Errata for

Bioelectricity: A Quantitative Approach, 3rd edition

by Robert Plonsey and Roger C. Barr (Springer 2007)

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Note: Textbook corrections start below. Corrections to the PDF file solutions begin on page 8.

1. page 4, the equation for $\overline{B}$: change $\overline{a_x}$ to $\bar{a}_x$
2. page 5, sentence preceding Eq. 1.10: add the word "differential" before "displacement" so the phrase reads "…then the differential displacement …"
3. page 5, Eq. 1.10: change $\overline{dl}$ to $d\bar{l}$
4. page 6, Eq. 1.13 and throughout Section 1.4.2 (in seven places): change $\overline{dl}$ to $d\bar{l}$
5. page 13, Eq. 1.43: in the denominator, r is the scalar magnitude of the radial position vector $\overline{r} = r\,\overline{a}_r$. Hence, remove the vertical bars so the denominator is simply r
6. page 17, Eq. 1.62: change a_θ to $\bar{a}_\theta$
7. page 18, last sentence: change to read: "In Eq. (1.69), $\bar{a}_r$ is a unit vector from $d\bar{S}$ to P, as illustrated in Figure 1.6."
8. page 25, line 3: change $R = \rho A/L$ to $R = \rho L/A$ (see Eq. 2.48 on page 39).
9. page 26, Eq. 2.6: replace $\nabla\Phi$ with $\nabla^2\Phi$
10. page 33, Figure 2.3: change (x', y', z) to (x', y', z').
11. page 35, paragraph above section 2.9.3: the bracketed term in Eq. 2.38 is equal to zero when $\cos\theta = 0$ (not one as stated); hence, change "equal to one when…" to "equal to zero when…"
12. page 37, in the two paragraphs above section 2.10: both paragraphs should refer to Figure 2.5 (not Figure 2.4).
13. page 37: delete the right bracket at the end of the sentence preceding section 2.10
14. page 41, in the line before Eq. 2.54: change "(2)" to "Eq. 2.47".
15. page 50, Table 3.3: remove the extra period in the expression for RT/F so it reads "8.314 × 300/96487"
16. page 56, paragraph beginning with "Figure 3.1" : add a space so the last sentence ends "…perhaps 30 Å thickness."

17. page 58, first line of the paragraph above section 3.10.2: add the subscript m to the capacitance so the denominator becomes "$(C_m d)$".

18. page 59, Eq. 3.20: in the left side term, i should be a subscript on $[C_p]$. The left side term should read:
$$\ln\left(\frac{[C_p]_i}{[C_p]_e}\right)$$

19. page 59, Eq. 3.21: replace "In" with "ln" (replace the uppercase I with a lowercase L) so the equation reads:
$$V_m^{\text{eq}} = \Phi_i - \Phi_e = \frac{-RT}{Z_p F}\ln\left(\frac{[C_p]_i}{[C_p]_e}\right)$$

20. page 59: Eq. 3.22: in the right-most term, add an open bracket in front of C_p so the term reads:
$$\frac{25}{Z_p}\ln\left(\frac{[C_p]_e}{[C_p]_i}\right)$$
In addition, note that Eq. 3.22 and Eq. 3.23 are valid only for a temperature of 17 °C.

21. page 61, Eq. 3.24: the computation and units are wrong. The correct answer is
$$Q = 0.628 \times 10^{-9} \text{ Coulombs}$$

22. page 61, Eq. 3.25: the units are wrong. The correct units are simply "Coulombs" (not Coulombs/cm). Also, in the sentence following, the relative charge depletion is $\approx$ 0.000017 (not 0.00017).

23. page 62, next-to-last paragraph in Section 3.12: change "the GHK equations (below)" to "the GHK equations (Sections 5.1.4 and 5.6)"

24. page 63: in the sentence above **Ionic Currents**, change the ending comma to a period.

25. page 64, line after Eq. 3.27: move the comma after the second E_{Na} to follow "positive", so the phrase reads "because E_{Na} is positive, $V_m \ldots$" .

26. page 67, third line from bottom of page: after 34.9 mV, change E_{CI} to E_{Cl} (the uppercase I should be a lowercase L)

27. page 69, first sentence of paragraph above Section 3.15: change $[K^+]e$ to $[K^+]_e$

28. page 69, section 3.15: two corrections in the units:
    - R has units of J/(°K-mole); true at all temperatures (not only 27 °C)
    - ϵ_0 has units of (Coulomb^2 sec^2)/(kg m^3) or (Farads/m)

29. page 76, sentence after Eq. 4.2: the units on Boltzmann's constant are J/°K.

30. page 83, panel (a) in Figure 4.8: change the resistor label from Γ_K to r_K

31. page 85, Table 4.1: the heading for the rightmost column should be "Channel Density"

32. page 88, sentence preceding Eq. 4.10: replace the semicolon with a comma

33. page 88, second paragraph after Eq. 4.11: in the first two sentences replace “boundary” with “initial”

34. page 93, Eq. 4.34: in the sentence following, note the authors suddenly redefine the meaning of N from the total number of channels to the total number of channels per unit area. This makes the units on the specific conductance $\overline{g}_{\mathrm{K}}$ work out to Siemens/cm^2

35. page 93, Eq. 4.37: the β_{K} terms should not be subscripts. The correct equations are:

$$\tau_n = 1/\left(\alpha_{\mathrm{K}} + \beta_{\mathrm{K}}\right) \text{ and } n_\infty = \alpha_{\mathrm{K}}/\left(\alpha_{\mathrm{K}} + \beta_{\mathrm{K}}\right)$$

36. page 94, Eq. 4.38: since g_{K} is Siemens/cm^2, this equation is really a current density, so replace I_{K} with J_{K}

37. page 94, sentence preceding Eq. 4.40: replace “conductivity” with “specific conductance”

38. page 94, Eq. 4.41: since g_{Na} is Siemens/cm^2, this equation is really a current density, so replace I_{Na} with J_{Na}

39. page 108, second to last paragraph: change P_{CI} to P_{Cl} (lowercase L in the subscript, not uppercase I).

40. page 109, third paragraph from bottom: Faraday’s constant should be 96,487 Coul/mol (consistent with other usage throughout the text)

41. page 114, last line: change $\partial/\partial t = 0$ to $\partial V_m/\partial t = 0$.

42. page 119, sentence preceding section 5.3: change “function” to “functions”

43. page 122, sentence preceding Eq. 5.18: the reference should be to Eq. 4.35, not 4.24

44. page 122, sentence preceding Eq. 5.19: the reference should be to Eq. 4.31, not 4.30

45. page 124, Eq. 5.33: change $\overline{g_{\mathrm{Na}}}$ to $\bar{g}_{\mathrm{Na}}$.

46. page 125, Eq. 5.36: delete the extra right parenthesis in the denominator of α_m

47. page 129, Eq. 5.46: on the right-side term, the m should be a subscript so the numerator is ΔV_m

48. page 129, Eq. 5.47: the signs on I_{Na} and I_{L} are wrong; the correct equation is:

$$\Delta V_m = \Delta t\left(I_m - I_{\mathrm{K}} - I_{\mathrm{Na}} - I_{\mathrm{L}}\right)/C_m$$

49. page 131, sentence after Eq. 5.49: the sentence should refer to Equation 5.49, not 5.48

50. page 133, in the middle line of Eq. 5.52: change $0 <= t < T$ to $0 \leq t < T$.

51. page 133, Eq. 5.55: the two variable conductivities should be:

$$g^i_{\text{K}} = \bar{g}^i_{\text{K}}\, n_i^4$$
$$g^i_{\text{Na}} = \bar{g}^i_{\text{Na}}\, m_i^3\, h_i$$

That is, add the maximum conductivities $\bar{g}^i_{\text{K}}$ and $\bar{g}^i_{\text{Na}}$ and delete the potential differences $(V^i_m - E_{\text{K}})$ and $(V^i_m - E_{\text{Na}})$.

52. page 135, next-to-last line in the caption of Figure 5.16: delete the comma after "Huxley".

53. page 137, next-to-last sentence in the first paragraph under **Units of calculation**: $\bar{g}_{\text{K}}$ is a conductance so reference tables could list it "in units of milliSiemens per square centimeter, i.e. conductance per unit area," not milliamperes and current.

54. page 149, Eq. 5.86: the natural logarithm should be $\ln$ (lowercase L, not the numeral 1), and the d (in the numerator) and the zero (in the denominator) should be subscripts on the potassium concentrations. The entire equation is better written as:

$$\frac{d}{V_m}\frac{RT}{F}\ln\left[\frac{\frac{V_m}{d}\frac{F}{RT}\,[K]_d - \frac{j_K}{D_K}}{\frac{V_m}{d}\frac{F}{RT}\,[K]_0 - \frac{j_K}{D_K}}\right] = d$$

55. page 150, start of section 5.6.7: change the second sentence, in parentheses, to "[For a passive membrane, J is the total ionic flux, as in (5.93).]"

56. page 157, first line in last paragraph: delete the open parenthesis before "the electrical".

57. page 157, Figure 6.3: in the third branch of the circuit (nearest the capacitor), g_{K} should be g_{L}, and E_1 should be E_{L}.

58. page 158, last line, the start of the parenthetical statement "Resistance r_e is low because…" In this chapter the authors are frustratingly sloppy with the terminology and symbols. Recall these definitions from Chapter 2:

   - resistance is R in Ω
   - resistivity is R_e in Ω-cm
   - resistance per unit length is r_e in Ω/cm
   - capacitance is C in μF
   - capacitance per unit area is C_m in μF/cm^2
   - capacitance per unit length is c_m in μF/cm

   Not defined in the book is "specific resistance", which is the product of resistance and area, or $\mathbb{R} = R \times$ area, with units of Ω-cm^2.

   Symbols used in this statement indicate resistance per unit length, so the statement should read "Extracellular resistance per unit length r_e is low because…" Alternatively, they could have said "Extracellular resistance $r_e \Delta x$ is low because…"

59. page 159, second line in item 4: change "leakage resistance r_m" to "leakage resistivity R_m", and after "capacitance" add "per unit area". This is a typical example of where the authors specify a certain quantity but give units for a different quantity.

60. page 160, last sentence: in Figure 6.5, I_e increases at location b (not a). Also, delete "even more" at the end of the same sentence.

61. page 162, line above Eq. 6.8: they have simplified Eq. 6.7, not 6.2.

62. page 164, first line in Section 6.2.3: add a period at the end.

63. page 168, third bullet in the list, second sentence: after dt change the closing parenthesis to a closing bracket.

64. page 182, first first sentence after Eq. 6.70: the experimental values are θ and a (not "θ and α").

65. page 183, Table 6.2: Specific resistance is Ω-cm^2, which is shown in the first column though the authors never define it. The second column shows capacitance per unit area, not "Specific capacitance" like the heading says.

66. page 184, second line after Eq. 6.72: to be consistent with Chapter 12, change "F-H equations" to "FH equations".

67. page 185, line before Eq. 6.73: I'm pretty sure they mean "room" temperature, not "frog" temperature.

68. page 186, footnote 5: at the end of the footnote, change "(see below)" to "see Section 6.3.4, page 177)".

69. page 215, Figure 7.7: the y axis label should be v_m, not V_m.

70. page 285, Figure 9.15: change "Endocarcium" to "Endocardium".

71. page 293, Eq. 9.29: the gates X and X_i are never defined in the text.

72. page 294, in the line above α_{K1} (Eq. 9.35): add an open parenthesis to the formula for $K1_\infty$ so it reads $K1_\infty = \alpha_{K1}/\left(\alpha_{K1} + \beta_{K1}\right)$

73. page 301, Eq. 9.66: lower the limit value so the left side term reads

$$-4\pi r^2 g_e \left.\frac{\partial \Phi_e}{\partial r}\right|_{r \to 0}$$

74. page 368, third line above Eq. 12.26: to be consistent throughout the text, change "F-H model" to "FH model".

75. page 391, Exercise 1: The components of vector $\bar{u}$ are (3, 9, 6) not (3.9, 6).

76. page 392, Exercise 5: in vector $\overline{A}$, change $\overline{2a_x}$ to $2\bar{a}_x$

77. page 392, Exercise 6, part b, second sentence: add the word "twice" so the sentence reads "The magnitude of the vector will be twice the area of that face."
78. page 397, Exercise 4: add units of mV/cm^2 on k
79. page 398, Exercise 9: vector $\overline{V}$ is wrong; it should read $\overline{V} = 2\bar{a}_x + 3\bar{a}_y$.
80. page 401, Exercise 22: the units on I_v should be mA/cm^2.
81. page 403, Exercise 24: as written, part b of the exercise is identical to part a. Change the end of the sentence in part b to read "...if the potential is computed by treating the two point sources as a dipole."
82. page 404, Exercise 29: wrong value for R_e; should be $R_e = 600\ \Omega$cm.
83. page 407: delete Exercise 7: it is the same as Exercise 6b.
84. page 411, Exercise 28: remove the extra parenthesis after "depth".
85. page 411, Exercise 30, part f: the mobility is u (letter u), not the Greek μ.
86. page 412, information for Exercises 31–33: membrane conductance units are mS/cm^2.
87. page 412, last paragraph: the information given is for Exercises 34–42 (not Exercises 34–45). Also, the third from last sentence should read
    "Membrane conductances are: $g_{\text{K}} = 0.375$ mS/cm^2, $g_{\text{Na}} = 0.01$ mS/cm^2, $g_{\text{L}} = 0.57$ mS/cm^2."
88. page 413, Exercise 42: assume $T = 300$ °K, $V_m = -40$ mV, and use the concentrations in shown on page 412.
89. page 414, Exercise 45: find dV_m/dt at time $t = 0$. The conductances should be
    "$g_{\text{K}} = 0.375$ mS/cm^2, $g_{\text{Na}} = 0.06$ mS/cm^2, $g_{\text{L}} = 0.8$ mS/cm^2."
    The correct answer is $dV_m/dt = -5.37$ mV/msec.
90. page 417, Exercise 5: the units on the potassium channel opening rate are msec^{-1}. Also, use the potassium concentrations listed in Table 3.1.
91. page 420, Exercise 16: the wording here is terrible. Rewrite the question as:
    Judging from the results of Exercises 14 and 15, if $V_m^2 > V_m^1$, is the K^+ current in phase 2 greater than that in phase 1?
92. page 420, Exercise 19: add to the problem statement:

    > Compare the potassium current before the transition to that immediately after; that is, compare I_{K}^1 (at $t = t_1$) to I_{K}^a (at $t = t_a$). What is the ratio $I_{\text{K}}^a / I_{\text{K}}^1$?

93. page 429, problem statement for Exercise 3: delete the second sentence "Also give formulas for the new values." This instruction is unnecessary because parts e through h directly ask for these formulas.

94. page 430, Exercise 5: the labels after part "d" are incorrect. The wrong labels run from c to g; change these to run from e to i.

95. page 432, Exercise 9: the labels after part "d" are incorrect. The wrong labels run from c to g; change these to run from e to i.

96. page 434, Exercise 14: instead of using their suggestion of the power-series expansion, use l'Hôpital's Rule.

97. page 436, part c of Exercise 15: change the wording to "What is ΔV_m 100 msec after the stimulus starts ..."

98. page 437, Exercise 18: two corrections:

    - Near the top of the problem statement, in the program's input values: delete the next-to-last line that shows `gK` and `gNa`. (Why are these unnecessary?)
    - In the paragraph above the output table, all references to v_m should be to V_m. While it is true that v_m $(= V_m - V_{\text{rest}})$ is used to compute the values of n, m, and h, the table shows V_m, not v_m. Since V_{rest} is constant, the time derivative of V_m and v_m are the same, so $V_{\text{dot}} = v_{\text{dot}}$

99. page 438, Exercise 20: Change "Reduce Δt" to "Reduce Δt to 2 μsec. Run a simulation to $t = 1000$ μsec." Note that all the other input values are the same as in Exercises 18 and 19.

100. page 438, Exercise 21: delete the text beginning with "Use these values to start the simulation." This part is not needed because here you are comparing the results from Exercises 19 and 20.

101. page 439, Exercise 23: After part b, add:
     Change the stimulus duration to 300 μsec and re-run the simulations.
     c. What is the just-above-threshold stimulus amplitude?
     d. What is the just-above-threshold membrane voltage?

102. page 440, Exercise 25: add "Use $I_s = 200$ μA/cm^2 for 150 μsec."

103. page 440, Exercise 26: in part a, change I_{Cl} to I_L. Use the same initial values for n, m, h, and V_m as in Exercise 19. In part b, change "Ex. 24" to "Ex. 25".

104. page 440, Exercise 27: change "Ex. 20" to "Ex. 19".

105. page 448, second sentence in Exercise 56: delete the extra negative sign before 105 mV.

106. page 449, Exercise 1: Edit parts a, b, and c to read:
     a. c_m, the core-conductor's membrane capacitance per unit length.
     b. R_m, the core-conductor's membrane resistivity.
     c. r_i, the core-conductor's intracellular resistance per unit length.

107. page 449, Exercise 2, three corrections:

     - To the problem statement add "Use Table 13.5 for initial conductance values."

- Part a: they are asking for the membrane's specific resistance (resistance $\times$ area, in Ω-cm^2), not the membrane's resistivity
- Part b: they are asking for membrane's resistivity

108. pages 449-450, Exercise 3, several corrections:
    - In the problem statement, the value 2,000 Ωcm^2 is the membrane's specific resistance, not its resistance.
    - Part a: they are asking for the membrane's resistivity, R_m.
    - Part b: they are asking for c_m.
    - Part c: they are asking for r_i.
    - Part d: they are asking for r_e.
109. page 450, Exercise 5: Assume you know the total current I, along with the cell radius a and resistances per unit length r_i and r_e.
110. page 451, Exercise 6: Assume you know the cell radius a, along with the resistances R and r.
111. page 452, Exercise 8: Change "crossing" to "pathway".
112. page 453, Exercises 10 and 11: Change "crossing" to "pathway".
113. page 525, index entries under "B": Add Boltzmann's constant, 76

Solutions in the PDF file on the CD-ROM

1. page 3, Answer 7: the units are μV.
2. page 3, Answer 11: the correct solution is

$$A(x) = \frac{I_o}{4\pi\sigma}\left[\frac{3\,(x-e)^2}{r^5} - \frac{1}{r^3}\right]$$

3. page 3, Answer 13a: add units mV/mm to $\nabla\Phi$
4. page 3, Answer 13b: If the units on σ are S/mm, then $\overline{J}$ is in mA/mm^2. The unit vector is also omitted; the correct solution is

$$\overline{J} = -\sigma\,\nabla\Phi = -k\,\sigma\,\text{sech}^2(x)\,\bar{a}_x \text{ mA/mm}^2$$

5. page 3, Answer 13c: negate the given answer and add correct units; the correct solution is

$$\nabla\cdot\nabla\Phi = -k\,\text{sech}^2(x)\,\tanh(x) \text{ mV/mm}^2$$

6. page 4, Answer 19a: units on k are μA/cm^3.

7. page 5, Answer 29b: add closing parenthesis:

$$r_e = 600/\left(3 \times 10^{-6}\right) = 2 \times 10^8 \text{ Ohms/cm}$$

8. page 6, Answer 3: mobility = $u_{\text{Na}} = 51.45 \times 10^{-9}\ \frac{\text{m/s}}{\text{V/m}}$
9. page 6, Answer 11: there are 0.2×10^{-12} moles of K^+ ions in volume 1, not 1×10^{-12}.
10. page 6, Answer 12: the particle flux is $j_K^D = 18.62 \times 10^{-6}$ mole/(cm^2-sec), not 7.448×10^{-6}
11. page 6, Answer 13: the particle flow is $i_K^D = 1.862 \times 10^{-9}$ moles/sec, not 2.98×10^{-9}
12. page 6, Answer 14:
13. page 6, Answer 15:
14. page 6, Answer 25: membrane resistance = 558,035,714 Ω or 558 MΩ
15. page 8, Answer 1: $R = 740.7\ \Omega$
16. page 8, Answer 5: $I_K = 5.59$ nA
17. page 8, Answer 13b: for $p = 0.95$, the minimum radius is $a = 0.46\mu$m.
18. page 8, Detailed answer 14a: in step 1, remove the extraneous "+1" so that

$$v_m = [-40 - (-60)] = 20 \text{ mV}$$

19. page 9, Answer 26: the ratio is 5.389, not 2.86
20. page 12, Answer 33c: $I_K^b = 5.29 \times 10^{-10}$ A $= 529$ pA, not 5.537×10^{-10} A
21. page 13, Answer 1g: the correct equation is

$$V_{\text{dot}} = \left(I_s - I_{\text{ion}}\right)/C_m$$

22. page 15, Answer 8: negative sign in the wrong place; the correct answer is:

$$I_s = I_{\text{ion}} = -209.4\ \mu\text{A/cm}^2$$

23. page 16, Answer 12: delete the two negative signs; the correct answer is:

$$I_s = I_{\text{ion}} = 40.73\ \mu\text{A/cm}^2$$

24. page 16, Answer 15c: ΔV_m = 5000 mV at $t_0 + t$ = 100 msec. This value is extremely large because the time (100 msec) is large
25. page 18, Answer 18: I disagree with some of these suggestions; we will discuss these in class.
26. page 18, paragraph within Answer 18: correct the spelling of "parameterized".

27. pages 20-21, Answers 20 through 26: the numbering is wrong for all of these answers. To correct this, add 1 to each answer number. For example, **Answer 20a** should be **Answer 21a**; **20b** should be **21b**; etcetera, so the last stated answer should be **Answer 27c**. The answer for Exercise 20 begins on page 19.

    After correcting the answer numbers, correct the given answers:

    - Answer 21a: change v_m to V_m. Also, the first number should be 42.692 (as shown in the output for Answer 20), not 49.692
    - Answer 22b: change "linear" to "nonlinear"
    - Answers 24a-24c: I get the peak times as 3150, 750, and 400 μsec
    - Answer 25: I get the stable return time as 24000 μsec.
    - Answers 27a-27c: I get the second AP times as 16750, 8050, and 4900 μsec

28. page 23, Answer 3a: $r_m = 63662$ Ω-cm, not 63700

29. page 23, Answer 3d: $r_e = 169765$ Ω/cm, not 169000