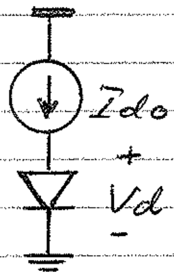
Problem We would like to use a pn junction as a thermal sensor.
To do so, we keep the junction current constant and measure the junction voltage as a function of temperature.
Use the diode equation



$$\boxed{I_d \cong I_s\, e^{\frac{V_d}{V_T}}} \quad \text{where } I_s = A\cdot q\, n_{io}^2 \frac{D_x}{L_x}\frac{1}{N_x}$$

to find an expression for the expected change of the diode voltage Vd

Solution

1. We assume that $\frac{D_x}{L_x}$ approximately remains constant over the temperature range of interest.

$$\therefore \boxed{\frac{dI_s}{dT} \cong \frac{dn_{io}^2}{dT}}$$

2. From $n_{io} = N_c(T)\, e^{-\frac{E_c - E_{Fi}}{kT}} = N_v(T)\, e^{-\frac{E_{Fi}-E_v}{kT}}$

we obtain

$$n_{io}^2(T) = N_c(T)\cdot N_v(T)\, e^{-\frac{E_c - E_v}{kT}}$$

where

$$N_c(T)\cdot N_v(T) = 4\left(\frac{2\pi kT}{h^2}\right)^3 \left(m_n^a m_p^a\right)^{3/2}$$

We can thus write

$$I_S(T) = I_{S0}\left(\frac{T}{T_0}\right)^3 e^{\frac{E_G}{kT_0}\left(1-\frac{T_0}{T}\right)}$$

The derivative of I_S w.r.t. T is

$$\frac{dI_S(T)}{dT} = \underbrace{I_{S0}\left(\frac{T}{T_0}\right)^3 e^{\frac{E_G}{kT_0}\left(1-\frac{T_0}{T}\right)}}_{I_S(T)}\left[\frac{3}{T} + \frac{E_G}{kT^2}\right]$$

$$\therefore \left\| \frac{dI_S(T)}{dT} = \frac{I_S(T)}{T}\left[3 + \frac{E_G}{kT}\right] \right\| \qquad (1)$$

<u>Note:</u> $\frac{E_G}{kT} = \frac{V_G}{V_T}$ where $V_G = \frac{E_G}{q}$ Bandgap voltage

3. Solving the original diode equation for the diode voltage yields

$$\left| V_d \cong V_T \ln\left(\frac{I_d}{I_S}\right) \right| \quad \text{where } I_d = I_{do}$$

$$\left\| \frac{dV_d(T)}{dT} = \frac{dV_T}{dT}\ln\left(\frac{I_{do}}{I_S}\right) - V_T\frac{1}{I_S}\frac{dI_S}{dT} \right\| \qquad (2)$$

4. Finally, inserting (1) into (2) yields

$$\frac{dV_d(T)}{dT} = \frac{V_T}{T}\ln\left(\frac{I_{do}}{I_S}\right) - \frac{V_T}{T}\left[3 + \frac{V_G}{V_T}\right]$$

or

$$\left\| \frac{dV_d(T)}{dT} = \frac{1}{T}\left[V_d - 3V_T - V_G\right] \right\| \qquad (3)$$

Numerical Solutions

Find the values for $\frac{dV_d(T)}{dT}$ between 300k and 400k if we know that V_d @ 300k is 0.7V

T [k]	dVd/dT [mV/k]	Vd [mV]
300	-1.659	700.0
320	-1.675	666.8
340	-1.690	633.3
360	-1.704	599.5
380	-1.718	565.4
400	-1.731	531.0

$V_{G_{Si}} = 1.12V$

Notes: The numerical values depend crucially on the initial diode voltage

second order approximation for diode voltage: (300k - 400k)

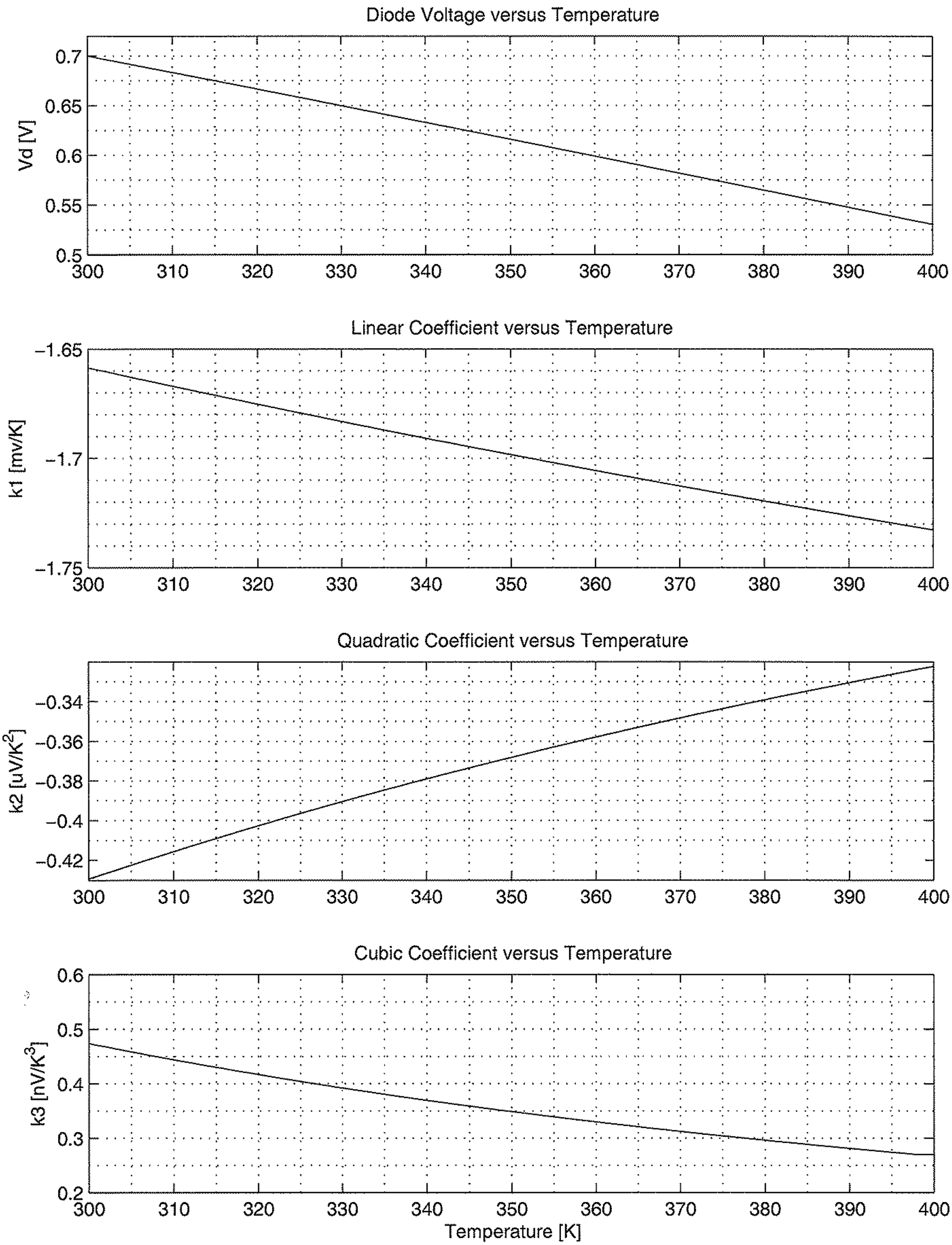
$$\left\| V_d(T) \cong V_{d0} + \Delta T k_1 + \Delta T^2 k_2 \right\|$$

where

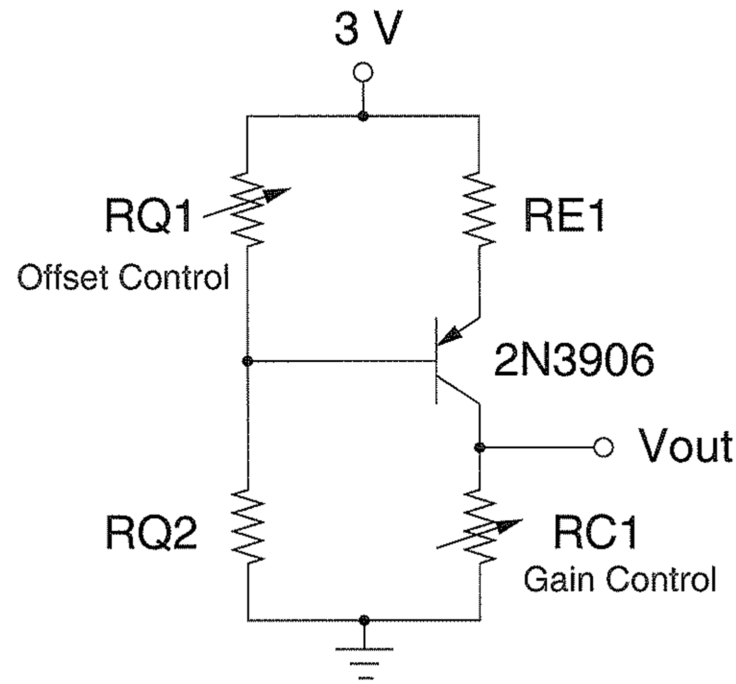
$k_1 \cong -1.697\ mV/k$	Values are referenced to 350K
$k_2 \cong -0.37\ \mu V/K^2$	

Notes: If one were to compensate for the linear deviation of the diode voltage (e.g. in a voltage reference circuit), one would end up with a quadratic (parabolic) error, i.e., $\Delta V_d = k_2 \cdot \Delta T^2$

VI - P4


Diode Voltage versus Temperature
Vd [V]
0.7
0.65
0.6
0.55
0.5
300 310 320 330 340 350 360 370 380 390 400
Linear Coefficient versus Temperature
k1 [mv/K]
−1.65
−1.7
−1.75
300 310 320 330 340 350 360 370 380 390 400
Quadratic Coefficient versus Temperature
k2 [uV/K^2]
−0.34
−0.36
−0.38
−0.4
−0.42
300 310 320 330 340 350 360 370 380 390 400
Cubic Coefficient versus Temperature
k3 [nV/K^3]
0.6
0.5
0.4
0.3
0.2
300 310 320 330 340 350 360 370 380 390 400
Temperature [K]

Single Transistor Thermal Sensor

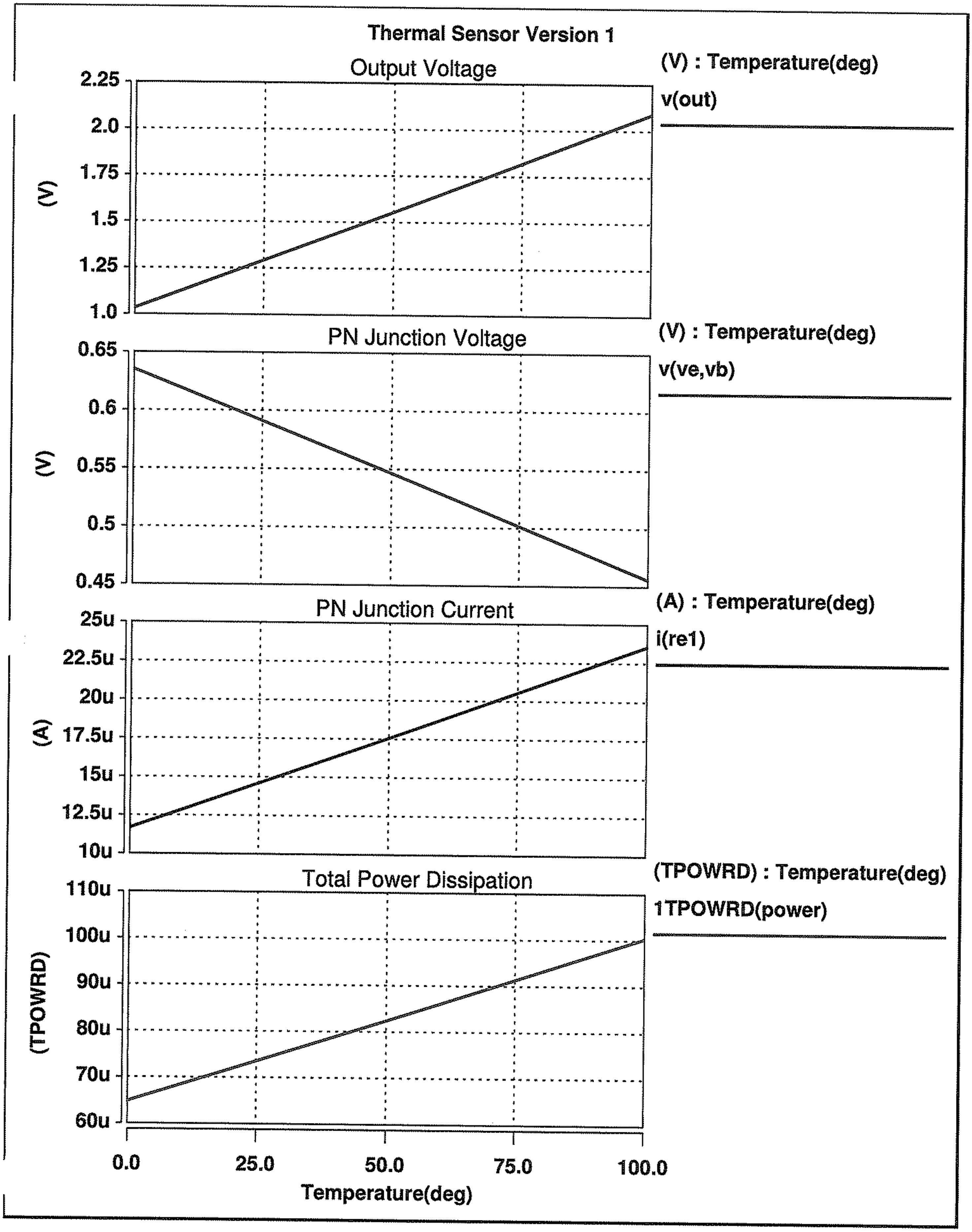


Component Power Dissipation

RQ1 = 82k	8.1 uW	RE1 = 15k	8.3 uW (100 C) 2.0 uW (0 C)
RQ2 = 220k	21.8 uW	RC1 = 90k	49.8 uW (100 C) 12.2 uW (0 C)
		2N3906	12.4 uW (100 C) 20.7 uW (0 C)

$$\boxed{Vout = k1\triangle T + k2\triangle T^2}$$

@ T=50 C   k1 = 10.67 mV/K   k2 = 2.6 uV/K^2


Thermal Sensor Version 1
Output Voltage
(V) : Temperature(deg)
v(out)
(V)
2.25
2.0
1.75
1.5
1.25
1.0
PN Junction Voltage
(V) : Temperature(deg)
v(ve,vb)
(V)
0.65
0.6
0.55
0.5
0.45
PN Junction Current
(A) : Temperature(deg)
i(re1)
(A)
25u
22.5u
20u
17.5u
15u
12.5u
10u
Total Power Dissipation
(TPOWRD) : Temperature(deg)
1TPOWRD(power)
(TPOWRD)
110u
100u
90u
80u
70u
60u
0.0
25.0
50.0
75.0
100.0
Temperature(deg)