Amplitude Modulation

Aim:

Modelling of an amplitude modulated (AM) signal; method of setting and measuring the depth of modulation; waveforms and spectra; trapezoidal display.

Introduction to Amplitude Modulation:

The amplitude modulated signal is defined as:

\[ AM = E(1 + m \cos \mu t) \cos \omega t \]  
\[ = A(1 + m \cos \mu t) \cdot B \cos \omega t \]  
\[ = \text{[low frequency term } a(t)\text{]} \times \text{[high frequency term } c(t)\text{]} \]

There are other methods of writing the AM equation; for example, by expansion, it becomes:

\[ AM = E.m \cos \mu t \cdot \cos \omega t + E \cdot \cos \omega t \]
\[ = \text{DSBSC + carrier} \]

Here:

‘E’ is the AM signal amplitude from eqn. (1). For modelling convenience eqn. (1) has been written into two parts in eqn. (2), where \(A.B = E\).

‘m’ is a constant, which, as you will soon see, defines the ‘depth of modulation’. Typically \(m < 1\). Depth of modulation, expressed as a percentage, is \(100.m\).

There is no inherent restriction upon the size of ‘m’ in eqn. (1).

‘\(\mu\)’ and ‘\(\omega\)’ are angular frequencies in rad/s, where \(\mu/(2.\pi)\) is a low, or message frequency, say in the range 300 Hz to 3000 Hz; and \(\omega/(2. \pi)\) is a radio, or relatively high, ‘carrier’ frequency. In TIMS the carrier frequency is generally 100 kHz.

Notice that the term \(a(t)\) in eqn. (3) contains both a DC component and an AC component. As will be seen, it is the DC component which gives rise to the term at \(\omega\) - the ‘carrier’ - in the AM signal. The AC term ‘\(m \cos \mu t\)’ is generally thought of as the message, and is sometimes written as \(m(t)\). But strictly speaking, to be
compatible with other mathematical derivations, the whole of the low frequency term \( a(t) \) should be considered the message.

Thus:

\[
a(t) = DC + m(t)
\]

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Figure 1 below illustrates what the oscilloscope will show if displaying the AM signal.

![Figure 1 - AM, with \( m = 1 \), as seen on the oscilloscope](image)

A block diagram representation of eqn. (2) is shown in Figure 2 below.

![Figure 2: generation of equation 2](image)

For the first part of the experiment you will model eqn. (2) by the arrangement of Figure 2. The depth of modulation will be set to exactly 100% \((m = 1)\). You will
gain an appreciation of the meaning of ‘depth of modulation’, and you will learn how to set other values of ‘m’, including cases where m > 1. The signals in eqn. (2) are expressed as voltages in the time domain. You will model them in two parts, as written in eqn. (3).

**Depth of Modulation:**

100% amplitude modulation is defined as the condition when m = 1. Just what this means will soon become apparent. It requires that the amplitude of the DC (= A) part of a(t) is equal to the amplitude of the AC part (= A.m). This means that their ratio is unity at the output of the ADDER, which forces ‘m’ to a magnitude of exactly unity.

The magnitude of ‘m’ can be measured directly from the AM display itself. Thus:

$$m = \frac{P - Q}{P + Q}$$

where P and Q are as defined in Figure 3.

![Figure 3: the oscilloscope display for the case m = 0.5](image)

**Spectrum:**

Analysis shows that the sidebands of the AM, when derived from a message of frequency μ rad/s, are located either side of the carrier frequency, spaced from it by μ rad/s.
You can see this by expanding eqn. (2). The spectrum of an AM signal is illustrated in Figure 4 (for the case \( m = 0.75 \)). The spectrum of the DSBSC alone was confirmed in the experiment entitled *DSBSC generation*. You can repeat this measurement for the AM signal. As the analysis predicts, even when \( m > 1 \), there is no widening of the spectrum. This assumes linear operation; that is, that there is *no hardware overload*.

**Procedure:**

**The low frequency term \( a(t) \):**

To generate a voltage defined by eqn. (2) you need first to generate the term \( a(t) \).

\[
a(t) = A.(1 + m.\cos \mu t)
\]

Note that this is the addition of two parts, a DC term and an AC term. Each part may be of any convenient amplitude at the *input* to an ADDER.

The DC term comes from the VARIABLE DC module, and will be adjusted to the amplitude ‘A’ at the *output* of the ADDER.

The AC term \( m(t) \) will come from an AUDIO OSCILLATOR, and will be adjusted to the amplitude ‘A.m’ at the *output* of the ADDER.

**The Carrier supply \( c(t) \):**

The 100 kHz carrier \( c(t) \) comes from the MASTER SIGNALS module.

\[
c(t) = B.\cos \omega t
\]

**To build the Model:**

The block diagram of Figure 2, which models the AM equation, is shown modelled by TIMS in Figure 6 below.
T1 first patch up according to Figure 5, but omit the input X and Y connections to the MULTIPLIER. Connect to the two oscilloscope channels using the SCOPE SELECTOR, as shown.

T2 use the FREQUENCY COUNTER to set the AUDIO OSCILLATOR to about 1 kHz.

T3 switch the SCOPE SELECTOR to CH1-B, and look at the message from the AUDIO OSCILLATOR. Adjust the oscilloscope to display two or three periods of the sine wave in the top half of the screen.

Now start adjustments by setting up a(t), as defined by eqn. (6), and with m = 1.

T4 turn both g and G fully anti-clockwise. This removes both the DC and the AC parts of the message from the output of the ADDER.

T5 switch the scope selector to CH1-A. This is the ADDER output. Set the sensitivity of the oscilloscope to about 0.5 volt/cm. Locate the trace on a convenient grid line towards the bottom of the screen. Call this the zero reference grid line.

T6 turn the front panel control on the VARIABLE DC module almost fully anticlockwise (not critical). This will provide an output voltage of about minus 2 volts. The ADDER will reverse its polarity, and adjust its amplitude using the ‘g’ gain control.
T7 whilst noting the oscilloscope reading on CH1-A, rotate the gain ‘g’ of the ADDER clockwise to adjust the DC term at the output of the ADDER to exactly 2 cm above the previously set zero reference line. This is ‘A’ volts.

You have now set the magnitude of the DC part of the message to a known amount. This is about 1 volt, but exactly 2 cm, on the oscilloscope screen. You must now make the AC part of the message equal to this, so that the ratio $\frac{A m}{A}$ will be unity.

T8 whilst watching the oscilloscope trace of CH1-A rotate the ADDER gain control ‘G’ clockwise. Superimposed on the DC output from the ADDER will appear the message sinewave. Adjust the gain G until the lower crests of the sinewave are EXACTLY coincident with the previously selected zero reference grid line.

The sine wave will be centred exactly A volts above the previously-chosen zero reference, and so its amplitude is A.

Now the DC and AC, each at the ADDER output, are of exactly the same amplitude A. Thus:

$$A = A . m$$

and so:

$$m = 1$$

You have now modelled $A.(1 + m \cos \mu t)$, with $m = 1$. This is connected to one input of the MULTIPLIER, as required by eqn. (2).

T9 connect the output of the ADDER to input X of the MULTIPLIER. Make sure the MULTIPLIER is switched to accept DC.

Now prepare the carrier signal:

$$c(t) = B \cos \omega t$$

T10 connect a 100 kHz analog signal from the MASTER SIGNALS module to input Y of the MULTIPLIER.

T11 connect the output of the MULTIPLIER to the CH2-A of the SCOPE SELECTOR. Adjust the oscilloscope to display the signal conveniently on the screen.

Since each of the previous steps has been completed successfully, then at the MULTIPLIER output will be the 100% modulated AM signal. It will be displayed on CH2-A. It will look like Figure 1.

T12 vary the ADDER gain G, and thus ‘m’, and confirm that the envelope of the AM behaves as expected, including for values of m > 1.
The Modulation Trapezoid:

With the display method already examined, and with a sinusoidal message, it is easy to set the depth of modulation to any value of ‘m’. This method is less convenient for other messages, especially speech.

The so-called trapezoidal display is a useful alternative for more complex messages. The patching arrangement for obtaining this type of display is illustrated in Figure 7 below, and will now be examined.

T13 patch up the arrangement of Figure 7. Note that the oscilloscope will have to be switched to the ‘X - Y’ mode; the internal sweep circuits are not required.
T14 with a sine wave message show that, as m is increased from zero, the display takes on the shape of a TRAPEZOID (Figure 8).

T15 show that, for m = 1, the TRAPEZOID degenerates into a TRIANGLE.

T16 show that, for m > 1, the TRAPEZOID extends beyond the TRIANGLE, into the dotted region as illustrated in Figure 8.

![Figure 8: the AM trapezoid for m = .5. The trapezoid extends into the dotted section as m is increased to 1.2 (120%).](image)

Questions:

Q1 there is no difficulty in relating the formula of eqn. (5) to the waveforms of Figure 6 for values of ‘m’ less than unity. But the formula is also valid for m > 1, provided the magnitudes P and Q are interpreted correctly. By varying ‘m’, and watching the waveform, can you see how P and Q are defined for m > 1 ?

Q2 derive eqn.(7), which relates the magnitude of the parameter ‘m’ to the peak-to-peak and trough-to-trough amplitudes of the AM signal.

Q3 if the AC/DC switch on the MULTIPLIER front panel is switched to AC what will the output of the model of Figure 5 become ?

Q4 in Task T6, when modelling AM, what difference would there have been to the AM from the MULTIPLIER if the opposite polarity (+ve) had been taken from the VARIABLE DC module ?