VIII. Noise in Integrated Circuits

1. Sources of Noise

A) Shot Noise

1. Causes: Carriers must jump potential barriers between a p-n junction (Random process). Hence, the effective current flowing through the junction fluctuates around its average value in a random way.

\[ I_{ph} = \text{Average current} \]

2. Spectral density:

\[ \frac{\dot{I}^2}{\delta f} = 2q \bar{I}_{ph} \delta f \quad \delta f < \frac{1}{T} \]

q: electron charge

T: Transit time of carrier

\[ I_{ph}: \text{Average current} \]

\[ \delta f: \text{Bandwidth in Hz} \]

3. Frequency distribution: White

Amplitude variation: Gaussian \((\overline{I}^2 = \sigma^2)\)

\[ \text{r.e. noise signal is } < \text{ t.s.d. in } 99.7\% \text{ of time} \]
3) Thermal Noise

1. Cause: Random Thermal Motion of Electrons in any conductor.

Note: Since $\mu_{\text{diff}} \ll T$, thermal noise is independent of current but proportional to $T$

$$V_{\text{th}} = \sqrt{\frac{4kT}{m_e}}$$

effective mass of charge in crystal lattice

e.g. $T = 300K$

$\frac{V_{\text{th}}}{m_e} \approx 10^{-5} m$  

2. Spectral Density

represented by a voltage source

$$V^2 = 4kT R \Delta f$$

represented by a current source

$$I^2 = \frac{4kT}{R} \Delta f$$

$\Rightarrow$ Frequency distribution: White

Amplitude variations: Gaussian ($I^2 = \sigma^2$, $V^2 = \sigma^2$)
C. Flicker Noise (1/f Noise)

1. Cause: Traps in a material capture and emit carriers in a random fashion.

2. Spectral Density:

   Empirical \( \overline{I^2} = K_1 \frac{I_0}{f^b} \af \)

   - \( I_0 \): constant
eq \( 0.5 \) to \( 2 \)

   - \( b \approx 1 \)

   The time constants associated with the capture and release of charge carriers give rise to a noise signal with energy concentrated at low frequencies.

   Frequency distribution

   \( \log \overline{I^2} vs. f \)

   \( \log \beta \)

   Amplitude variation: Often not Gaussian
1. \( T_d = \left[ \frac{dI_d}{dV_d} \right]^{-1} = \frac{kT}{qI_0} \) Modelling resistor (produces no noise).

2. \( R_s \) physical series resistor (produces noise).

3. \( \overline{V_s^2} = 4kT R_s \alpha f \) Thermal noise due to \( R_s \).

4. \( \overline{i^2} = 2qZ_0 \alpha f + k \overline{\frac{I_0^2}{p f}} \) Shot noise and flicker noise of a p-n junction and semiconductor material.
1. \( V_B = 4kT_0 \alpha_f \) due to base series resistor \( r_B \) (thermal)

2. \( I_B = 2qI_o \alpha_f + I_n \) af holes jumping into emitter (shot)

Flicker Recombination Current
in Base (shot)

3. \( I_C = 2qI_CAF \) af, EL jumping from emitter (shot)

Recombination Current in Base (shot)

4. Neglect noise source associated with \( I_C \) (small)

Note: If recombination current in base is small, then all noise sources are statistically independent.
C) Field Effect Transistor

\[ i_\beta^2 = 2q \bar{I}_\beta \delta f \quad \text{Shot Noise due to reverse leakage in JFET} \]

Note: \( i_\beta^2 = 0 \) for MOSFET

\[ i_d^2 = 4kT \left( \frac{1}{\pi \Delta f} \right) \delta f + k \frac{\bar{I}_d}{\Delta f} \]

Thermal Noise, Flicker Noise

Channel resistance, \( R_{ch} \):

Recall: \( \tau_T = \frac{2}{\mu} \cdot \frac{W}{C_{ox} \cdot L} (V_{gs} - V_T) \quad \text{Total charge} \)

\[ \frac{1}{\bar{\tau}_S} = \frac{\mu q}{L} \int_0^{V_T} \frac{x}{V_s - V_T} \, dx \quad \text{sheet resistance} \]

\[ \Rightarrow \frac{1}{\bar{\tau}_S} = \frac{2}{3} \frac{\mu}{C_{ox} \cdot L} (V_{gs} - V_T) \]

\[ \bar{\tau}_{ch} = \frac{L}{W}, \quad \bar{\tau}_S = \frac{2}{3} \frac{\mu C_{ox} \cdot W}{L (V_{gs} - V_T)} \]

Since \( g_m = \frac{\mu C_{ox} \cdot W}{L} (V_{gs} - V_T) \)

\[ \Rightarrow \bar{\tau}_{ch} = \frac{1}{\bar{\tau}_S} \cdot g_m \]

\[ i_d^2 = 4kT \frac{2}{3} g_m \Delta f + k \frac{\bar{I}_d}{\Delta f} \]

MOSFET: \[ i_d^2 = 4kT \frac{2}{3} g_m \Delta f + k \frac{\bar{I}_d}{\Delta f} \]
3. Circuit Noise Calculations

Given: Noise current source with spectral density

\[ S(f) = \frac{\varepsilon^2}{\Delta f} \]

In a small bandwidth \( \Delta f \), the mean-square value of the noise current is given by

\[ i = \sqrt{S(f) \Delta f} \]

Thus, the noise current in bandwidth \( \Delta f \) can be represented approximately by a sinusoidal current generator with rms value \( i \) as shown above.

If the noise current in bandwidth \( \Delta f \) is now applied as an input signal to a circuit, its effect can be calculated by substituting the sinusoidal generator and performing circuit analysis in the usual fashion. This method of noise calculations reduce to familiar sinusoidal circuit analysis calculations.
Procedure:

1. Replace $\frac{i_n^2}{\omega_n}$ with sinusoidal source of magnitude $i_n$ ($V_n$) and frequency $\omega_0$.

2. Calculate $V_{on}(\omega_0)$ due to $i_n(\omega_0)$.

3. $V_{on}^2$ is output power spectrum density at $\omega_0$.

4. If more than 1 source and sources independent, then add sum of squares $V_0^2 = \sum_{n} V_{on}^2$.

Example:

Find $V_0^2$.
1. \[ \bar{V}_1^2 = 4kT \bar{I}_1 \alpha P \]
\[ \bar{V}_2^2 = 4kT \bar{I}_2 \alpha P \]

2. \[ V_0 = \bar{V}_1 + \bar{V}_2 \Rightarrow \bar{V}_0^2 = \bar{V}_1^2 + \bar{V}_2^2 + 2 \bar{V}_1 \bar{V}_2 \]

Since \( V_1 \) is statistically independent of \( V_2 \)
\[ \bar{V}_1 \bar{V}_2 = 0 \] (orthogonality)

3. \[ \bar{V}_0^2 = \bar{V}_1^2 + \bar{V}_2^2 = 4kT (\bar{I}_1 + \bar{I}_2) \alpha P \]

⇒ equivalent network

4. Equivalent Input Noise Generators

Idea: Replace noisy network by a noiseless network with equivalent input noise sources
1. If $Z_s = 0$, $\overline{I_{eq}^2}$ is shorted.
   $\overline{V_{eq}^2}$ is single noise source.

2. If $Z_s \to \infty$, $\overline{V_{eq}^2}$ is in open-loop.
   $\overline{I_{eq}^2}$ is single noise source.

3. Finite $Z_s$. Both $\overline{V_{eq}^2}$ and $\overline{I_{eq}^2}$ contribute to output noise.

How to find $\overline{V_{eq}^2}$ and $\overline{I_{eq}^2}$?

a) Short input of both networks and equate output noise of both cases.
   Solve for $\overline{V_{eq}^2}$.

b) Open input of both networks and equate output noise of both cases.
   Solve for $\overline{I_{eq}^2}$.

Example 1: JFET

Noisy network

\[
\begin{align*}
\overline{V_{eq}} & \quad \overrightarrow{g_m v_{gs}} \quad C_{gs} \quad \overrightarrow{g_m v_{gs}} \quad v_o \\
\overrightarrow{r_{eq}} & \quad \overrightarrow{v_{gs}} \quad C_{gs} \quad \overrightarrow{g_m v_{gs}} \quad v_o \\
\end{align*}
\]

Noisless network with eq. input noise sources

\[
\begin{align*}
\overline{V_{eq}} & \quad \overrightarrow{g_m v_{gs}} \quad C_{gs} \quad \overrightarrow{g_m v_{gs}} \quad v_o \\
\overrightarrow{I_{eq}} & \quad \overrightarrow{v_{gs}} \quad C_{gs} \quad \overrightarrow{g_m v_{gs}} \quad v_o \\
\end{align*}
\]
A) Short inputs to find $\overline{V_{eq}^2}$

noisy NW  \quad \text{noisless NW}

$V_{out} = \overline{i_d \cdot 0}$  \quad $V_{out} = \overline{-f \cdot f_{eq} \cdot 0}$

\[ \overline{i_d} = \overline{-f \cdot f_{eq}} \]

\[ \overline{V_{eq}^2} = \frac{\overline{i_d^2}}{f \cdot m^2} = \frac{4 \pi f^2}{3} \cdot m \cdot \alpha \cdot f + \frac{\overline{i_{eq}^2}}{f \cdot m^2} \cdot \alpha \cdot f \]

5) Open inputs to find $\overline{i_{eq}^2}$

noisy NW  \quad \text{noisless NW}

$V_{out} = \overline{(i_d - f \cdot m \cdot \frac{i_d}{\omega C_f}) \cdot 0}$  \quad $V_{out} = \overline{-f \cdot f_{eq} \cdot 0}$

\[ \overline{i_d - f \cdot m \cdot \frac{i_d}{\omega C_f}} = \overline{-f \cdot f_{eq}} \]

\[ \overline{i_{eq}^2} = \frac{i_{eq}^2}{f \cdot m} \cdot f_{eq} \]

since $i_f$ and $i_d$ are statistically independent

\[ \overline{i_{eq}^2} = \overline{i_f^2} + \frac{\overline{\omega C_f^2}}{f \cdot m^2} \overline{i_d^2} \]

or

\[ \overline{i_{eq}^2} = 2q \cdot \alpha \cdot f + \overline{\omega C_f^2} \cdot \overline{V_{eq}^2} \cdot \alpha \cdot f \]

Hospet

$\overline{i_{eq}^2} \approx 0$

\[ \overline{i_{eq_{Hos}}^2} \approx \omega C_f \cdot \overline{V_{eq}^2} \cdot \alpha \cdot f \]

small at low to moderate frequencies
Comments:
1. $V_{eq}$ is much higher than bipolar because $g_m$ is lower for FETs.
2. $i_{eq}$ is much lower than bipolar at low to moderate frequencies ($\omega C_C << 1$) since $I_a << I_Q$.

Note: MOSFET $I_a = 0 \Rightarrow i_{eq} = 0$ at low to mod. freq.

\[ V_{eq} = 4kTb_i a f + 2kTg_m a f \]

\[ i_{eq} = 2qI_Q a f + k_i \frac{I_c}{\beta} a f + 2qI_C \frac{a f}{\sqrt{\beta(\omega j)^2}} \]

where $\beta(\omega j) = \frac{g_m T_f}{(1 + j\omega \tau' C_p)}$ and $g_m = I_c \frac{q}{kT}$

Note:
- at low frequencies: $\frac{i_{eq}^2}{V_{eq}^2} \approx \frac{i_b^2}{V_b^2}$ independent sources
- at high frequencies: $\frac{i_{eq}^2}{V_{eq}^2} \approx \frac{i_b^2}{V_b^2} + \frac{i_c^2}{g_m^2}$ dependent sources
Example 2: Differential input stage of CMOS Opamp

1. Replace circuit elements by noiseless elements and their equivalent noise sources.

\[
\begin{align*}
MOS : \quad & \frac{K}{V_L C_{eq}} \\
\text{V}_{\text{eq}}^2 = & \left( \frac{8}{3} \frac{kT}{g_m} + \frac{kT}{g_m} \right) \Delta f \\
\text{V}_{\text{eq}}^2 = & \omega C_{gs}^2 \text{V}_{\text{eq}}^2 \approx 0
\end{align*}
\]

2. Apply a small signal analysis to the circuit with the equivalent noise model and solve for the output noise power.
Output noise power: (assume \( V_1^2 = V_2^2 \) and \( V_x^2 = V_o^2 \))

\[
V_o^2 = \left( \frac{g_{m1}}{g_{o1} + g_{o3}} \right)^2 (2V_1^2 + 2 \left( \frac{g_{m3}}{g_{m1}} \right)^2 V_3^2)
\]

Note: \( \frac{g_{m1}}{g_{o1} + g_{o3}} \) = gain of differential stage

3. Calculate equivalent input noise source that produces the same output noise power

\[
\overline{V}_{\text{ieq}}^2 = \frac{V_o^2}{A_d^2}
\]

where \( A_d = \left( \frac{g_{m1}}{g_{o1} + g_{o3}} \right) \)

\[
\Rightarrow \overline{V}_{\text{ieq}}^2 = 2V_1^2 + 2 \left( \frac{g_{m3}}{g_{m1}} \right)^2 V_3^2
\]

Note: \( g_{m3} \ll g_{m1} \) (for high gain-bandwidth product)

Thus noise contribution of \( M1 \) and \( M2 \) is dominant
5. Sampled Noise (J. Fischer: Jour. SSC, pp. 742-752, Aug. 1982)

Example: Inverting SC integrator

What is noise response of this system?

This circuit performs a track and hold function (tracing during phase 1 and holding during phase 2.)

Track and hold equivalent circuit

We assume input noise is white with equivalent bandwidth $B_{IN}$ and power density $N_0$.

Note: equivalent noise bandwidth

$$ \int_{-\infty}^{\infty} S_n(f) \, df = N_0 \cdot B_{IN} $$
a) **Sample-and-Hold Noise Model**

If we assume an ideal S/H operation with holding time 
\[ T_h = T_{SH} \cdot \frac{\tau_{SH}}{T} \] 
we obtain the following output power density

\[ P_0(f) \leq \frac{R_n}{T} \cdot \tau_{SH}^2 \cdot \sin^2\left(\frac{\tau_{SH}^2}{T}\right) \quad \text{if } \frac{1}{2} < \frac{T}{T_{SH}} \]

\[ P_0(f) \leq \frac{2R_n}{T} \cdot \tau_{SH}^2 \cdot \sin^2\left(\frac{\tau_{SH}^2}{T}\right) \quad \text{if } \frac{1}{2} \leq \frac{T}{T_{SH}} \]

Follower effect due to undersampling of input noise

b) **Track-and-Hold Noise Model**

Since the track-and-hold operation occupies the entire sampling period, we can denote the tracking time 
\[ T_T = T - T_{SH} \]
by \( (1 - T_{SH}) T \). The output power density is then equal to

\[ P_0(f) \leq \frac{R_n}{T} \left[ \tau_{SH}^2 \cdot \sin^2\left(\frac{\tau_{SH}^2}{T}\right) + (1 - T_{SH}) \right] \quad \text{if } \frac{1}{2} < \frac{T}{T_{SH}} \]

\[ P_0(f) \leq \frac{2R_n}{T} \cdot \tau_{SH}^2 \cdot \sin^2\left(\frac{\tau_{SH}^2}{T}\right) + (1 - T_{SH}) \quad \text{if } \frac{1}{2} \leq \frac{T}{T_{SH}} \]
Examples:

a) $T_{SH} = 0$ (fully tracking + no S/H term)

$$\hat{N}_0(f) = n_n \quad \forall f$$

b) $T_{SH} = 1$ (ideal S/H + no tracking term)

$$\hat{N}_0(f) \leq n_n \text{sinc}^2\left(\frac{f}{f_s}\right) \quad \frac{\Delta f_n}{2} f_s < \frac{1}{2} f_s$$

$$\hat{N}_0(f) \leq n_n 2 \frac{\Delta f_n}{f_s} \text{sinc}^2\left(\frac{f}{f_s}\right) \quad \frac{\Delta f_n}{2} f_s \geq \frac{1}{2} f_s$$

Output noise spectrum exhibits familiar $\sin x/x$ envelope.

c) $T_{SH} = \frac{1}{2}$

$$\hat{N}_0(f) \leq n_n \frac{1}{2} \left[ \frac{\Delta f_n}{f_s} \text{sinc}^2\left(\frac{f}{2 f_s}\right) + 1 \right] \quad \frac{\Delta f_n}{2} f_s < \frac{1}{2} f_s$$

$$\hat{N}_0(f) \leq n_n \frac{1}{2} \frac{\Delta f_n}{f_s} \left[ \text{sinc}^2\left(\frac{f}{2 f_s}\right) + 1 \right] \quad \frac{\Delta f_n}{2} f_s \geq \frac{1}{2} f_s$$

In case of undersampled noise ($f_s \ll \frac{\Delta f_n}{2} f_s$) the low frequency noise is dominated by the S/H term ($\hat{N}_0(f \to 0) \approx \frac{1}{2} n_n \frac{\Delta f_n}{f_s}$) while the transmission gate term sets the high-frequency noise floor ($\hat{N}_0(f \to \infty) \approx \frac{1}{2} n_n$).
Concept of Equivalent Input Noise

Example Noninverting amplifier

Equivalent Circuit for Noise Analysis

\[
\begin{align*}
| V_{on}^2 &= V_A^2 (1 + \frac{R_2}{R_1})^2 + \bar{I}_n^2 R_2 + \frac{V^2_{ref}}{R_1} + V_{n_2}^2 + V_{n_3}^2 | \\
\text{where} & \\
| V_A^2 & \approx 4 kT \frac{Z}{5} f_{min} + Kn \frac{I}{(W-L C_{ox}) f} \\
| \bar{I}_n^2 & \approx V_A^2 \omega C_{gs}^2 | \\
| V_{n_2}^2 & \approx 4 kT R_2 \\
| V_{n_3}^2 & \approx 4 kT R_3
\end{align*}
\]
\[ \bar{V}_{ieq}^2 = \frac{V_{on}}{Q_{in}} = V_A^2 + \frac{\bar{R}_2^2}{(1 + \omega C_f \bar{R}_c)^2} + 4kT \bar{R}_1 \]

Inserting the actual amplifier noise sources yields:

\[ \bar{V}_{ieq}^2 = V_A^2 \left[ 1 + \frac{\omega C_f \bar{R}_c^2}{(1 + \omega C_f \bar{R}_c)^2} \right] + 4kT \bar{R}_1 \frac{1}{(1 + \omega C_f \bar{R}_c)} \]

Opamp Contrib. \hspace{1cm} External Res.

If \( \bar{R}_2 \gg \bar{R}_c \)

\[ \bar{V}_{ieq}^2 = V_A^2 \left[ 1 + \omega C_f \bar{R}_c^2 \right] + 4kT \bar{R}_1 \]

\[ \omega_n = \frac{1}{C_f \bar{R}_c} \]

\( C_f = 100 \Omega \cdot \mu F \)

\( \bar{R}_c = 1 \text{ k}\Omega \)

\( \omega_n = 2\pi \cdot 1.6 \text{ GHz} \)

Note: Opamp pair roll-off prevents high-frequency noise increase
Total Noise Power

Example noninverting amplifier

\[ \overline{V_{\text{eq}}^2} \approx \overline{V_A^2} + 4kT \eta \]

\( \eta = 1 \text{K} \)

\[ 4kT \eta = 16.5 \times 10^{-18} \frac{V^2}{\text{Hz}} \]

\[ \sqrt{4kT \eta} = 4.07 \frac{\Delta V}{\text{VHz}} \]

\[ \begin{array}{c}
\overline{V_A^2} \\
V_A^2 \end{array} \]

\[ \begin{array}{c}
\frac{[V^2/\text{Hz}]}{10^{-18}} \\
25 \end{array} \]

\( f_0 = 10 \text{kHz} \)

\( \log f \)

Question: What is the total noise power in the frequency band from 10kHz to 100kHz?

\[ V_{\text{eq}}^2 \approx \int_{f_1}^{f_2} V_{\text{eq}}^2(f) \, df \]

\[ \begin{array}{c}
\frac{13^2}{f_1} \\
\frac{13^2}{f_2} \end{array} \]

\( f_1 = 10 \text{kHz} \)

\( f_2 = 100 \text{kHz} \)

\[ = \int_{f_1}^{f_2} (25 + 16.5) \times 10^{-18} \, df + \int_{f_1}^{f_2} 25 \times 10^{-18} \frac{f_0}{f} \, df \]

\[ V_{\text{eq}}^2 \approx 41.6 \times 10^{-15} [V^2] + 25 \times 10^{-14} \cdot 4 \cdot \ln(0.01) [V^2] \]

\[ V_{\text{eq}}^2 \approx 41.6 \times 10^{-15} [V^2] + 25 \times 10^{-14} [V^2] \]

\[ V_{\text{eq}}^2 \approx \text{thermal contribution} + \text{1/f contribution} \]

\[ \therefore V_{\text{eq}}^2 \approx 2.54 \mu V \]
Low Noise CMOS Opamp

Design Guidelines

A) HfR Noise

\[ V_{\text{HI R}}^2 \propto \frac{1}{V(W/L)_{\text{IN}}} \left[ 1 + \mu L \frac{V_{(W/L)_{\text{IM}}}}{V_{\text{DIFF}}} \right] \left[ 1 + \frac{\mu L}{\sqrt{2}} \right] \]  

(1)

B) Ip Noise

\[ V_{\text{IP}}^2 \propto \frac{1}{V_{(W/L)_{\text{IN}}}^2} \left[ 1 + \frac{\mu L^2}{L_{\text{DE}}^2} \right] \]  

(2)

where \( \mu \) = \( \begin{cases} k_p \mu_p & \text{n-channel input stage} \\ k_n \mu_n & \text{p-channel input stage} \end{cases} \)

\( n\text{-channel input} \quad \mu_{n,\text{IP}} \approx 0.15 - 0.35 \)

\( p\text{-channel input} \quad \mu_{p,\text{IP}} \approx 3 - 7 \)

To keep Ip contribution of local stage within reason, one should select \( L_{\text{ID}} > \sqrt{2} \mu L_{\text{IN}} \).

c.f. \( n\text{-channel input} \): \( L_{\text{ID}} > 0.72 L_{\text{IN}} \)

\( p\text{-channel input} \): \( L_{\text{ID}} > 3.2 L_{\text{IN}} \).