Implementation of Sallen-Key and Multi-Feedback (MFB) Architecture for Higher Order Butterworth Filters

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Abstract— Filter design specifications can be manifest in all manner of necessity, whether they are high-pass, low-pass, band-pass, or stop-band filters. In addition, the parameters for such filters are defined for a very specific application. A design for a low-pass filter with a pass-band of 2k Hz and a stop-band of 4k will be discussed. The further properties of the filter are to allow for a 48 dB drop between the pass-band and the stop-band, which results in the need to implement an 8th order filter. This paper will compare two very different filter architectures to meet these requirements: Sallen-Key and Multi-Feedback (MFB).

I. INTRODUCTION

Filters can be constructed in a multitude of ways. Combinations of capacitors and resistors, as well as operational amplifiers and transistors, have been used effectively for many years to produce reliable and functional filters. Just as there are a number of different elements from which a filter can be made, there are a number of different configurations those elements can be arranged to produce virtually identical results. However, some configurations produce better results, based on stability, susceptibility to noise, coupling, and the number of elements involved. We look to two architecturally similar designs for a low-pass filter. Sallen-Key filters and MFB filters have both proven to be very reliable and stable filter designs [1]. Although the MFB configuration consists of more resistors, the benefits or detractions from such a design are not as readily seen. A design specification requiring a pass-band to 2k Hz and a stop-band at 4k Hz is implemented on the two circuits. In addition, the circuit must attenuate the signal by 48 dB in that range. This results in the need for an 8th order filter, constructed as the cascade of 4 second order filters.

II. METHODS

The overall methodology developed for this experiment is consistent with the conventional methodology used for testing circuits. This methodology includes, but is not limited to, PSpice simulation, MATLAB and mathematical analysis, and realization of the physical circuit to extract experimental data. The recording of the experimental data is facilitated by the use of SONY/TEKTONIC circuit analysis equipment. More specifically, we used the AFG 310 arbitrary function generator to provide the input sine

waves of varying frequencies, the PS 501-2 to provide the \pm 9V rails for the op amps, and the TDS 210 two channel digital real time oscilloscope to simultaneously record the frequency response of the circuit as well as the input signal. The individual circuits were treated identically, producing similar results.

Using the Q values for an eighth order Butterworth filter of 2.56 for stage 1, 0.899 for stage 2, 0.601 for stage 3, and 0.509 for stage 4, we can derive the following equations necessary to determine the capacitor values to facilitate cutoff frequency of 2kHz for Sallen-Key implementation [1].

$$\begin{aligned} C_{11} &= \frac{2Q_1}{\omega_0 R} = \frac{2.56x2}{(2\pi x 2kHzx10k\Omega)} = 40.7nF \\ C_{21} &= \frac{1}{2xQ_1\omega_0 R} = \frac{1}{(2.56x2)(2\pi x 2kHzx10k\Omega)} = 1.55nF \\ C_{12} &= \frac{2Q_1}{\omega_0 R} = \frac{0.899x2}{(2\pi x 2kHzx10k\Omega)} = 14.3nF \\ C_{22} &= \frac{1}{2xQ_1\omega_0 R} = \frac{1}{(0.899x2)(2\pi x 2kHzx10k\Omega)} = 4.425nF \\ C_{13} &= \frac{2Q_1}{\omega_0 R} = \frac{0.601x2}{(2\pi x 2kHzx10k\Omega)} = 9.56nF \\ C_{23} &= \frac{1}{2xQ_1\omega_0 R} = \frac{1}{(0.601x2)(2\pi x 2kHzx10k\Omega)} = 6.6nF \\ C_{14} &= \frac{2Q_1}{\omega_0 R} = \frac{0.509x2}{(2\pi x 2kHzx10k\Omega)} = 8.1nF \\ C_{24} &= \frac{1}{2xQ_1\omega_0 R} = \frac{1}{(0.509x2)(2\pi x 2kHzx10k\Omega)} = 7.81nF \end{aligned}$$

Table 3.

Similarly, using the Q values for an eighth order Butterworth filter of 2.56 for stage 1, 0.899 for stage 2, 0.601 for stage 3, and 0.509 for stage 4, we can derive the following equations necessary to determine the capacitor values to facilitate cutoff frequency of 2kHz for MFB (Multiple Feedback) implementation.

$$Q = \frac{\sqrt{R2 * R3 * C1 * C2}}{R3 * C1 + R2 * C1 + R3 * C1 * (-K)}, \text{ where } -K \text{ is equal to } \frac{-R2}{R1}. \text{ Since all resistors}$$

are kept at the same value, $10k\Omega$, this equation reduces to
$$Q = \frac{10k\Omega\sqrt{C1 * C2}}{10k\Omega(3C1)} = \frac{\sqrt{C1 * C2}}{3C1}.$$

To find the value of the combination of C1*C2, we set the cutoff frequency equal to

$$f_{cutoff} = \frac{1}{2\pi\sqrt{R1*R2*C1*C2}} [2].$$
 Since R1=R2=10k Ω , this reduces to

$$2000Hz = \frac{1}{2\pi10k} \frac{1}{2\pi10k} \frac{1}{\sqrt{C1*C2}}$$

$$C1*C2 = \left(\frac{1}{2\pi10k} \frac{1}{\sqrt{C1*C2}}\right)^2$$

$$C1*C2 = 6.33E^{-17}$$

$$C2 = \frac{6.33E^{-17}}{C1}$$

ForQ = 2.56	ForQ = 0.601
$2.56 = \frac{\sqrt{C1^*C2}}{3C1}$	$0.601 = \frac{\sqrt{C1 * C2}}{3C1}$
$(2.56)^2 * (3C1)^2 = C1 * C2$	$(0.601)^2 * (3C1)^2 = C1 * C2$
59.1C1 = C2	3.25C1 = C2
$59.1C1 = \frac{6.33E^{-17}}{C1}$	$3.25C1 = \frac{6.33E^{-17}}{C1}$
$C1^2 = \frac{6.33E^{-17}}{59.1}$	$C1^2 = \frac{6.33E^{-17}}{3.25}$
C1 = 1nF	C1 = 4.4 nF
C2 = 59.1nF	C2 = 14.3nF
ForQ = 0.899	ForQ = 0.509
$0.899 = \frac{\sqrt{C1^*C2}}{3C1}$	$0.509 = \frac{\sqrt{C1 * C2}}{3C1}$
$(0.899)^2 * (3C1)^2 = C1 * C2$	$(0.509)^2 * (3C1)^2 = C1 * C2$
7.27C1 = C2	2.33C1 = C2
$7.27C1 = \frac{6.33E^{-17}}{C1}$	$2.33C1 = \frac{6.33E^{-17}}{C1}$
$C1^2 = \frac{6.33E^{-17}}{7.27}$	$C1^2 = \frac{6.33E^{-17}}{2.33}$
C1 = 2.9nF	C1 = 5.2nF
C2 = 21.4nF	C2 = 12.1 nF

Table2.

A. Analysis and measurements of the Sallen-Key Architecture

PSpice simulations and MATLAB provide the theoretical analysis for our design. Figure 1 shows the circuit design for Sallen-Key, with the capacitor values derived in Table 1.



Figure 1. Sallen-Key implementation of an 8th order Butterworth Low-Pass filter [2].

The capacitor values derived are not readily available. Every attempt was made to approximate the necessary value by putting capacitors in series or parallel, depending on the value needed as per the design. Below, figure 2, the changes in the capacitor values have been made according to the actual capacitors used. The frequency response for each design shows very good agreement and can be seen in figures 3 and 4.



Figure2. Circuit with capacitor values as constructed.



Figure 3. Frequency response (Bode Plot) of the Sallen-Key as designed.



Figure 4. Frequency response of the Sallen-Key as built. Notice the bump at about 1.5k due to the pole location that has been shifted as a result of the capacitor values not being exactly as designed.

B. Analysis and measurements of the MFB architecture

Again we use PSpice simulations and MATLAB to provide the theoretical analysis for our design. Figure 5 shows the circuit design for Multi-Feedback (MFB) architecture, with the capacitor values derived in Table 2.



Figure 5. MFB implementation of an8th order Butterworth Low-Pass filter.

As in the Sallen-Key implementation, the capacitor values derived are not readily available. Below, figure 6, the changes in the capacitor values have been made according to the actual capacitors used. The frequency response for each design show very good agreement and can be seen in figures 7 and 8.



Figure6. Sallen-Key implementation of an8th order Butterworth Low-Pass filter.



Figure 7. Frequency response (Bode Plot) of the MFB as designed.



Figure 8. Frequency response of the MFB as built. Differences can easily be seen at 6k where the attenuation is -80dB whereas the designed response is only down to -75dB.

III. RESULTS

A. Analysis and measurements of the Sallen-Key filter

As mentioned earlier excellent results were achieved performing this circuit analysis. Much of the data obtained through measurement was in very good agreement with the PSpice model. Figure 9 and Table 3 show the results as measured using the Sallen-Key circuit. Notice that the pass-band frequency occurs at a lower frequency than the design specifications. This is most likely due to the mismatched capacitor values.

dB Plot for Sallen-Key Implementation 8th Order LPF



Figure 9 and 9a. Frequency response of the Sallen-Key as built. Figure 9 is data recorded from the cicuit (above.) figure 9a is the PSpice simulation offered for comparison (below). Differences can easily be seen as the pass-band occurs at 1k Hz instead of 2kHz, most likely due to the imperfect capacitor values.



Freq					
(Hz)	Vin	Vout	dB		
10	0.71	0.71	0		
20	0.71	0.71	0		
50	0.71	0.71	0		
100	0.714	0.714	0		
200	0.714	0.714	0		
500	0.7	0.73	0.364496		
600	0.692	0.714	0.271842		
700	0.686	0.668	-0.23095		
800	0.678	0.58	-1.35603		
900	0.67	0.47	-3.07954		
1000	0.66	0.36	-5.26483		
1500	0.628	0.106	-15.4531		
1600	0.621	0.082	-17.5856		
1700	0.613	0.063	-19.7624		
1800	0.607	0.049	-21.8599		
1900	0.6	0.038	-23.9674		
2000	0.593	0.029	-26.2131		
2100	0.586	0.023	-28.1234		
2200	0.58	0.0178	-30.2602		
2300	0.573	0.0139	-32.3028		
2400	0.567	0.0108	-34.4032		
2500	0.56	0.0084	-36.4782		
2600	0.554	0.0064	-38.7466		
3000	0.529	0.0022	-47.6207		
3100	0.523	0.0016	-50.2876		
3200	0.518	0.0011	-53.4587		
3300	0.512	0.0008	-56.1236		
3400	0.507	0.0005	-60.1208		
3500	0.501	0.0003	-64.4543		
3600	0.497	0.0002	-67.9065		
4000	0 478	0 00005	-79 6092		



B. Analysis and measurements of the MFB filter

As mentioned earlier excellent results were achieved performing this circuit analysis. Much of the data obtained through measurement was in very good agreement with the PSpice model. One notable exception is the shift, once again, of the pass band to 100 Hz, very far removed from the projected 2k pass-band. Certainly the capacitor values are affecting this to some degree; however, we must conclude also that there are inherent difficulties in constructing this type of filter. There are many benefits to this type of filter





Freq	<i></i>	., .	15		
(HZ)	Vin	Vout	aB		
10	0.731	0.731	0		
20	0.72	0.72	0		
30	0.705	0.705	0		
40	0.709	0.682	-0.3372		
50	0.711	0.627	-1.0920		
60	0.712	0.554	-2.1794		
70	0.713	0.494	-3.1872		
80	0.713	0.42	-4.5968		
90	0.713	0.389	-5.2628		
100	0.713	0.359	-5.9599		
120	0.713	0.309	-7.2626		
140	0.713	0.268	-8.4990		
160	0.713	0.233	-9.7146		
180	0.713	0.203	-10.911		
200	0.713	0.178	-12.053		
400	0.7	0.057	-21.784		
600	0.688	0.023	-29.517		
800	0.675	0.013	-34.307		
1000	0.661	0.0072	-39.257		
1200	0.648	0.0041	-43.975		
1400	0.634	0.0025	-48.083		
1600	0.619	0.0015	-52.312		
1800	0.605	0.0009	-56.550		
2000	0.591	0.0006	-59.868		

Table 4.

IV. SIGNAL PROCESSING

A. MATLAB Graphical Analysis-Analog

We now turn our attention to further extracting information regarding the functionality of the filter. MATLAB enables the user to perform extremely complex mathematical analysis efficiently. In addition, MATLAB has many functions capable of generating exceptionally insightful graphs, many of which will be discussed.

Since this is an 8th order system, it can be realized by the cascade of four 2nd order filters.[3] Each of the 2nd order filters has a characteristic equation of the form (s² + $2Q\omega s + \omega^2$), where ω is the natural frequency, and the Q values are provided from filter tables (See appendix 1). By convolving the four characteristic equations we will generate and 8th order equation that describes the filter [5].

```
>> LP1=[1 2*0.5098*w w^2];LP2=[1 2*0.6013*w w^2];LP3=[1 2*0.8999*w w^2];
LP4=[1 2*2.5628*w w^2];
>> LP4th=conv(LP1,LP2)
LP4th =
    1.0000    2.2222    3.2262    2.2222    1.0000
>> LP6th=conv(LP4th,LP3)
LP6th =
    1.0000    4.0220    8.2257    10.2509    8.2257    4.0220    1.0000
>> LP8th=conv(LP6th,LP4)
LP8th =
    1.0000    9.1476    29.8408    56.4344    68.9932    56.4344    29.8408    9.1476    1.0000
```

For simplicity, w has been set to 1, but can easily be moved out to 2000Hz as the specifications require. By setting w = 1, we do not have to multiply by a gain factor to maintain unity gain. With this information we can easily generate a transfer function:

>> lpf=tf(b,LP8th)

Transfer function:

s^8 + 9.148 s^7 + 29.84 s^6 + 56.43 s^5 + 68.99 s^4 + 56.43 s^3 + 29.84 s^2 + 9.148 s + 1

1

With our transfer function it is now possible to generate our zero, pole, gain information:

```
>> [z,p,k]=tf2zp(b,LP8th)
z =
 Empty matrix: 0-by-1
p =
-4.9224
-0.8999 + 0.4361i
-0.8999 - 0.4361i
-0.5098 + 0.8603i
-0.5098 - 0.8603i
-0.6013 + 0.7990i
-0.6013 - 0.7990i
-0.2032
k =
  1
>> lpf=tf(b,LP8th)
Transfer function:
                                 1
     _____
                                              ------
```

s^8 + 9.148 s^7 + 29.84 s^6 + 56.43 s^5 + 68.99 s^4 + 56.43 s^3 + 29.84 s^2 + 9.148 s + 1

In turn, with our transfer function it is now possible to generate our zero, pole, gain information:



Figure 11. All the poles are positioned in the left plane, confirming that this is indeed a low-pass filter.

This information makes it very easy to graph the pole, zero plot, figure 11. We can now use this information to generate a Bode plot (figure 12) [5].



A fundamental analysis of many circuits includes both the impulse response and the step response. They are shown here as figures 13 and 14.



Figure 13.

Figure 14.

There are two other graphs that are not as fundamental to circuit analysis but yield a great deal of information regarding the circuit's performance. The Root Locus, rlocus, computes the Evans root locus of a SISO open-loop model.[6] The root locus gives the closed-loop pole trajectories as a function of the feedback gain k (assuming negative feedback). Root loci are used to study the effects of varying feedback gains on closed-loop pole locations. In turn, these locations provide indirect information on the time and frequency responses. Rlocus(sys) calculates and plots the root locus of the open-loop SISO model sys. [6]. The root locus plot can be seen in figure 15.



The other insightful graph is the Nyquist Diagram, which directly correlates the circuit stability to the region enclosed by the graph. If the point -1 lies within the smaller encircled region (see figure 16), the system is considered to be unstable. As can be easily seen, the encircled region stops at -0.4 and does not approach -1. This is a stable system.



Figure 16. If the encircled area includes -1 on a Nyquist digram, the system is unstable. Inspection reveals that this is a very stable system.

B. MATLAB Graphical Analysis-Digital

Although MATLAB provides exceptional analog filter analysis, it is in the discrete world that it really performs. By using a bilinear transformation to take the analog filter and make it digital, we can use similar analysis in the digital domain.



Figure 17. Notice that the locations of the poles have moved and we are now introducing 8 zeros at negative $1, \pi$ on the frequency plot.

V. FAILURE ANALYSIS

A. Monte Carlo Simulations

Resistor values can vary by as much as 20 percent. Typically this value is given in the indicator bands around the resistor. For instance a resistor with a gold tolerance band has a 5% margin of error, for silver it is 10%, and if no tolerance band is present, the margin of error is 20%. It is important in circuit design and analysis to account for such variability. In PSpice, this is done by using Monte Carlo simulations to perform a given number of analysis runs while varying the resistor through the possible tolerance values. Figure 18 shows such an analysis using 20% Gaussian distribution on the Sallen-Key architecture. Figure 19 shows the analysis for the MFB architecture using the same distribution. Notice the range of values over the stop band at 2K Hz.



Figure 18. The output voltage varies by 75% from 1V to approximately 1.75V.



Figure 19. The output voltage varies by almost 100% from 0.7V to approximately 1.4V.

By using a better resistor value, that is one with a better tolerance, we see a tighter response (Figure 20). There is not the same scattered distribution as with the 20% tolerance resistors. But what is interesting is that the output voltage is climbing to almost 2-and-a-half volts. Of course, this difference would not be quite as extreme on a Bode plot, since the difference between them is only 7.95dB.



Figure 20. The 5% tolerance resistors narrow the distribution of possible output voltage values.

Another useful analysis tool is the histogram plot. It allows you to look quickly at a range of values and determine their distribution. An example can be seen in figure 21, the histogram of the Sallen-Key bandwidth samples.



Figure 21. The histogram tells us that 5% of the samples indicate the bandwidth is 355Hz, 10% are represented at 372Hz, etc.

A. Worst Case Scenario

As reported earlier, resistor values can vary by as much as 20 percent. Monte Carlo simulations give us an idea of how the circuit will behave if the resistors are uniformly or Gaussian distributed. However, the possibility exists that all of the resistors will be either at the lower end of their possible tolerance values or the higher end. Even worse, there is the possibility that half will be at the absolute low end and the other half at the absolute high end, creating the greatest (or worst) case scenario. We use the PSpice Worst Case Scenario to help us predict how the circuit will behave if these conditions exist. Figure 22 illustrates the worst case scenario of the Sallen-Key architecture with 5% Gaussian distribution of resistor values.



Figure 22.

Since this is the single worst case scenario, only one trace is output. It is, however, the expected value based on the Monte Carlo distribution of 5% Gaussian distribution resistor values. Note that the values is identical to the highest output in the Monte Carlo simulation.

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APPENDIX A

FILTER ORDER	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
	FSF	Q								
2	1.000	0.7071								
3	1.000	1.0000	1.000							
4	1.000	0.5412	1.000	1.3065						
5	1.000	0.6180	1.000	1.6181	1.000					
6	1.000	0.5177	1.000	0.7071	1.000	1.9320				
7	1.000	0.5549	1.000	0.8019	1.000	2.2472	1.000			
8	1.000	0.5098	1.000	0.6013	1.000	0.8999	1.000	2.5628		
9	1.000	0.5321	1.000	0.6527	1.000	1.0000	1.000	2.8802	1.000	
10	1.000	0.5062	1.000	0.5612	1.000	0.7071	1.000	1.1013	1.000	3.1969

Table 1. Butterworth Filter Table

APPENDIX B FLOW GRAPH Eighth Order Low Pass Filter



Signal emerges with a total of 80 dB/dec. Filters are designed for unity gain.

APPENDIX C **Op-Amp Specifications**

National Semiconductor LM124/LM224/LM324/LM2902 Low Power Quad Operational Amplifiers General Description Advantages Eliminates need for dual supplies The LM124 series consists of four independent, high gain,

internally frequency compensated operational amplifiers which were designed specifically to operate from a single power supply over a wide range of voltages. Operation from split power supplies is also possible and the low power supply current drain is independent of the magnitude of the power supply voltage.

Application areas include transducer amplifiers, DC gain blocks and all the conventional op amp circuits which now can be more easily implemented in single power supply systems. For example, the LM124 series can be directly operated off of the standard +5V power supply voltage which is used in digital systems and will easily provide the required interface electronics without requiring the additional ±15V power supplies.

Unique Characteristics

- In the linear mode the input common-mode voltage range includes ground and the output voltage can also swing to ground, even though operated from only a single power supply voltage
- The unity gain cross frequency is temperature compensated
- The input bias current is also temperature compensated

- Four internally compensated op amps in a single
- package Allows directly sensing near GND and V_{OUT} also goes
- to GND
- Compatible with all forms of logic Power drain suitable for battery operation

Features

OUT

- Internally frequency compensated for unity gain
- Large DC voltage gain 100 dB Wide bandwidth (unity gain) 1 MHz (temperature compensated) Wide power supply range: 3V to 32V Single supply or dual supplies $\pm 1.5V$ to $\pm 16V$ ■ Very low supply current drain (700 µA)-essentially independent of supply voltage Low input biasing current 45 nA (temperature compensated) Low input offset voltage 2 mV and offset current 5 nA Input common-mode voltage range includes ground
- Differential input voltage range equal to the power supply voltage 0V to V+ - 1.5V Large output voltage swing







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APPENDIX D Picture of circuit and analysis



Here we see a protoboard with two circuits; the top one is Sallen-Key the bottom (being analyzed in this picture), is the MFB architecture. Notice the increase in hardware from SK to the MFB design. The two voltmeters show from left to right, Vin and Vout respectively.