Problem 18
The four coplanar vectors shown in the figure have the following magnitudes (in cm):

\[
\|\vec{A}\| = 4 \quad \|\vec{B}\| = 7 \quad \|\vec{C}\| = 3 \quad \|\vec{D}\| = 5
\]

A. Compute the \(x\) and \(y\) components of each vector.
B. Write each vector in terms of the coordinate unit vectors \(\hat{i}\) and \(\hat{j}\) (in the \(x\) and \(y\) directions, respectively).
C. Compute the vector \(\vec{E} = \vec{A} + \vec{D}\).
D. Compute the vector \(\vec{F} = \vec{A} + \vec{B} + \vec{C} + \vec{D}\).
E. Compute the magnitude of \(\vec{F}\).
F. Compute the angle between \(\vec{F}\) and the \(x\) axis.
G. Compute the smallest angle between \(\vec{E}\) and \(\vec{F}\).

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{vector_diagram.png}
\caption{Vector diagram showing the four coplanar vectors.}
\end{figure}

Answers
B. \(\vec{A} = 4\hat{i} \text{ cm}, \vec{B} = 6.06\hat{i} + 3.50\hat{j} \text{ cm}, \vec{C} = 3\hat{j} \text{ cm}, \vec{D} = -3.54\hat{i} - 3.54\hat{j} \text{ cm}\)
C. \(\vec{E} = 0.464\hat{i} - 3.54\hat{j} \text{ cm}\)
D. \(\vec{F} = 6.52\hat{i} + 2.96\hat{j} \text{ cm}\)
E. \(\|\vec{F}\| = 7.17 \text{ cm}\)
F. \(24.42^\circ\)
G. \(106.96^\circ\)
Problem 19
The two coplanar vectors have the magnitudes $\|\vec{A}\| = 7 \text{ m}$ and $\|\vec{B}\| = 5 \text{ m}$.

Compute the:

A. unit vectors in the direction of $\vec{A}$ and $\vec{B}$;
B. dot product $s = \vec{A} \cdot \vec{B}$;
C. dot product $t = (-\vec{A}) \cdot (-\vec{B})$;
D. cross product $\vec{C} = \vec{A} \times \vec{B}$;
E. cross product $\vec{D} = \vec{B} \times \vec{A}$.

**Answers**
A. $\hat{u}_A = 0.906\hat{i} + 0.423\hat{j} \text{ m}$, $\hat{u}_B = 0.259\hat{i} + 0.956\hat{j} \text{ m}$
B. $s = 22.50 \text{ m}^2$
C. $t = 22.50 \text{ m}^2$
D. $\vec{C} = 26.80\hat{k} \text{ m}^2$
E. $\vec{D} = -26.80\hat{k} \text{ m}^2$
Problem 20
The figure shows two forces, $E$ and $F$, applied to a bracket on a physical therapy machine. Determine the angle $\theta$ at which $E$ must be applied to have the combined effect of $E$ and $F$ be equal to that of a single 200 N force.

\[
\begin{align*}
E &= 100 \text{ N} \\
F &= 150 \text{ N}
\end{align*}
\]

Answers $75^\circ 31'$ or $75.52^\circ$

Problem 21
A force vector $\vec{F} = -7\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}$ N has a line of action that passes through point $p$, which has coordinates of $x = 3$, $y = 6$, and $z = 5$ (all in cm).

A. A second point $q$ is located at coordinates $x = 1$, $y = 2$, and $z = 2$ (cm). Compute the components of the unit vector $\hat{r}$ that originates at $q$ and is directed at $p$.

B. Compute the cross product $\vec{M} = \hat{r} \times \vec{F}$.

C. Compute the angle between $\hat{r}$ and each of the three coordinate axes.

D. Find $\vec{F}_\parallel$, the component of $\vec{F}$ that is parallel to $\hat{r}$.

E. Find $\vec{F}_\perp$, the component of $\vec{F}$ that is orthogonal (perpendicular) to $\hat{r}$. Check your answer by showing the dot product of $\vec{F}_\perp$ with $\hat{r}$ is zero, and $\vec{F}_\parallel + \vec{F}_\perp = \vec{F}$.

F. Compute the vector $\vec{U}$ that is orthogonal (perpendicular) to the plane containing vector $\hat{r}$ and vector $\vec{F}$.

Answers

A. $\hat{r}_x = 0.371$, $\hat{r}_y = 0.743$, $\hat{r}_z = 0.557$ cm

B. $\vec{M} = -2.785\mathbf{i} - 5.014\mathbf{j} + 8.542\mathbf{k}$ N-cm

C. Angle between $\hat{r}$ and the $x$ axis: $68.20^\circ$  
   $y$ axis: $42.03^\circ$  
   $z$ axis: $56.15^\circ$

D. $\vec{F}_\parallel = 2.138\mathbf{i} + 4.276\mathbf{j} + 3.207\mathbf{k}$ cm

E. $\vec{F}_\perp = -9.138\mathbf{i} + 4.724\mathbf{j} - 0.207\mathbf{k}$ cm

F. $\vec{U} = -2.785\mathbf{i} - 5.014\mathbf{j} + 8.542\mathbf{k}$ N-cm
Problem 22
Zoologists estimate the jaw of a lion is subjected to a force $P$ as large as 800 N. What forces $T$ and $M$ must be exerted by the temporalis and masseter muscles, respectively, to support this value of $P$?

Answers $M = 874.7$ N, $T = 763.2$ N
Problem 23
The left figure shows an athlete executing a warm-up exercise while holding a 5 lb weight in each hand. The athlete’s shoulders are parallel with the x axis, and the lower right leg and torso are parallel with the y axis. The idealized stance is shown on the right; tables below give the lengths and angles of the body segments.

A. Compute the x, y coordinates for the location of each weight, $W_R$ and $W_L$.
B. Compute the position vector $\vec{P}$ from the origin ($x = 0$, $y = 0$) to the center of the shoulders.
C. Compute the cross product of the position vector $\vec{P}$ with the force vector $\vec{W}_L$.
D. Sketch the five external forces, with their proper directions, acting on the athlete’s body.
   Assume the athlete’s weight is concentrated at the midpoint of the torso. Just sketch the forces – do not compute their magnitudes.

### Lengths (inches)

<table>
<thead>
<tr>
<th>Body Part</th>
<th>Length</th>
</tr>
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<tbody>
<tr>
<td>torso</td>
<td>A = 24</td>
</tr>
<tr>
<td>neck-shoulder</td>
<td>D = 8</td>
</tr>
<tr>
<td>thigh</td>
<td>B = 18</td>
</tr>
<tr>
<td>upper arm</td>
<td>E = 10</td>
</tr>
<tr>
<td>lower leg</td>
<td>C = 21</td>
</tr>
<tr>
<td>lower arm</td>
<td>F = 13</td>
</tr>
</tbody>
</table>

### Angles (degrees)

<table>
<thead>
<tr>
<th>Angle</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>56</td>
</tr>
<tr>
<td>$\theta$</td>
<td>43</td>
</tr>
<tr>
<td>$\beta$</td>
<td>51</td>
</tr>
<tr>
<td>$\phi$</td>
<td>18</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>37</td>
</tr>
</tbody>
</table>

**Answers**

A. $W_R$ is located at $x = 5.3$, $y = 43.7$ inches; $W_L$ at $x = 40.4$, $y = 70.2$ inches
B. $\vec{P} = 14i + 56.3j$ inches
C. $\vec{P} \times \vec{W}_L = -70k$ lbf-in
Problem 24
To operate properly the traction bar shown in figure (i) requires a torque \( T \) with a certain magnitude about point \( p \). The torque is created by the 20 N force \( F \) applied at point \( a \) as shown in figure (ii).

(i) \[ \begin{array}{c}
\text{4 cm} \\
\text{8 cm} \\
\text{4 cm} \\
\text{7 cm}
\end{array} \]

(ii) \[ \begin{array}{c}
\text{F = 20 N}
\end{array} \]

(iii) \[ \begin{array}{c}
\text{G}
\end{array} \]

(iv) \[ \begin{array}{c}
\text{H}
\end{array} \]

(v) \[ \begin{array}{c}
\text{45°}
\end{array} \]

(vi) \[ \begin{array}{c}
\text{30°}
\end{array} \]

A. Compute the magnitude and direction of the torque \( T \) about \( p \) due to the force \( F \) in figure (ii).

B. If force \( F \) is replaced by force \( G \), which is located at point \( b \) as shown in figure (iii), compute the magnitude of \( G \) required to produce the same torque \( T \). Why is \( G \) different from \( F \)?

C. If the force \( F \) is instead replaced by force \( H \) located at point \( c \) as shown in figure (iv), compute the magnitude of \( H \) required to produce the same torque. How does the magnitude of \( H \) compare with that of \( F \) and \( G \) ?

D. Figure (v): now \( F \) is to be replaced by force \( I \), which is oriented 45° up from the horizontal. Compute the magnitude of \( I \) required to produce the same torque. How does the magnitude of \( I \) compare with that of \( F \) and \( G \)? Why are they different?

E. Figure (vi): compute the torque \( Q \) about point \( p \) due to the force \( J = 56 \) N. Force \( J \) is much larger than the forces above; why isn’t the magnitude of the torque \( Q \) also much larger?

**Answers**
A. \( T = 0.80 \) N-m (ccw); B. \( G = 11.4 \) N; C. \( H = 20 \) N;
D. \( I = 14.14 \) N; E. \( Q = -0.82 \) N-m (cw)
Problem 25
A man has a body mass of $m = 70$ kg and an average density of 1.1 g/cm$^3$.

A. Compute the man’s volume.
B. If the man is modeled as a sphere with the same volume, compute his radius and diameter.
C. If the man is modeled as a right circular cylinder with a height $h = 172$ cm, find the radius and diameter of the cylinder.
D. If the man (with the same height) is modeled as a rectangular solid with a square cross-section, find the length of a side of the square.
E. Repeat part D if the square cross-section is instead a rectangular cross-section, where the long and short cross-section dimensions have a ratio of either 3:1, 5:1, or 7:1.
F. An allometric formula to estimate a person’s body surface area $A$ (in cm$^2$) from their body mass $m$ (kg) and height $h$ (cm) is$^a$:

$$ A = 71.84 m^{0.425} h^{0.725} $$

For each case above (parts B-E), compute the surface area of the geometric model and compare it to the man’s body surface area as predicted by the formula. Which geometric model is best in terms of approximating the body surface area?

G. What is the percent error in body surface area of the best geometric model versus the allometric formula?

**Answers**

A. volume = 63,636 cm$^3$
B. radius = 24.8 cm
C. radius = 10.9 cm
D. length = 19.2 cm
E. For the 3:1 ratio the short-side length is 11.11 cm
F. The empirical formula gives body surface area = 18,251 cm$^2$.
   For the 3:1 ratio model the surface area is 16,020 cm$^2$.
   The 5:1 ratio model is best in terms of surface area.
G. The 5:1 ratio model has 1.31% error.

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$^a$D. DuBois and E.F. DuBois: *A formula to estimate the approximate surface area if height and weight be known.* Archives of Internal Medicine 17:863-871 (June 1, 1916).
Problem 26
Consider a thin circular disk with a radius of $r_o = 1.5$ inches.

A. Compute, using proper unit conversions, the:
   i. area of the disk
   ii. perimeter of the disk
   iii. first moment of inertia given by
   \[ Q = \frac{2}{3} r_o^3 \]
   iv. area moment of inertia using
   \[ I = \frac{\pi}{4} r_o^4 \]
   v. polar moment of inertia from
   \[ J = \frac{\pi}{2} r_o^4 \]

B. An annulus is created from the disk above by drilling a 1.2 inch diameter hole in the center. Compute the:
   i. area of the annulus
   ii. perimeter of the inner edge
   iii. first moment of inertia given by
   \[ Q = \frac{2}{3} (r_o^3 - r_i^3) \]
   iv. area moment of inertia using
   \[ I = \frac{\pi}{4} (r_o^4 - r_i^4) \]
   v. polar moment of inertia from
   \[ J = \frac{\pi}{2} (r_o^4 - r_i^4) \]

Answers A. area = 4,560 mm$^2$
   perimeter = 239 mm
   first moment of inertia = 36.9 cm$^3$
   area moment of inertia = 165 cm$^4$
   polar moment of inertia = 3,309,937 mm$^4$