IV. Nonlinear Circuit Applications

1. Rectifiers

a) Half-wave Rectifier

\[ V_s = V_d + V_c \]

\[ V_c = V_s - V_d \]

DC Component of \( V_c \):

\[ V_{c\text{dc}} = \frac{1}{2\pi} \int_{0}^{\pi} V_c \sin(\theta) \, d\theta = \frac{V_s}{2\pi} \left(1 - \cos \frac{\pi}{2}\right) = \frac{V_s}{2} \]

Half-wave Rectifier with Hold Capacitor
\[ \overline{V} - 2 \]

\[ \Delta V_c = \overline{V}_c (1 - e^{-\frac{t}{\tau}}) \approx \overline{V}_c \frac{\Delta V_c}{\tau} \]

where \( \tau = R_C C \)

At \( t = \tau \)

\[ \Delta V_c = \overline{V}_c \frac{\Delta V_c}{R_C C} \]

To obtain a smooth output voltage with little ripple,

\( R_L C \gg \tau \)

Important application of a half-wave rectifier

**AM Demodulator (Envelope detector)**

Basic Configuration

```
\[ V_{AM} \]
\[ R \]
\[ C \]
\[ V_{out} \]
\[ \tau = \pi C \]
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\[ V_{out} \quad \text{Signal} \]

\[ V_{AM} \quad \text{VAM} \]

\[ V_{AM} = A(t) \cos R_C t \]

\( R_C \): carrier frequency

\[ A(t) = A_0 \cos \omega_0 t \]

\( \omega_0 \): momentary sign. freq.

The RC time constant \( \tau \) of the detector must be chosen such that the maximum slope of the output voltage is at least as large as the maximum slope of the envelope, i.e. the AM input signal before modulation.
Thus
\[
\text{Max} \left\{ \frac{d}{dt} \left[ A_0 \cos \omega_0 t \right] \right\} < \text{Max} \left\{ \frac{d}{dt} \left[ A_0 e^{-\frac{t}{RC}} \right] \right\}
\]

\[A_0 \omega_{\text{max}} < A_0 \frac{1}{RC}\]

\[\therefore \quad RC < \frac{1}{\omega_{\text{max}}}\]

On the other hand, the \(RC\) time constant should not be smaller than the period of the carrier, since then the hold effect would be very weak.

\[\therefore \quad RC > \frac{\tau C}{\text{rmin}}\]

We thus obtain the following condition for the time constant \(\tau = RC\) of the envelope detector

\[
\frac{\tau C}{\text{rmin}} < RC < \frac{1}{\omega_{\text{max}}}
\]

AM Radio. \(0.5\) MHz \(\leq F_C \leq 1.6\) MHz \(f_0 \leq 4.5\) kHz

\[\therefore \quad 2\mu s < RC < 35.4\mu s\]

Choose \(RC\) as the geometric mean of the upper and lower bounds

\[RC \approx 8.4\mu s\]

e.g.: \(\tau = 1\) kHz
\[C = 0.2\mu F\]
b) **Full-wave Rectifier**

b1) **Bridge Full-wave Rectifier**

![Diagram of bridge full-wave rectifier with a floating source note.]

Note: This circuit requires a floating source.

In order to obtain a completely smooth dc output voltage, one generally applies a voltage regulator circuit to the output of the rectifier. An alternative solution is to employ a low-pass filter which cuts off all harmonics of the input frequency.
2. Voltage Regulators

Basic Configuration (with Zener diode)

A voltage regulator can be used to remove a ripple from an input voltage and (or) to maintain a constant output voltage over a range of loads.

Example 1

\[ V_s = 20\text{V (const.)} \]
\[ V_z = 10\text{V} \]

Determine \( R_z \) so that \( V_z \) remains constant at 10 V while the load resistor \( R_L \) varies between \( R_{\text{min}} = 100\Omega \) and \( R_{\text{max}} = 1k\Omega \).
Solution: \( V_c = I_s \cdot R_s + V_e \)

\[ R_s = \frac{V_c - V_e}{I_s} \]

Here: \( I_s = I_e + I_z = \text{const.} \) (for \( V_c = \text{const.} \))

Condition that \( V_c = V_e \) \( \rightarrow \) \( I_e > 0 \) \( \rightarrow \) otherwise \( V_c < V_e \)

\[ I_s > I_{z, \text{max}} = \frac{V_e}{R_{\text{min}}} \]

\[ I_s < \frac{V_c - V_e}{I_{z, \text{max}}} = \frac{V_c - V_e}{V_e} \frac{R_{\text{min}}}{R_s} \]

\( R_s < 100 \Omega \) \( \rightarrow \) Note: If \( I_s = I_{z, \text{max}} \), we obtain a minimum load current, hence \( I_e > 0 \) is met under all conditions

If we select \( R_s = 90 \Omega \), then \( I_{e, \text{min}} = 11.11 \text{mA} \)
\( I_{e, \text{max}} = 101.11 \text{mA} \)

The Zener diode must therefore be capable of dissipating a maximum power of \( P_{z, \text{max}} = V_c \cdot I_{z, \text{max}} = 1.011 \text{W} \)

\[ P_{z, \text{max}} = V_c \cdot \left( \frac{V_c - V_e}{R_s} - \frac{V_e}{R_{\text{min}}} \right) \]

If \( P_{z, \text{max}} < 1 \text{W} \) then increase \( R_s \) \( R_s = 95 \Omega \) \( I_{e, \text{max}} = 6.3 \text{mA} \)
\( I_{e, 0} = 95.3 \text{mA} \)

\[ P_{z, \text{max}} = 0.85 \text{mW} \]
Example 2: \( 6\, \text{V} \leq V_s \leq 7.5\, \text{V} \)
\(\quad 100 \, \Omega \leq R_s \leq \infty\)
\(\quad V_e = 5\, \text{V}\)

Determine \( R_s \) and the max power dissipated by the Zener diode.

Solution. KVL: \( V_s = I_s R_s + V_e \)

KCL: \( I_s = I_e + I_z \)

Condition to maintain \( V_e = V_z \)

\[
R_s \leq \frac{V_s \text{min} - V_e}{I_{e, \text{max}}} \geq \frac{V_e}{I_{e, \text{min}}} = \frac{5}{200} = 0.025 \Omega
\]

Select \( R_s = 15 \Omega \)

\[
I_{e, \text{max}} = \frac{V_s \text{max} - V_e}{R_s} = 0.167 \text{A}
\]

\[
P_{e, \text{max}} = V_e (I_{e, \text{max}} - \frac{V_e}{R_{e, \text{max}}}) = 0.033 \text{W}
\]

Check: \( P_{e, \text{min}} = \frac{V_s \text{min} - V_e}{R_s - \frac{V_e}{R_{e, \text{min}}}} = 0.017 > 0\)

Thus \( V_e = V_z \) under worst case condition!
1.3.4 Clipping and Clamping

Clipping circuits are used to limit voltage excursions.

\[ V_i \sim \pi \]

\[ V_i \pm V_1 \]

\[ V_0 \]

\[ t \]

Clamping is used to make sure a voltage never goes negative (or positive).

\[ V_i \sim \pi \]

\[ V_i \pm V_c \pm V_0 \]

\[ \Rightarrow V_0 = V_i - V_c \]
cascade of clamping circuit with simple rectifier.

Voltage doubling

Voltage multiplier