Scattering of Plane Electromagnetic Waves at the Junction Formed by a PEC half-plane and a half-plane with Anisotropic Conductivity

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Abstract

In this study, scattering of plane electromagnetic waves at the junction formed by a PEC half-plane and a half-plane with anisotropic conductivity is investigated. By using Fourier Transform technique the problem is formulated into a matrix Wiener-Hopf system and an exact closed-form solution is obtained for the most general case by factorizing a 2 × 2 Wiener-Hopf matrix. Also, four different special cases are examined which in all, by using Fourier Transform technique the problem is formulated into a pair of simultaneous Wiener-Hopf equations which are decoupled via a polynomial transformation and solved through the standard procedure. The diffracted field is expressed in a form suitable for GTD applications and the effects of the resistivities of the anisotropic half-plane on the diffraction coefficient are also investigated.

I. Introduction

As is known, the scattering from any body is a function of both its geometrical and material properties. In recent years there has been a renewed interest in investigating the influence of material properties on edge diffraction. In particular, the edge diffraction by a half plane of finite, isotropic conductivity has been studied by severel authors [1, 2]. In these works, the electromagnetic property of imperfectly conducting surface is specified by its scalar resistivity $R = (1/\sigma t)$ with σ being the conductivity, and t is the thickness which is assumed to be small compared with the wavelength. In the most general case, as noted by Senior[3], the surface resistivity may be anisotropic supporting electric current sheets in directions parallel to both axes of the plane. For such a plane located at $y = 0$, a constant resistivity tensor can be written in the following dyadic form:

$$\overline{\overline{R}} = R_1\widehat{x}\widehat{x} + R_2\widehat{z}\widehat{z} \tag{1}$$

where $\widehat{x}$ and $\widehat{z}$ denote the unit vectors in Cartesian coordinate system. Here, R_1 and R_2 represent

$$R_1 = (1/\sigma_x t) \quad \text{and} \quad R_2 = (1/\sigma_z t) \tag{2}$$

with σ_x and σ_z being the conductivities in the x and z directions, respectively. In this work, scattering of plane electromagnetic waves at the junction formed by a PEC half-plane and a half-plane with anisotropic conductivity is considered for the oblique incidence case. The PEC half-plane is located at $y = 0$, $x < 0$ and at $y = 0$, $x > 0$ another half-plane is located with anisotropic resistivity where the half-plane is of finite resistivities R_2 and R_1 in the directions z and x, respectively (see Fig.-1). The structure is simulated by standard resistive boundary conditions and the problem is formulated by Fourier transform technique. The formal solution is derived for the diffraction problem by employing Daniele-Khrapkov method. While an exact closed form solution is obtained by factorizing a 2 × 2 Wiener-Hopf matrix, it is quite complicated not only algebraically, but also it contains no less than six transcendental functions. This complexity is not too surprising for this type of anisotropy. The explicit expressions of the solution can be obtained for the special case when $R_1 \cdot R_2 << 1$. Also, 4 different special cases are examined which in all, by using Fourier transform technique, the problem can be formulated into a pair of simultaneous Wiener-Hopf equations which are decoupled via a polynomial transformation and solved through the standard procedure.

II. Formulation of the Problem

A plane electromagnetic wave given by

$$\vec{E}^i(x,y,z) = (A_x^i, A_y^i, A_z^i)e^{-ik\sin\theta_o(x\cos\phi_o + y\sin\phi_o)}e^{ikz\cos\theta_o} \tag{3}$$

and satisfying

$$\vec{k}\cdot\vec{A^i} = 0 \tag{4}$$

is incident upon the $y = 0$ plane where the negative half of which ($x < 0$) is PEC, while the positive half ($x > 0$) has anisotropic conductivity at oblique incidence. The time dependence is assumed as $exp(-i\omega t)$ and the z-dependence as $exp(ik_z)$ of the incident field, which are common to all field

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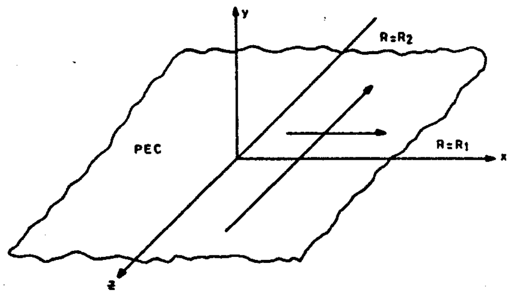

Figure-1. Geometry of the problem.

quantities, will be suppressed throughout the anlysis. In the above expressions θ_o is the measure of obliquity with $\theta_o = \pi/2$ corresponding to the incidence in a plane perpendicular to the edge. k is the free-space wave number of the medium and ω is the angular frequency of the field. To make the incident and consequently the scattered field Fourier integrable with respect to x, we assume that the wave number has a small positive imaginary part. Then the lossless case can be obtained by making $\mathrm{Im}\, k \longrightarrow 0$ in the final expressions.

The half-plane which assumed to have a finite conductivity of R_1 in the x-direction and of R_2 in the z-direction can be characterised by the following general anisotropic resistivity conditions given by Senior[3] :

$$\widehat{y} \times \left[\vec{E}\,(x,+0) - \vec{E}\,(x,-0)\right] = 0 \quad , \quad x > 0 \qquad (5)$$

$$\widehat{y} \times \left[\widehat{y} \times \vec{E}\,(x,+0)\right] = -\overline{\overline{R}}\,\widehat{y} \times \left[\vec{H}\,(x,+0) - \vec{H}\,(x,-0)\right] \quad , \quad x > 0\ . \qquad (6)$$

where $\widehat{y}$ is the unit vector directed along the y-axis. Here, $\vec{E}$ and $\vec{H}$ denote the total fields which are written as the sum of the incident and scattered field components

$$\vec{E}\,(\vec{H}) = \vec{E}^{i}\,(\vec{H}^{i}) + \vec{E}^{s}\,(\vec{H}^{s}) \qquad (7)$$

for all y.

As is known, to obtain the scattered fields, it is sufficient to consider the $x-$ and z-components of the electric field. In order to be able to use the above boundary conditions, the tangential components of the magnetic field must also be known. These components can be derived easily from E_x^s and E_z^s via Maxwell's equations, and in order to determine the representaion for E_y, $div\,\vec{D} = 0$ will be used.

For E_x^s and E_z^s which satisfy the reduced wave equation of the half-plane, one can assume the following integral representations:

$$E_z^s = \int_L A_{\pm}(\alpha)\, e^{-i\alpha x \pm i\Gamma(\alpha)y} d\alpha + E_z^r\,(E_z^t) \quad , \quad y \gtrless 0 \qquad (8)$$

$$E_x^s = \int_L B_{\pm}(\alpha)\, e^{-i\alpha x \pm i\Gamma(\alpha)y} d\alpha + E_x^r\,(E_x^t) \quad , \quad y \gtrless 0 \qquad (9)$$

where $\Gamma(\alpha) = \sqrt{N^2 - \alpha^2}$ with $N = \sqrt{k^2 - k_z^2} = k \sin\theta_0$. The square root function $\Gamma(\alpha)$ is defined in the complex α-plane cut as shown in Fig.-2, such that $\Gamma(0) = N$.

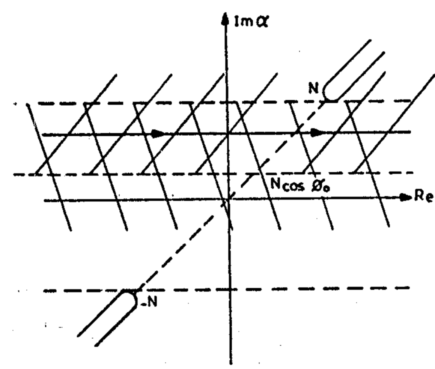

Figure-2. Complex α-plane and position of integration line L, where the regularity band is determined by $\mathrm{Im}(\alpha) < \mathrm{Im}(N)$ and $\mathrm{Im}(\alpha) > \mathrm{Im}(N\cos\phi_0)$.

The terms E_x^r and E_z^r are defined by

$$E_x^r\,(E_x^t) = R_x\,(T_x)\, A_x^i e^{-ik_x x \pm ik_y y} \quad , \quad y \gtrless 0 \qquad (10)$$

$$E_z^r\,(E_z^t) = R_z\,(T_z)\, A_z^i e^{-ik_x x \pm ik_y y} \quad , \quad y \gtrless 0\ . \qquad (11)$$

In the eqs. $(8-11)$, the $(+)$ signs together with the reflection terms are used in $y > 0$ half space while the $(-)$ signs together with the transmission terms are used in $y < 0$ half space. $R_x\,(T_x)$ and $R_z\,(T_z)$ denote the reflection(transmission) coefficients related to the x and z components of the electric field that would be reflected(transmitted) if the whole plane $y = 0$ were characterized by a constant surface resistance R.

By using Maxwell's equations and eqns. $(8-11)$, H_z^s and H_x^s can now be obtained as

$$H_z^s = \frac{1}{\omega\mu} \int_L \left\{ \frac{\alpha}{\pm\Gamma(\alpha)} k_z A_\pm(\alpha) - \left[\pm\Gamma(\alpha) + \frac{\alpha^2}{\pm\Gamma(\alpha)} \right] \cdot B_\pm(\alpha) \right\} e^{-i\alpha x \pm i\Gamma(\alpha) y} d\alpha - \frac{1}{\pm k_y \omega\mu} \cdot \left\{ \left(k_x^2 + k_y^2\right) R_x(T_x) A_x^i - k_x k_z R_z(T_z) A_z^i \right\} e^{-ik_x x \pm i k_y y} \tag{12}$$

$$H_x^s = \frac{1}{\omega\mu} \int_L \left\{ \left[\pm\Gamma(\alpha) + \frac{k_z^2}{\pm\Gamma(\alpha)} \right] A_\pm(\alpha) - \frac{\alpha}{\pm\Gamma(\alpha)} k_z B_\pm(\alpha) \right\} e^{-i\alpha x \pm i\Gamma(\alpha) y} d\alpha + \frac{1}{\pm k_y \omega\mu} \cdot \left\{ \left(k_y^2 + k_z^2\right) R_z(T_z) A_z^i - k_x k_z R_x(T_x) A_x^i \right\} e^{-ik_x x \pm i k_y y} \tag{13}$$

for $y > 0$ and $y < 0$.

The spectral coefficients $A_\pm$ and $B_\pm$, appearing in equations (8, 9) and (12, 13) are to be determined with the aid of the boundary conditions. To obtain a unique solution, it is also necessary to take into account the following edge conditions as $x \to 0$:

$$E_z = O(\sqrt{x}) \tag{14}$$

$$E_x = O(1/\sqrt{x}) \tag{15}$$

$$H_z = O(1) \tag{16}$$

Now by substituting the scattered field expressions into the boundary conditions and inverting the resulting integral equations

$$A_+(\alpha) = \Phi_1^+(\alpha) \tag{17}$$

$$B_+(\alpha) = \Phi_2^+(\alpha) \tag{18}$$

$$B_+(\alpha) = B_-(\alpha) \tag{19}$$

$$A_+(\alpha) = A_-(\alpha) \tag{20}$$

$$\frac{\alpha k_z}{\omega\mu\Gamma} \left[A_+(\alpha) + A_-(\alpha)\right] + \left[\frac{-N^2}{\omega\mu\Gamma} + \frac{1}{R_1}\right] B_+(\alpha) - \frac{-N^2}{\omega\mu\Gamma} B_-(\alpha) = \Phi_1^-(\alpha) - \frac{h}{2\pi i(\alpha - k_x)} \tag{21}$$

$$\left[\frac{-\left(k^2-\alpha^2\right)}{\omega\mu\Gamma} + \frac{1}{R_2}\right] A_+(\alpha) - \frac{\left(k^2-\alpha^2\right)}{\omega\mu\Gamma} A_-(\alpha) + \frac{\alpha k_z}{\omega\mu\Gamma} \left[B_+(\alpha) + B_-(\alpha)\right] = \Phi_2^-(\alpha) - \frac{j}{2\pi i(\alpha - k_x)} \tag{22}$$

are obtained, with

$$h = \frac{1}{k_y \omega\mu} \left\{ \left(-N^2 - k_y^2\right) A_x^i + k_x k_y A_y^i + k_x k_z A_z^i \right\} \tag{23}$$

and

$$j = \frac{1}{k_y \omega\mu} \left\{ \left(-2k_y^2 - k_z^2\right) A_z^i + k_x k_z A_x^i - k_y k_z A_y^i \right\}. \tag{24}$$

In the above expressions, $\Phi_{1,2}^+(\alpha)$ and $\Phi_{1,2}^-(\alpha)$ are yet unknown functions regular in the half-plane $\operatorname{Im}\alpha > \operatorname{Im} k_x$ and $\operatorname{Im}\alpha < \operatorname{Im} N$, respectively.

The elimination of $A_\pm(\alpha)$ and $B_\pm(\alpha)$ between $(17-22)$ gives a matrix Wiener-Hopf equation written in the strip $\operatorname{Im} k_x < \operatorname{Im}\alpha < \operatorname{Im} N$ as follows:

$$\mathbf{G}(\alpha)\mathbf{\Phi}^+(\alpha) = \mathbf{\Phi}^-(\alpha) + \mathbf{V}(\alpha) \tag{25}$$

where

$$\mathbf{G}(\alpha) = \begin{bmatrix} \dfrac{2\alpha k_z}{\omega\mu\Gamma} & \dfrac{-2N^2}{\omega\mu\Gamma} + \dfrac{1}{R_1} \\ \dfrac{-2\left(k^2-\alpha^2\right)}{\omega\mu\Gamma} + \dfrac{1}{R_2} & \dfrac{2\alpha k_z}{\omega\mu\Gamma} \end{bmatrix}. \tag{26}$$

and

$$\mathbf{V}(\alpha) = \begin{bmatrix} \dfrac{-h}{2\pi i(\alpha - k_x)} \\ \dfrac{-j}{2\pi i(\alpha - k_x)} \end{bmatrix} \tag{27}$$

where h and j are given by eqs. (23) and (24). Here, μ denotes the magnetic permeability of the surrounding medium. $\mathbf{\Phi}^+(\alpha), \mathbf{\Phi}^-(\alpha)$ and $\mathbf{V}(\alpha)$ are column vectors, where the terms $\mathbf{\Phi}^+(\alpha)$, $\mathbf{\Phi}^-(\alpha)$ are unknown functions that would be determined later and $\mathbf{V}(\alpha)$ corresponds to the contributions of the incident and reflected fields as given in (27)

III. Solution of the Wiener-Hopf System

The formal solution is derived for the diffraction problem by employing Daniele-Khrapkov method. While an exact closed form solution is obtained by factorizing a 2×2 Wiener-Hopf matrix, it is quite complicated not only algebraically, but also it contains no less than six transcendental functions. This complexity is not too surprising for this type of anisotropy[4]. The explicit expressions of the solution can be obtained for the special case when $R_1 \cdot R_2 << 1$. Also, 4 different special cases (Case for $R_2 \to \infty$, Case for $R_1 \to \infty$, Case for $R_1 = 0$, and Case for $R_2 = 0$) are examined which in all, by using Fourier transform technique, the problem can be formulated into a pair of simultaneous Wiener-Hopf equations which are decoupled via a polynomial transformation and solved through the standard procedure. Here, in this paper we are going to give the solution of only one special case of the scalar Wiener-Hopf system; i.e. case for $R_2 \to \infty$.

Letting $R_2 \to \infty$, the problem reduces to two simultaneous Wiener-Hopf equations:

$$\frac{2\alpha k_z}{\omega\mu\Gamma(\alpha)} \Phi_1^+ + \left[-2N^2 + \frac{\omega\mu\Gamma}{R_1}\right] \frac{\Phi_2^+}{\omega\mu\Gamma(\alpha)} = \Phi_1^- + V_1 \tag{28}$$

and

$$\frac{1}{\omega\mu\Gamma(\alpha)} \left[-2\left(k^2-\alpha^2\right)\Phi_1^+ + 2\alpha k_z \Phi_2^+\right] = \Phi_2^- + V_2. \tag{29}$$

Since Φ_1^+ and Φ_2^+ are both regular functions in the upper half-plane, the sum of these two functions in (29) is also regular in the upper half-plane. Let

$$\Psi^+(\alpha) = -2\left(k^2-\alpha^2\right)\Phi_1^+ + 2\alpha k_z \Phi_2^+ \quad (30)$$

So (29) is reduced into the below form;

$$\frac{1}{\omega\mu\Gamma(\alpha)}\Psi^+(\alpha) = \Phi_2^- + V_2 \quad (31)$$

where V_2 is given by (27).

As is seen, the polynomial transformation given in (30) reduced the simultaneous system of equations in $(28-29)$ into two scalar Wiener-Hopf equations. Therefore, first eq. (31) will be solved and Φ_1^+ is expressed in terms of Φ_2^+. Then, eq. (28) will also be reduced to a scalar equation involving only Φ_2^+ and Φ_1^-.

Eqn. (31) is a scalar Wiener-Hopf equation whose solution is

$$\Phi_1^+(\alpha) = \frac{\alpha k_z}{(k^2-\alpha^2)}\Phi_2^+(\alpha) - \frac{1}{2(k^2-\alpha^2)}\Psi^+(\alpha) \quad (32)$$

with

$$\Psi^+(\alpha) = \frac{-jkZ_0}{2\pi i(\alpha-k_x)}\sqrt{N-k_x}\sqrt{N+\alpha}. \quad (33)$$

Now, substituting this in eqn. (28) and rearranging yields

$$\frac{T(\alpha)}{\omega\mu\Gamma(\alpha)(k^2-\alpha^2)}\Phi_2^+ = \Phi_1^- + M(\alpha) \quad (34)$$

where

$$T(\alpha) = \frac{kZ_0\sigma_1\sigma_2}{R_1N^2}\cdot\frac{\Gamma^3}{\chi(\xi_1,\alpha)\chi(\xi_2,\alpha)}, \quad (35)$$

with

$$\xi_1 = N/\sigma_1 \quad \text{and} \quad \xi_2 = N/\sigma_2, \quad (36)$$

and

$$\sigma_{1,2} = -\frac{kR_1}{Z_0} \pm k\sqrt{\left(\frac{R_1}{Z_0}\right)^2 - \cos^2\theta_0}, \quad (37)$$

and where

$$M(\alpha) = V_1 + \frac{\alpha k_z}{\omega\mu\Gamma(\alpha)}\frac{\Psi^+(\alpha)}{(k^2-\alpha^2)}. \quad (38)$$

with V_1 given by (27). The function $\chi(\xi,\alpha)$ given by eqn. (35) is defined as

$$\chi(\xi,\alpha) = \frac{\Gamma(\alpha)}{N+\xi\Gamma(\alpha)} = \chi^+(\xi,\alpha)\chi^-(\xi,\alpha) \quad (39)$$

which is factorized in terms of Maliuzhinetz function[5]. Then performing Wiener-Hopf decomposition, one obtains

$$\Phi_2^+ = \frac{\chi^-(\xi_1,k_x)\chi^-(\xi_2,k_x)\chi^+(\xi_1,\alpha)\chi^+(\xi_2,\alpha)}{2\pi i(\alpha-k_x)(N-k_x)}$$
$$\cdot\frac{(k-k_x)\sin^2\theta_0 R_1(k+\alpha)}{\cos^2\theta_0(N+\alpha)}h - j\frac{\sin^2\theta_0\sqrt{R_1}(k+\alpha)}{2\pi i(\alpha-k_x)(k+k_x)}$$
$$\cdot\frac{\chi^-(\xi_1,k_x)\chi^-(\xi_2,k_x)\chi^+(\xi_1,\alpha)\chi^+(\xi_2,\alpha)k_xk_z}{\cos^2\theta_0(N+\alpha)(N-k_x)}. \quad (40)$$

Since Φ_1^+ were expressed in terms of Ψ^+ and Φ_2^+ in (32), by using (33)

$$\Phi_1^+ = j\frac{kZ_0\sqrt{N-k_x}\sqrt{N+\alpha}}{4\pi i(\alpha-k_x)(k^2-\alpha^2)} + \frac{\alpha k_z}{(k^2-\alpha^2)}\Phi_2^+ \quad (41)$$

is obtained. Since $\Phi_1^+(\alpha)$ and $\Phi_2^+(\alpha)$ are now completely determined, the spectral coefficients $A_\pm$ and $B_\pm$ can be written from (17-20) to give

$$A_\pm(\alpha) = \Phi_1^+(\alpha), \quad (42)$$

$$B_\pm(\alpha) = \Phi_2^+(\alpha). \quad (43)$$

IV. Conclusions

Now, using the expressions of the spectral coefficients, the diffracted fields given by (8) and (9) can be obtained by using Steepest Descent method:

$$E_x^d(\rho,\phi) \sim D_x(\theta_0,\phi_0,\phi)\frac{e^{iN\rho}}{N\rho} \quad (44)$$

where

$$D_x(\theta_0,\phi_0,\phi) =$$
$$= \frac{e^{-i\pi/4}}{i\sqrt{2\pi}}\cdot\frac{\chi^+(\xi_1,-N\cos\phi)\chi^+(\xi_2,-N\cos\phi)}{(1-\cos\phi)(1-\cos\phi_0)}\frac{R_1}{Z_0}Y$$
$$\cdot\frac{\chi^-(\xi_1,N\cos\phi_0)\chi^-(\xi_2,N\cos\phi_0)(1-\sin\theta_0\cos\phi)\sin\phi}{(\cos\phi+\cos\phi_0)\cos\phi_0} \quad (45)$$

with

$$Y = \frac{(1-\sin\theta_0\cos\phi_0)}{\cos\theta_0\sin\phi_0}\cdot\left\{-\sin\theta_0 - \sin\theta_0\sin^2\phi_0\right.$$
$$\left.+\sin\theta_0\sin\phi_0\cos\phi_0 + \cos\theta_0\cos\phi_0\right\} + \left\{-2\sin^2\theta_0\sin^2\phi_0\right.$$
$$\left.-\cos^2\theta_0 + \sin\theta_0\cos\theta_0\cos\phi_0 - \sin\theta_0\cos\theta_0\sin\phi_0\right\}$$
$$\times\frac{\cos\phi_0}{(1+\sin\theta_0\cos\phi_0)\sin\theta_0\sin\phi_0}. \quad (46)$$

Here, $\chi(\xi,\alpha)$ is the function given in (39) and $\xi_{1,2}$ are defined as follows:

$$\xi_{1,2} = \frac{\sin\theta_0}{-\frac{R_1}{Z_0} \pm \sqrt{\left(\frac{R_1}{Z_0}\right)^2 - \cos^2\theta_0}}. \quad (47)$$

Also, in a similar manner

$$E_z^d(\rho,\phi) \sim D_z(\theta_0,\phi_0,\phi)\frac{e^{iN\rho}}{\sqrt{N\rho}} \quad (48)$$

is obtained, where

$$D_z(\theta_0,\phi_0,\phi) = -\frac{e^{-i\pi/4}}{2\sqrt{2\pi i}}\frac{\sin\phi\sqrt{1-\cos\phi_0}}{\sin\phi_0(\cos\phi+\cos\phi_0)}$$
$$\frac{\sqrt{1-\cos\phi}}{(1-\sin^2\theta_0\cos^2\phi)}\cdot\left\{-2\sin^2\theta_0\sin^2\phi_0\right.$$

$$-\cos^2\theta_0+\sin\theta_0\cos\theta_0\cos\phi_0-\sin\theta_0\cos\theta_0\sin\phi_0\}$$
$$-\frac{\sin\theta_0\cos\phi\cos\theta_0}{1-\sin^2\theta_0\cos^2\phi}D_x(\theta_0,\phi_0,\phi). \quad (49)$$

with $D_x(\theta_0,\phi_0,\phi)$ given by (45-47).

Some numerical results have been obtained for the diffracted fields where the ambient medium was taken as free space. The incident plane wave field strength was assumed to be 1 V/m. the diffracted field expressions involve the split functions $\chi^{\pm}(\alpha)$, which can be written in terms of the Maliuzhinetz functions, as is done by Uzgören et al.[5]. By using approximate formula given by Volakis and Senior[6], the Maliuzhinetz functions and the diffracted fields are computed.

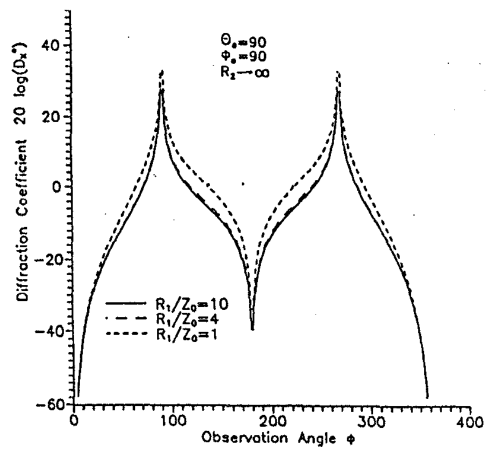

Figure-3. Variation of the diffraction coefficient ($20\log_{10}D_x^e$) with respect to the observation angle for different incidence angles and different resistivities.

Figures 3-4 illustrate the variation of the amplitude of the x-component (z-component) of the diffracted field $20\log_{10}\left(|u_d|\times\sqrt{N\rho}\right)$ by the observation angle for the different values of the normalized resistance R/Z_0. The diffracted field expressions given are not uniform and it is expected that the field will take very large values in the transition regions. As seen from the Figs., the transition boundaries are determined by the incidence angle as $(\pi-\phi_0)$ and $(\pi+\phi_0)$.

The problem of diffraction from a discontinuity formed by a PEC and an anisotropic resistive half-planes is considered for the first time in this work. Therefore there has been no opportunity to compare the results on Figs. 3-4 with some previously obtained results to validate the accuracy of our high-frequency solution. But it should be noted that, for $R_1\,(R_2)=0$, the z-component (x-component) of the scattered field involves only the reflected term as expected and this may validate the accuracy of the solution.

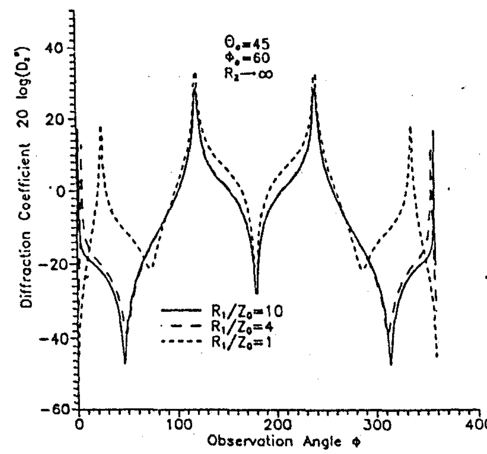

Figure-4. Variation of the diffraction coefficient ($20\log_{10}D_z^e$) with respect to the observation angle for different incidence angles and different resistivities.

For $\theta_0=\pi/2$ (normal incidence case) and $R_1,R_2\to\infty$, the z-component of the diffracted electric field reduces to the following well-known result for the PEC half-plane problem in the E_z polarization case:

$$E_z^d=-\frac{e^{i\pi/4}}{\sqrt{2\pi}}\frac{\sqrt{1+\cos\phi_0}\sqrt{1+\cos\phi}}{\cos\phi+\cos\phi_0}\cdot\frac{e^{iN\rho}}{\sqrt{N\rho}}$$

Although this is necessary but not sufficient to establish the accuracy of our problem, this may be considered as another check to validate the solution.

V. References

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