BME 207 Introduction to Biomechanics Spring 2017

Homework 3

Problems 4.1–4.16 in the textbook.

Problem 17
The suspended cast shown is held in equilibrium by the tensions in the knee cable and ankle cable, \( T_K \) and \( T_A \) respectively. The weight of the lower leg and cast is \( W \), which acts through the center of gravity. The horizontal distance from the center of gravity to the knee cable is \( q = 0.25 \) m; the distance to the ankle cable is \( r = 15 \) cm. In the vertical direction the knee cable is attached \( s = 150 \) mm higher than the ankle cable. The leg and cast together weigh \( W = 150 \) N. The ankle cable makes a \( 50^\circ \) angle with the horizontal.

A. Show that the tension \( T_A \) does not act perpendicular to the upper edge of the shin.
B. Compute the cable tensions \( T_K \) and \( T_A \) and the angle of the knee cable \( \phi \) with respect to the horizontal.

Answers  B. \( T_K = 98.9 \) N, \( T_A = 93.1 \) N, \( \phi = 52.8^\circ \)
Problem 18
The diagram shows the right foot of a 120 lb person standing on “tip-toe.” The tension in the Achilles tendon $T$ makes an angle $\alpha = 57^\circ$ with the horizontal.

A. Neglecting the weight of foot ($W_F = 0$), compute the tension in the Achilles tendon and the magnitude and direction of the reaction force $R$ at the distal end of the tibia.

B. Repeat part A, but include the weight of foot (using the anthropometric data for the Standard Human). Assume the foot’s center of gravity is 2 inches below the end of the tibia.

C. Compare the answers in parts A and B. What percent error is introduced by omitting the foot weight? Is it reasonable to neglect the foot weight for this analysis?

Answers  
A. $T = 118.2$ lb, $R = 171.7$ lb, $\phi = 68.0^\circ$ down from the horizontal  
B. $T = 118.2$ lb, $R = 170.1$ lb, $\phi = 67.8^\circ$ down from the horizontal  
C. percent error in $R$ is 0.95%, percent error in $\phi$ is 0.32%
Problem 19
During a shallow crouch, the patellar ligament, which is attached near the proximal end of the tibia, generates a tension $T$ at angle $\theta$ with the horizontal. Directly above the ligament attachment, the reaction force $R$ acts on the tibial condyles. For a 130 pound female, the lower leg and foot weight is $W = 8$ pounds, and the normal force acting on the foot is $N = 65$ pounds. If $\theta = 35^\circ$ calculate the magnitude of the ligament tension $T$ and the components of the reaction force $R$ using the dimensions shown.

\[ T = 339.3 \text{ lb}, \quad R_X = 277.9 \text{ lb}, \quad R_Y = 251.6 \text{ lb} \]
Problem 20
An athlete holds a weight $B$ such that the lower arm is vertical, the upper arm makes an angle $\theta$ with the horizontal, and the deltoid muscle force $F_D$ is horizontal. The lower arm weight is $W_L$. The upper arm weight $W_U$ acts at $e/2$ along the humerus.

A. Use the equilibrium equations to develop expressions for $F_D$, $R_X$, and $R_Y$ in terms of the forces, angles, and distances shown.

B. Compute $F_D$, the reaction force $R$, and its angle $\phi$ when:

- $B = 10.0 \text{ lb}$
- $d = 4.3 \text{ in}$
- $W_U = 4.7 \text{ lb}$
- $e = 14 \text{ in}$
- $W_L = 3.3 \text{ lb}$
- $g = 12 \text{ in}$
- $\theta = 16^\circ$
- $h = 19 \text{ in}$

Answers $F_D = 177.7 \text{ lb}$, $R = 178.6 \text{ lb}$, $\phi = 5.78^\circ$
Problem 21

The act of pushing a door requires flexion of the shoulder and extension of the elbow. The triceps muscles exert a force $F_T$ on the olecranon process located along the ulna a distance $f$ proximal to the elbow joint at E; for the position shown, this force is parallel to the humerus long axis. The force $F_D$, exerted by the anterior deltoid muscle on the humerus at point D, is oriented at an angle $\theta$ from the humerus long axis. The upper arm and lower arm make angles $\beta$ and $\phi$, respectively, with the horizontal. The force exerted on the wrist by the door $F_W$ acts horizontally, and the weight of the upper and lower arms are $W_U$ and $W_L$, respectively. The shoulder joint reaction force $R$ is oriented at an angle $\alpha$ with the horizontal. The points O, D, C, and E lie on the long axis of the humerus; the points A, B, and E lie on the long axis of the ulna.

Calculate the individual moment vectors about point D in terms of the forces, lengths, and angles shown in the free body diagram.

Answers

for $R$: $\vec{M} = dR (\cos \beta \sin \alpha - \sin \beta \cos \alpha) \hat{k}$
for $F_W$: $\vec{M} = F_W [a \sin \phi - (e - d) \sin \beta] \hat{k}$
for $F_T$: $\vec{M} = -fF_T (\cos \beta \sin \phi + \sin \beta \cos \phi) \hat{k}$
for $W_U$: $\vec{M} = -(c - d) W_U \cos \beta \hat{k}$
for $W_L$: $\vec{M} = W_L [(d - e) \cos \beta - b \cos \phi] \hat{k}$
Problem 22
The astronaut transports space junk supported by the right hand and left knee. The rectus
temoris and vastus lateralis muscles generate forces $F$ and $V$, which are in equilibrium with
with the reaction forces $R$ on the greater trochanter, and $K$, which acts perpendicular to the
femoral long axis just above the condyles.

A. Write the equations of equilibrium relative to the X-Y coordinate system.
B. Write the equations of equilibrium relative to the x-y coordinate system.
C. Compute the magnitude of the reaction forces $K$ and $R$, and the orientation angle $\phi$ of $R$ with
respect to the femoral long axis if:

\[
\begin{align*}
F &= 20 \text{ N} \\
V &= 15 \text{ N} \\
a &= 15 \text{ cm} \\
b &= 13 \text{ cm} \\
l &= 33 \text{ cm} \\
\alpha &= 61^\circ \\
\beta &= 57^\circ \\
\gamma &= 41^\circ
\end{align*}
\]

Answers

A. $\Sigma F_x = F \sin \alpha + V \cos \beta - R \cos \phi = 0$
   $\Sigma F_y = F \cos \alpha + V \sin \beta - K - R \sin \phi = 0$
   $\Sigma M_K = (l - a - b)F \cos \alpha + (l - b)V \sin \beta - lR \sin \phi = 0$
B. $\Sigma F_x = F \cos(\gamma + 90 - \alpha) + V \cos(\gamma + \beta) + K \sin \gamma - R \cos(\gamma + \phi)$
   $\Sigma F_y = F \sin(\gamma + 90 - \alpha) + V \sin(\gamma + \beta) - K \cos \gamma - R \sin(\gamma + \phi) = 0$
   $\Sigma M_K = (l - a - b) \cos \gamma F \sin(\gamma + 90 - \alpha) + (l - b) \cos \gamma V \sin(\gamma + \beta) + l \sin \gamma R \cos(\gamma + \phi) - (l - a - b) \sin \gamma F \cos(\gamma + 90 - \alpha) - (l - b) \sin \gamma V \cos(\gamma + \beta) - l \cos \gamma R \sin(\gamma + \phi) = 0$
C. $K = 13.18 \text{ N}, R = 27.23 \text{ N}, \phi = 19.51^\circ$