrix}$	$[h_c] \cong \begin{bmatrix} \beta r_d & 1 \\ -\beta & \frac{\beta}{r_{cb}} \end{bmatrix}$
$A_V \cong - \frac{R_c \parallel R_L}{r_d}$	$A_V \cong \frac{R_c \parallel R_L}{r_d + r_e}$	$A_V \cong \frac{1}{1 + \frac{r_d}{r_e \parallel R_L}}$
$A_i \cong \frac{\beta}{(1 + \frac{R_L}{r_c})(1 + \beta \frac{r_d}{r_e})}$	$A_i \cong - \frac{1}{1 + \frac{r_e}{r_c}}$	$A_i \cong - \frac{\beta}{(1 + \frac{R_L}{r_e})(1 + \beta \frac{r_d \parallel R_L}{r_e})}$
$R_i \cong r_e \parallel \beta r_d$	$R_i \cong r_d$	$R_i \cong r_e \parallel (\beta r_e \parallel R_L)$
$R_o \cong r_c$	$R_o \cong r_c$	$R_o \cong r_d + \frac{r_e}{\beta}$

$$r_d = \frac{V_T}{I_{EQ}} \cong \frac{V_T}{I_{CQ}}$$