MOS Device Characteristics

1. The MOS Capacitor

![Diagram of MOS Capacitor]

**Assumptions:**

1. Gate is equipotential region
2. Oxide is perfect insulator
3. No charge carriers in oxide
4. Semiconductor is uniformly doped
5. Semi-conductor is sufficiently thick so that a field free region is formed before reaching the back contact
6. Back contact is ohmic
7. Capacitor is a one-dimensional structure in x
8. \( \Phi_m = \Phi_s + (E_p - E_F) \)

Work function or affinity of metal
Energy-Band Diagrams.

Zero Bias ($V_g = 0$)

Vacuum (free electron)

$E_v$

Metal

Inductor (Oxide)

Fermi

Semi-conductor (N-type)

Zero Bias

a) $V_g' > 0$

Vacuum

$qV_g$

$E_v$

Metal

Inductor

Vacuum

Free carrier accumulation

E Fermi

b) $V_g' < 0$ but $qV_g < (E_F - E_F)$

Vacuum

$qV_g$

$E_v$

Metal

Inductor

Vacuum

Free carrier depletion

E Fermi

$E_{F0}$ denotes intrinsic Fermi level in field-free (bulk) Si.
charge distribution for different bias voltages

a) $V_a^i > 0$

b) $V_a^i < 0$ but $q\alpha V_s < (E_p - E_{10})$

c) $V_a^i < 0$ but $q\alpha V_s > (E_p - E_{10})$
Inversion is achieved if

\[ \bar{V}_1 = 4 \]

Inversion

or

\[ -q\phi_1 > 2(E_F - E_{cv}) \quad (p\text{-type } S) \]

The gate voltage \( \bar{V}_1 \) that must be applied to achieve inversion is called the threshold voltage \( V_T^' \). Since \( V_T^' = \Delta V_0 + \Delta V_x \), we obtain:

\[ V_T^' = -2 \frac{(E_F - E_{cv})}{q} + \Delta V_x = +2\phi_1 + \Delta V_x \quad (p\text{-type } S) \]

Under inversion (\( \bar{V}_1 = V_T^' \)), the hole concentration at the surface of the semiconductor is equal to the donor concentration \( N_D \) (Def. 2.6)

Hence

\[ p_T = N_D = \frac{N_D}{e^{kT/q} + 1} \quad \sigma_T = \frac{q}{kT} \cdot \frac{\sigma_{ox}}{C_{ox}} \]

and

\[ \Delta V_x = \frac{qN_D \cdot W_T}{C_{ox}} \quad \left( C_{ox} = \frac{E_{ox}}{C_{ox}} \right) \]

Solving for \( \phi_1 \) and \( V_T^' \) yields:

\[ \phi_1 = -\frac{kT}{q} \ln \left[ \frac{N_D}{n_i} \right] \quad \left( p\text{-type } S \right) \]

\[ V_T^' = +2\phi_1 - \frac{q \cdot N_D \cdot W_T}{C_{ox}} \quad \text{p-channel device} \]

In analogy to this result, we obtain for a \( p \)-doped semiconductor:

\[ \phi_1 = +\frac{kT}{q} \ln \left[ \frac{N_A}{n_i} \right] \quad \left( p\text{-type } S \right) \]

\[ V_T^' = +2\phi_1 + \frac{q \cdot N_A \cdot W_T}{C_{ox}} \quad \text{n-channel device} \]
2. The MOS Field Effect Transistor

Basic Structure (n-channel)

Operation (enhancement mode)

$V_G > V_T \quad (V_D = 0)$
Threshold Voltage \( V_{th} \) \((V_{th} < 0)\)

\[
V_{th} = V_{FB} + 2 \Phi_p + \frac{Q_{sd}, V_{th}}{C_{ox}} - \frac{Q_{sd}, V_{th}}{C_{ox}} - \frac{Q_n}{C_{ox}} \quad (-C_{ox} = \frac{E_{ox}}{C_{ox}})
\]

where \( V_{FB} = \phi_s - \frac{Q_n}{C_{ox}} - \frac{Q_n}{C_{ox}} - \frac{Q_n}{C_{ox}} \quad (-C_{ox} = \frac{E_{ox}}{C_{ox}}) \)

and \( Q_{sd} = q \cdot N_A \cdot W \cdot \sqrt{2 \Phi_p} \cdot 2 \Phi_p \quad (\phi = \frac{K}{2} \cdot \frac{e}{C_{ox}} \left[ \frac{V}{N_A} \right]) \)

\[ V_{FB} \neq 0 \quad (Body-Effect) \]

\[
V_t = V_{FB} + 2 \Phi_p + \frac{Q_{sd}, V_{th}}{C_{ox}} - \frac{Q_{sd}, V_{th}}{C_{ox}} - \frac{Q_n}{C_{ox}} \quad (-C_{ox} = \frac{E_{ox}}{C_{ox}})
\]

\[ = V_{t0} - \frac{y}{2} \left[ \sqrt{2 \Phi_p + V_{th}} - \sqrt{2 \Phi_p} \right] \]

\[ = V_{t0} - \frac{y}{2} \Phi_p \left[ \sqrt{1 + \frac{V_{th}}{2 \Phi_p}} - 1 \right] \]

where \( y = \frac{\sqrt{2 \Phi_p} \cdot q \cdot N_A}{C_{ox}} \quad (Body-effect increases with \frac{V_{th}}{C_{ox}}) \)

and \( V_{t0} = V_{FB} + 2 \Phi_p + \sqrt{2 \Phi_p} = V_{FB} + \sqrt{2 \Phi_p} \left[ \sqrt{2 \Phi_p} + y \right] \)

Example: \( N_A = 5 \times 10^{22} \text{ m}^{-2} \)

\( V_{FB} = -0.6 \text{ V} \)
\( \Phi_p = 0.4 \text{ V} \)

\[ \phi = 0.507 \text{ V} \]

\[ V_{t0} = 0.565 \text{ V} \]

\( C_{ox} = 2.5 \times 10^{-2} \frac{e}{\text{m}^2} \)

\( \Phi_p = 0.45 \text{ V} \)

\[ V_t(V_{th}) = 0.65 + 0.45 \left[ \sqrt{2 \Phi_p} - 1 \right] \]
Quantitative Relationships

A) Square Law Theory (simple but not quite accurate)

Current density in conducting channel:

\[ J_y = q \mu_n n E_y = -q \mu_n n \frac{dV}{dy} \] (drift current)

\[ \mu = \frac{q \nu}{m} \]

Integrating the current density over cross-sectional area of channel yields:

\[ I_0 = \int J_y \, dx \, dz = +W \int J_y \, dx \]

\[ = \left( W \frac{dV}{dz} \right) \left( -q \int_0^{x_0} \mu_n n \, dx \right) = -W \frac{dV}{dz} \mu_n n x_0 \]

Since \( I_0 \) is constant over full channel length, we can write:

\[ \frac{1}{W} \int_0^{x_0} I_0 \, dx = I_0 L = -W \mu_n \int_0^{x_0} A_n \, dV \]

And finally:

\[ I_0 = -\mu_n n \int_0^{x_0} A_n \, dV \]
Calculation of channel charge \( Q_N \):

Assumption: change added to the gate of an MOS-Cop is balanced by increase in the minority charge \( V_{ox} \) \( V_f \)

we assume \( V_a = V_{ox} + 2Q_f + V_{PP} \)

Thus \( Q_N = Q_f \) \( \frac{V_{ox} + V_f}{V_{ox}} \) over of inversion

\[ Q_N = \frac{Cox(V_{ox} - V_{th}) - Cox(V_f - 2Q_f - V_{th})}{V_{ox} - V_{th}} \]

\[ = Cox(V_{ox} - V_{th}) \]

\[ \Rightarrow Q_N = -Cox(V_{ox} - V_f) \]

Finally, by replacing \( V_f - 2Q_f - V_{th} \) with \( V_{ox} - 2Q_f - V_{PP} - V_{th} \)

where \( 0 < V < V_a \) we obtain:

\[ Q_N(V_f) = -Cox[V_{ox} - V_f - V_f(V_f)] \quad (V_a > V_f) \]

and so:

\[ \Phi = \frac{W}{2} Cox \int_{V_a}^{V_f} (V_{ox} - V_f - V_f) dV \quad (0 < V_{ox} \leq V_{ox}) \]

\[ = \frac{W}{2} Cox \int_{V_a}^{V_f} V_{ox} dV \quad V_f \leq \frac{1}{2} V_{ox}^2 \]

pre-pinch-off characteristics
post-pinch-off characteristics:

\[ I_D \bigg| V_D > V_{Dsat} = I_D \bigg| V_D = V_{Dsat} = \frac{W}{L} \mu_n C \frac{1}{2} \left( V_G - V_t \right) V_{Dsat}^2 \]

\[ \text{(assumption: constant current remaining constant)} \]

\[ I_D \bigg| V_D > V_{Dsat} = \frac{W}{L} \mu_n C \frac{1}{2} \left( V_G - V_t \right) V_{Dsat}^2 \]

Since \( Q_n(L) = 0 \) when \( V_D = V_{Dsat} \), we can write:

\[ Q_n(L) = -C \left( V_G - V_t - V_{Dsat} \right) = 0 \]

\[ V_{Dsat} = \left[ V_G - V_t - V_{Dsat} \right] = V_{off} \]

According to this eq, the saturation current varies as the square of the gate voltage above turn-on, thus called "square-law" dependence.
Channel Length Modulation

If $V_{DS} > V_{TH}$, the drain side of the conductivity channel is pinched off. The effective channel length is then given by

$$L_{eff} = L - W(V_{DS}) \quad \text{and} \quad V_{eff} = (V_{DS} - V_{TH})$$

where

$$W(V_{DS}) = \sqrt{\frac{2 \varepsilon_e \epsilon_0 (V_{TH} + V_{DS} - V_{eff})}{Q}}$$

\[\frac{dI_D}{dV_{DS}} = \frac{dI_D}{dL_{eff}} \frac{dL_{eff}}{dV_{DS}} = -\frac{1}{2} \frac{W}{L_{eff}} \mu C_{ox} V_{eff}^2 \frac{dL_{eff}}{dV_{DS}}\]

\[\frac{dL_{eff}}{dV_{DS}} = -\frac{dW}{dV_{DS}} = -\sqrt{\frac{\varepsilon_r \epsilon_0}{2Q N_{sc}(Q_{ox} + V_{DS} - V_{eff})}}\]

\[\frac{dI_D}{dV_{DS}} = \frac{1}{2} \frac{W}{L_{eff}} \mu C_{ox} V_{eff}^2 \frac{1}{L_{eff}} \sqrt{\frac{\varepsilon_r \epsilon_0}{2Q N_{sc}(Q_{ox} + V_{DS} - V_{eff})}}\]

\[I_D = \frac{1}{2} \frac{W}{L_{eff}} \mu C_{ox} \frac{V_{eff}^2}{2} \left(1 + \Lambda [V_{DS} - V_{eff}]\right)\]

where

$$\Lambda = \frac{1}{L_{eff}} \sqrt{\frac{\varepsilon_r \epsilon_0}{2Q N_{sc}(Q_{ox} + V_{DS} - V_{eff})}}$$

\[I_D = \frac{1}{2} \mu C_{ox} \frac{V_{eff}^2}{2} (1 + \Lambda [V_{DS} - V_{eff}])\]
Summary MOSFET Equations

Threshold Voltage

\[ V_t = 2 \Phi_p + \frac{l}{C_{ox}} \sqrt{4 \varepsilon_s \Phi_p q N_{sd}} + V_{FS} \]  
\[ \Phi_p = \frac{kT}{q} \ln \left( \frac{N_{sd}}{n_i^2} \right) \]
\[ C_{ox} = \frac{C_{ox}}{t_{ox}} \]

\[ V_t = -2 \Phi_p - \frac{l}{C_{ox}} \sqrt{4 \varepsilon_s \Phi_p q N_{sd}} + V_{FS} \]

Drain-Source Current (Square-Law Th.)

\[ \text{Ohmic Region} \quad V_{DS} \leq (V_{gs} - V_t) \]
\[ I_D = \frac{W}{L} \mu C_{ox} \int_0^L (V_{gs} - V_t - V_x) \, dx \]
\[ = \frac{W}{L} \mu C_{ox} \left[ (V_{gs} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \]

\[ \text{Saturation Region} \quad V_{DS} > (V_{gs} - V_t) \]
\[ I_D = \frac{1}{2} \frac{W}{L} \mu C_{ox} \left[ V_{gs} - V_t \right]^2 \left( 1 + \lambda V_{DS} \right) \]
\[ \lambda = \frac{1}{L} \sqrt{\frac{\varepsilon_s}{2q N_{sd} (\Phi_p + V_{DS})}} \quad \text{ch. length mod. factor} \]
\[ \varepsilon_s \approx 1.03 \times 10^{-10} \frac{A^2}{V \text{m}} \quad \Phi_p \approx 900 \text{mV} \]
MOSFET Operating Modes (N-channel Dev)

A) **Cutoff**

\[
\begin{align*}
\text{Model: } & V_{GS} < V_t \\
\text{Model: } & I_D = 0 \\
\text{no channel formed}
\end{align*}
\]

B) **Triode or Ohmic Region**

\[
\begin{align*}
\text{Model: } & V_{GS} > V_t, \ V_{DS} < (V_{GS} - V_t) \\
\text{Model: } & I_D = \mu C_{ox} \frac{W}{L} \left( (V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right) \\
\text{Conductive channel between Drain - Source} \\
\text{Voltage controlled} \\
\end{align*}
\]

\[
\frac{\partial I_D}{\partial V_{DS}} = \mu C_{ox} \frac{W}{L} \left( V_{GS} - V_t - V_{DS} \right)
\]

As long as \( V_{DS} \) is small with regard to \( V_{GS} - V_t \), the \( I/V \) characteristic of the MOSFET in the Triode region is approximately linear.

**Example:** \( \mu C_{ox} \frac{W}{L} = 200 \mu A/V^2 \) \( V_{GS} - V_t = 2 \) \( V_{DS} = 0.1 \) \( G_{DS} = 400 \mu S \) \( V_{DS} = 2.5 \) \( R_S \)
C) Forward active or Saturation Region

\[ V_{ds} > V_t \quad V_{ds} > (V_{gs} - V_t) \]

Channel is pinched-off at drain side of MOS.

The depleted channel on the drain side creates a situation similar to a PN Junction, where the free carriers can still cross the imposed barrier as long diffusion forces transport them to the very edge of the depletion region.

**I/V Characteristic**

\[ I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} \left[ V_{gs} - V_t \right]^2 \left[ 1 + \frac{1}{2} V_{ds} \right] \]

channel length modulation factor

The I/V characteristic of a MOSFET in saturation is approximately parabolic.

If the MOSFET channel is comparatively short, the depletion region on the drain side shortens the ohmic region of the channel noticeably, which in turn increases the current.

**Note:** \( V_A \) is the equivalent of the Early Voltage \( V_A \) in conjunction with \( IS \).
Since the (drain) current of a MOSFET is essentially controlled by its Gate-Source voltage $V_{gs}$, we can think of the device acting like a voltage-controlled current source. We therefore introduce the trans-conductance $g_{m}$ as follows:

\[ g_{m} = \frac{\partial I_{d}}{\partial V_{gs}} = \mu C_{ox} \frac{W}{L} [V_{gs}-V_{T}] \]

or

\[ g_{m} = \frac{\partial I_{d}}{\partial V_{gs}} = \sqrt{2I_{dsat} \mu C_{ox} \frac{W}{L}} \]

The dependence of the (saturation) current on the applied Drain-Source voltage $V_{ds}$ can be described by the conductance $g_{ds}$ defined as

\[ g_{ds} = \frac{\partial I_{d}}{\partial V_{ds}} = \frac{1}{2} \mu C_{ox} \frac{W}{L} [V_{gs}-V_{T}] \]

or

\[ g_{ds} = \frac{\partial I_{d}}{\partial V_{ds}} = \lambda I_{dsat} \]

Due to the insulating layer between the gate and the channel, the Gate-Source and the Drain-Source junctions act like (ideal) capacitors.
The low frequency model of the MOSFET in saturation therefore looks like

\[
\begin{align*}
\text{2-Port} & \quad \text{at} \\
V_{gs} & \quad \text{+} \\
\quad & \quad \text{D} \\
\quad & \quad \text{S} \\
\quad & \quad \text{V}_{ds} \\
\end{align*}
\]

\[
\begin{align*}
\text{+} & \quad \text{at} \\
\quad & \quad \text{D} \\
\quad & \quad \text{S} \\
\quad & \quad \text{V}_{ds} \\
\end{align*}
\]

Note: The above model for the MOSFET (in common-source configuration) is a Y-Parameter model with

\[
\begin{align*}
Y_{uu} & \approx 0 \\
Y_{ul} & \approx 0 \\
Y_{21} & = g_m \\
Y_{32} & = g_{ds} \\
V_m & = \sqrt{2\mu C_{ox} \frac{V_t}{I_{ds}}} \\
g_{ds} & = \lambda \cdot I_{ds}
\end{align*}
\]

Numerical Example:

Device Parameters:

\[
\begin{align*}
\mu C_{ox} \frac{W}{L} & = 200 \mu A/V^2 \\
\lambda & = 0.05 \ V/V \\
V_t & \equiv 1 \ V
\end{align*}
\]

DC Biasing:

\[
\begin{align*}
V_{ds} & = 2 \ V \\
V_{gs} & > V_{ds} - V_t
\end{align*}
\]

\[
\begin{align*}
I_{ds} & \approx 100 \mu A \\
g_m & \approx 200 \ \mu S \\
g_{ds} & \approx 10 \ \mu S
\end{align*}
\]

saturation current
transconductance
output conductance