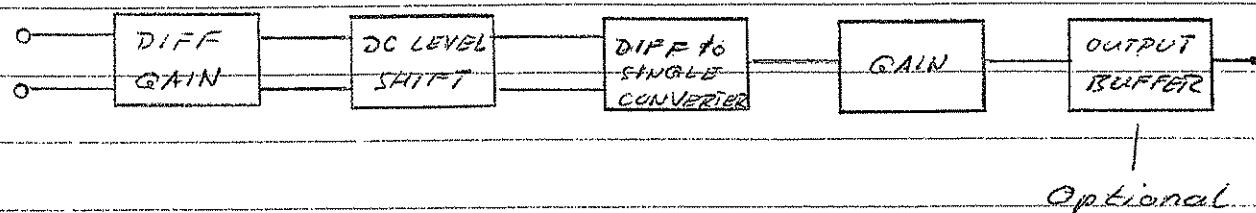


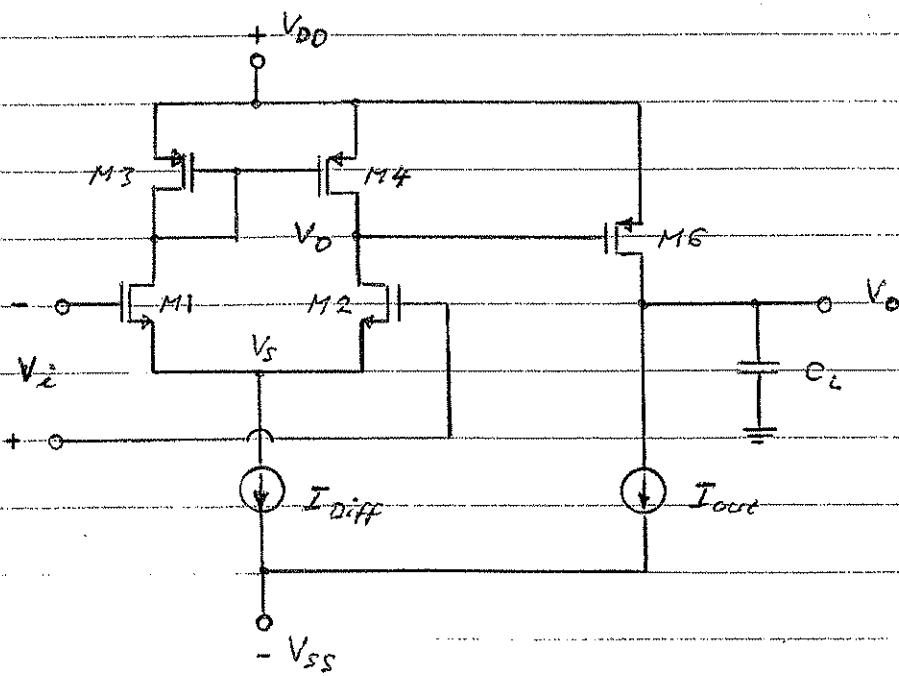
IV Operational Amplifiers and Comparators

1. Overview

opamp internal functions

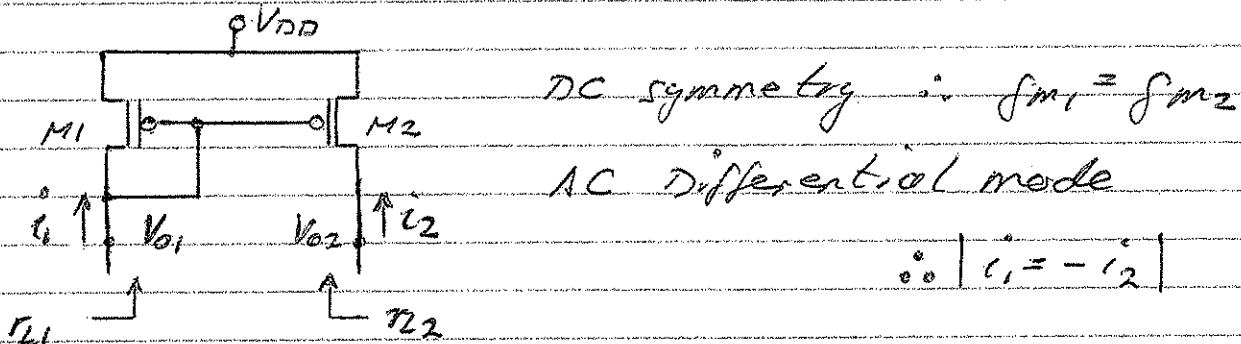


Basic 2-stage CMOS Opamp

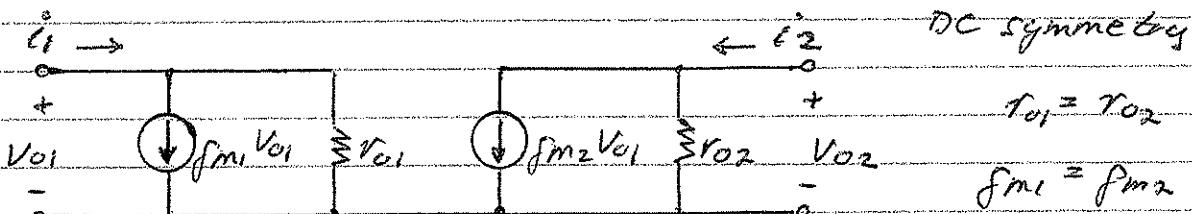


Differential to Single-ended Conversion

Basic Configuration (p-channel load)



Linear equivalent circuit ($V_{DD} \approx \text{Gnd}$)



Equations

$$V_{D1} = (i_1 - f_{m1} V_{D1}) r_{D1} \quad (1)$$

$$V_{D2} = (i_2 - f_{m2} V_{D1}) r_{D2} \quad (2)$$

$$i_1 = -i_2 \quad (3)$$

$$\text{From (1)} \quad \| r_{L1} = \frac{V_{D1}}{i_1} = \frac{V_{D1}}{1 + f_{m1} r_{D1}} \approx \frac{1}{f_{m1}} \|$$

$$(1) + (2) \quad \| V_{D2} = (i_2 - i_1 \frac{f_{m2} r_{D1}}{1 + f_{m1} r_{D1}}) r_{D2} \quad (4)$$

$$(3) + (4) \quad \| \frac{V_{D2}}{i_2} = r_{L2} = \left(1 + \frac{f_{m2} r_{D1}}{1 + f_{m1} r_{D1}} \right) r_{D2} \approx 2 r_{D2} \quad (5)$$

$$\text{Diff. Gain: } \| V_{D1,dm} = -\frac{1}{2} V_{dm} g_{min} r_{L1} \approx -\frac{1}{2} V_{dm} \frac{g_{min}}{f_{m1}} \|$$

$$\| V_{D2,dm} = \frac{1}{2} V_{dm} g_{min} r_{L2} \approx V_{dm} g_{min} r_{D2} \|$$

Note: Under common mode (cm) input conditions, eq. (3) has to be replaced by

$$i_{1,cm} = i_{2,cm} = \frac{1}{2} \frac{V_{cm}}{r_{ss}} \quad (3')$$

where r_{ss} denotes the output resistance of the tail current source I_{SS} .

Equation (5) thus changes to

$$\frac{V_{os}}{i_{2,cm}} = r_{2,acm} = \left(1 - \frac{g_{m2}r_{o1}}{1 + g_{m1}r_{o1}}\right)r_{o2} \approx \frac{1}{g_{m1}} \quad (5')$$

Therefore

$$V_{o1,cm} = \frac{1}{2} V_{cm} \frac{r_{o1}}{r_{ss}} = \frac{1}{2} V_{cm} \frac{1}{g_{m1}r_{ss}}$$

$$V_{o2,cm} = \frac{1}{2} V_{cm} \frac{r_{o2}}{r_{ss}} \approx \frac{1}{2} V_{cm} \frac{1}{g_{m1}r_{ss}}$$

Summary

Diff. Gain	$A_{dm} = g_{min} r_{o2}$
------------	---------------------------

CM Gain	$A_{cm} = \frac{1}{2} \frac{1}{g_{m1}r_{ss}}$
---------	---

CMTZ	$\frac{A_{dm}}{A_{cm}} = 2 g_{min} r_{o2} g_{m1} r_{ss}$
------	--

2 Design for DC

A) DC Biasing ($V_i = 0$)

$$\frac{1}{2} I_{DFF} = \frac{1}{2} \mu C_{ox} \left(\frac{w}{l} \right) [V_{GS1} - V_{T1}]^2$$

$$V_{GS1} = -V_S \quad (V_{L1} = V_{L2} = 0)$$

$$\Rightarrow V_S = -[V_{T1} + \sqrt{\frac{2I_{DFF}}{\mu C_{ox} (w/l)} V_{T1}}]$$

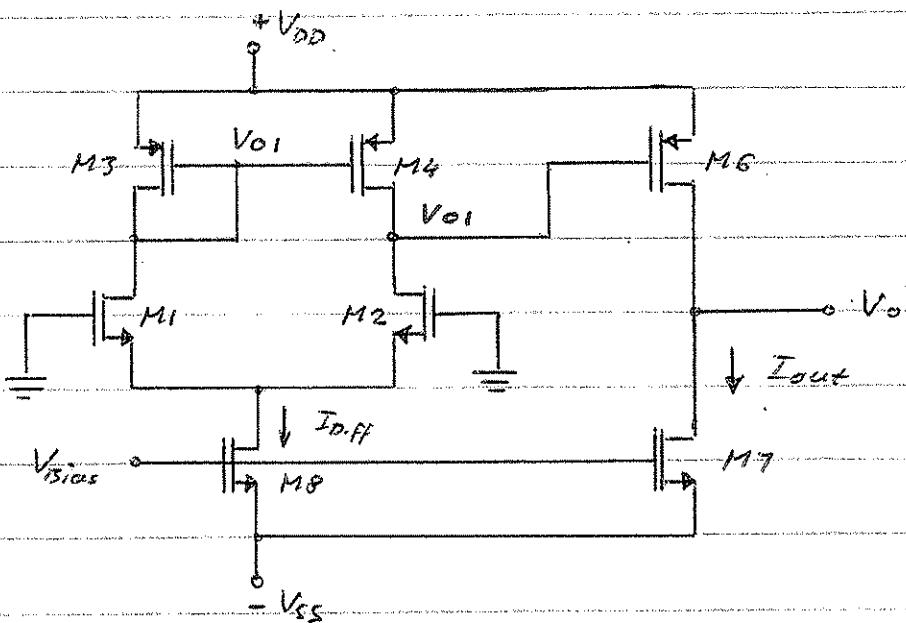
$$\frac{1}{2} I_{DFF} = \frac{1}{2} \mu C_{ox} \left(\frac{w}{l} \right) [-V_{GS3} + V_{T3}]^2$$

$$V_{GS3} = -(V_{DD} - V_0)$$

$$\Rightarrow V_0 = V_{DD} - [-V_{T3} + \sqrt{\frac{I_{DFF}}{\mu C_{ox} (w/l)} V_{T3}}]$$

Notes To achieve a large input common mode range
 keep V_S close to zero and V_0 close to
 V_{DD} → choose a small value for I_{DFF} .
 As we shall see, this also yields a max
 gain. The price for these improvements is
 a reduction in the amplifier speed.

B) Systematic offset Voltage



To obtain zero systematic offset, $V_0|_{V_{G1}=V_{G2}} = 0$. This implies that $I_3 = I_4 = \frac{1}{2} I_{D,off}$

Since M_3 , M_4 and M_6 are all p-type transistors with the same V_{OS} , we can write:

$$\left| \frac{I_{\text{diff}}}{I_{\text{out}}} = \frac{2(w/l)_3}{(w/l)_6} = \frac{2(w/l)_4}{(w/l)_6} \right| \quad (1)$$

Similar arguments applied to H7 and H8 yields

$$\frac{I_{D,eff}}{I_{out}} = \frac{(w/L)g}{(w/L)_7} \quad | \quad (2)$$

thus, to achieve zero systematic offset, we must satisfy the following equation:

$$\frac{2(w/L)_3}{(w/L)_6} = \frac{2(w/L)_4}{(w/L)_6} = \frac{(w/L)_8}{(w/L)_7}$$

c) Random Offset Voltage

If offsets in V_T and (w/L) in the differential input stage only are considered, a straightforward analysis gives:

$$V_{OS} = \alpha V_{T,1-2} + \alpha V_{T,3-4} + \frac{g_m 3-4}{2} + \frac{1}{2} [V_{OS} - V_T]_{1-2} \left[\frac{\Delta(w/L)_{1-2}}{(w/L)_{1-2}} + \frac{\Delta(w/L)_{3-4}}{(w/L)_{3-4}} \right]$$

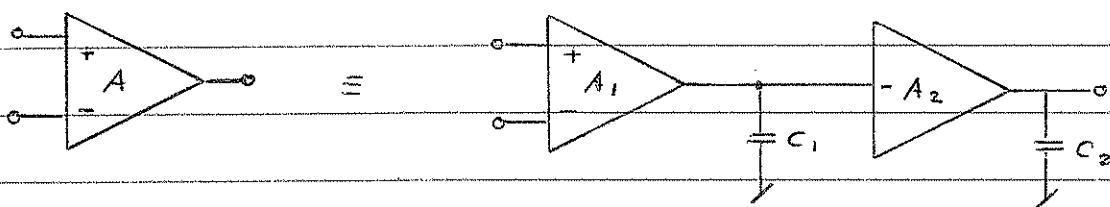
$\sqrt{\frac{e_{noise}}{K_{1-2}}}$

Mismatch of Input V_T Mismatch of Load g_m (w/L) Mismatch of Inputs + Loads

- Operate input stage at low bias to minimize offset
- Realize small ratio of $g_m 3 / g_m 1$

5. Design for AC

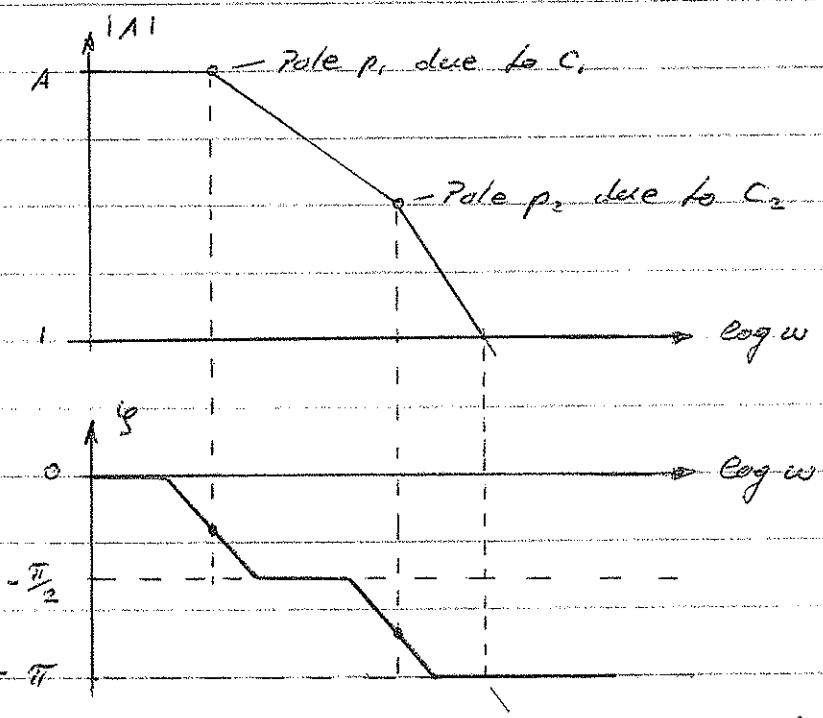
A) Compensation



$$A = A_1 \cdot A_2$$

C_1, C_2 : Parasitic Capacitances

Gain Phase Plot

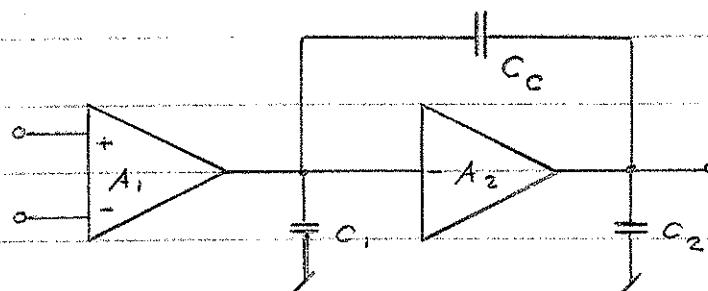


zero phase margin

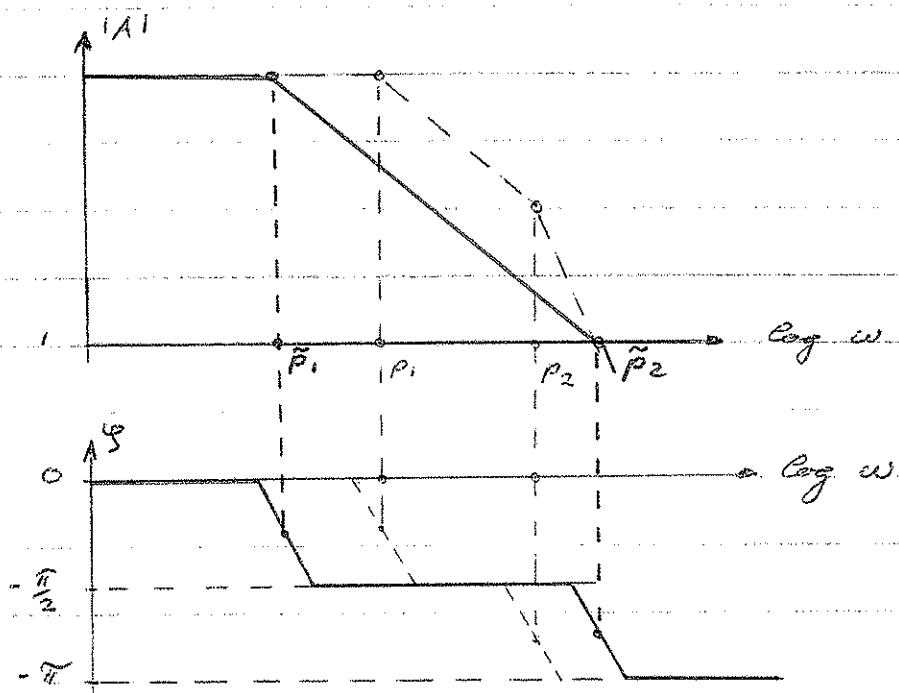
Opamp in Feedback configuration:

- 180° phase shift due to neg. feedback
- 180° phase shift due to parasitic poles
- ⇒ total phase shift: 360° i.e. neg. feedback becomes positive
 - ⇒ instabilities
- ⇒ Additional phase compensation required

Pole splitting Compensation

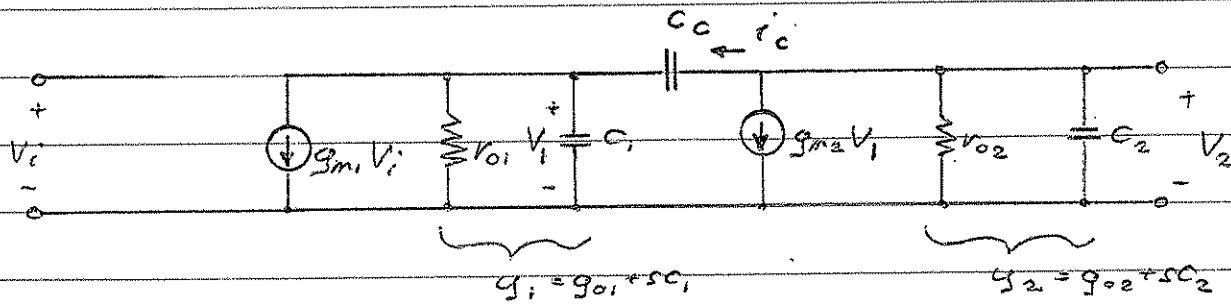
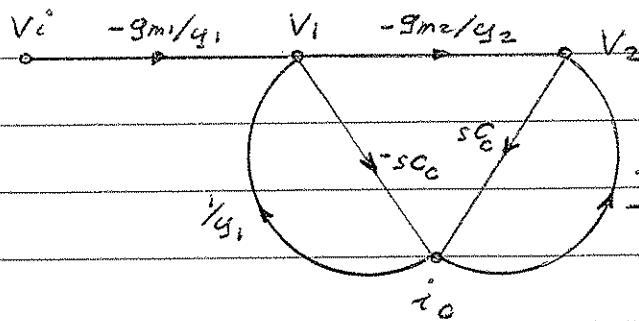


C_c pushes pole 1 lower in frequency due to Miller multiplication
pushes pole 2 higher in frequency



$$\text{at } |A| = 1 \quad \varphi = -\frac{3}{4}\pi$$

$$\Rightarrow \text{Phase margin} = +\frac{1}{4}\pi \hat{=} 45^\circ$$

Quantitative DescriptionSmall Signal model of OpampSFG

This feedback is eliminated with a source follower in the C_C branch and sC_C has to be replaced by $\frac{sC_C}{1+sC_C/g_{m2}}$.

$$\Rightarrow \frac{V_2}{V_i} = \frac{g_{m1}(g_{m2} - sC_C) + (1 + \frac{sC_C}{g_{m2}})}{g_1 g_2 + sC_C(g_{m2} + g_1 + g_2)}$$

$$\frac{V_2}{V_i} = \frac{g_{m1} g_{m2}}{(1 + \frac{sC_C}{g_{m2}})(g_{o1} g_{o2} (1 + sL \frac{C_1 + C_2 + C_C + C_0 + C_0 + C_C g_{m2}}{g_{o1} g_{o2}} + \frac{C_C g_{m2}}{g_{o1} g_{o2}}) + s^2 (\frac{C_1 C_2}{g_{o1} g_{o2}} + C_C \frac{C_1 + C_2}{g_{o1} g_{o2}}))}$$

$$\boxed{\frac{V_2}{V_i} = \frac{g_{m1} g_{m2}}{g_{o1} g_{o2} (1 + sL \frac{C_1 + C_2 + C_C + C_0 + C_0 + C_C g_{m2}}{g_{o1} g_{o2}} + \frac{C_C g_{m2}}{g_{o1} g_{o2}}) + s^2 (\frac{C_1 C_2}{g_{o1} g_{o2}} + C_C \frac{C_1 + C_2}{g_{o1} g_{o2}})}}$$

Source Follower

Note: without C_C ($C_C = 0$)

$$\text{Zero: } \tilde{z}_1 = \frac{g_{m2}}{C_C} - \frac{g_{m2}}{C_C}$$

$$z_1 \rightarrow \infty$$

$$\text{Poles: } \tilde{p}_1 \approx -\frac{g_{o1} g_{o2}}{g_{m2} C_C}$$

$$p_1 \approx -\frac{g_{o1}}{C_1} = \tilde{p}_1 \frac{C_C}{C_1} \frac{g_{m2}}{g_{o2}}$$

$$\tilde{p}_2 \approx -\frac{g_{m2} C_C}{C_1 C_2 + C_C (C_1 + C_2)}$$

$$p_2 = -\frac{g_{o2}}{C_2} = \tilde{p}_2 (1 + \frac{C_1}{C_2} + \frac{C_1}{C_C}) \frac{g_{o2}}{g_{m2}}$$

$$\tilde{p}_3 \approx -\frac{g_{m2}}{C_1} \quad \tilde{p}_3 \approx -\frac{1}{C_1 R_E}$$

$$\tilde{p}_1 = p_1 \frac{C_1}{C_C} \frac{g_{o2}}{g_{m2}}$$

$$\tilde{p}_2 = p_2 \frac{1}{(1 + \frac{C_1}{C_2} + \frac{C_1}{C_C})} \frac{g_{m2}}{g_{o2}}$$

Right-half-plane zero: Adds additional neg. phase to transfer function
 \Rightarrow decreases phase margin

Solution: zero must be shifted over to left half-plane
 \Rightarrow phase margin is increased

A) with a source-follower in the feedback path

zero:

$$\tilde{z}_1' \rightarrow 0$$

eliminates RHP zero

costs:
 - Area

- Power

B) with a resistor R_z in the feedback path

zero:

$$\tilde{z}_1' = -\frac{g_m}{C_o(g_m R_z - 1)}$$

g_m : transcond. of output stage

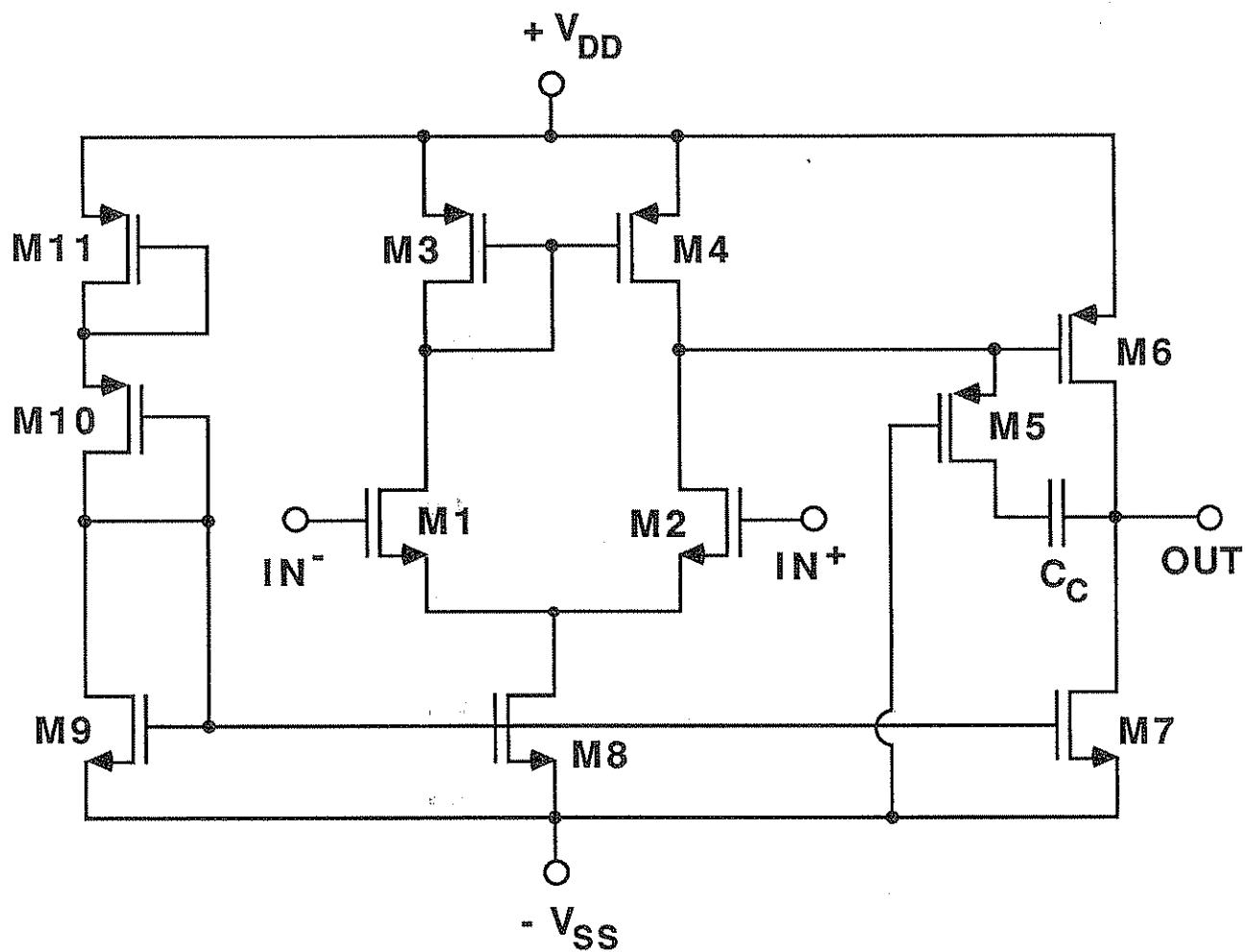
if $g_m R_z = 1$ zero cancelled

if $g_m R_z > 1$ LHP zero

- consumes no additional power

- requires little additional area

2-Stage CMOS Amplifier



Two-stage Differential Amplifier

By replacing all transistors by their linear equivalent circuits and using first-order analysis techniques, the amplifier differential gain A_d and the common mode rejection ratio CMRR turn out to be:

$$A_d = \frac{g_{m1}r'_{03}g_{m6}r'_{06}(1 + sC_c[r_{05} - \frac{1}{g_{m6}}])}{1 + s[C_c r'_{03} g_{m6} r'_{06} + C_L r'_{06}] + s^2 C_c C_L r'_{03} r'_{06}}$$

$$CMRR = 2g_1r_{08}g_{m3}r'_{03}$$

$$\text{where } g_1 = g_{m1} + g_{mb1} \quad r'_{03} = \frac{r_{01}r_{03}}{r_{01}+r_{03}} \quad r'_{06} = \frac{r_{06}r_{07}}{r_{06}+r_{07}}$$

The two parameters g_m and g_{bm} denote the transistor gate-source transconductance and body-source (or backgate) transconductance, respectively.

The two poles and the zero that are present in the gain expression are approximately located at:

$$\omega_z = -\frac{g_{m6}}{C_c[g_{m6}r_{05} - 1]}$$

$$\omega_{p1} = -\frac{1}{C_c r'_{03} g_{m6} r'_{06}}$$

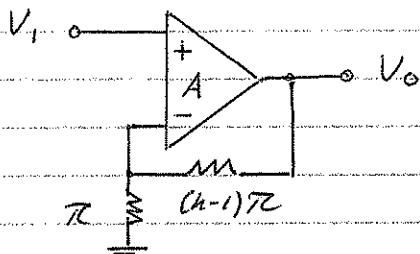
$$\omega_{p2} = -\frac{g_{m6}}{C_L}$$

Note that in order to guarantee a left half-plane zero, the product $g_{m6}r_{05}$ has to be greater than 1.

Most often, the nondominant second pole and the zero of the amplifier are kept equal to or greater than the unity-gain frequency GB. The frequency response can then be modeled by a first-order lowpass circuit. The unity-gain frequency is then approximately equal to:

$$GB = |A_d \omega_{p1}| = \frac{g_{m1}}{C_c}$$

Amplifier in Feedback Configuration



$$V_o = \left(V_i - \frac{V_o}{k} \right) - A \quad k \geq 1$$

$$\left| \frac{V_o}{V_i} = \frac{A}{1 + \frac{A}{k}} \right|$$

$$\left| A(s) = \frac{A_0}{(1 + \frac{s}{p_1})(1 + \frac{s}{p_2})(1 + \frac{s}{p_3})} \right|$$

p_1, p_2 & p_3 are neg.
real poles

- Is this circuit stable for all possible values of k ?

- Find poles of closed loop system as a function of feedback parameter k

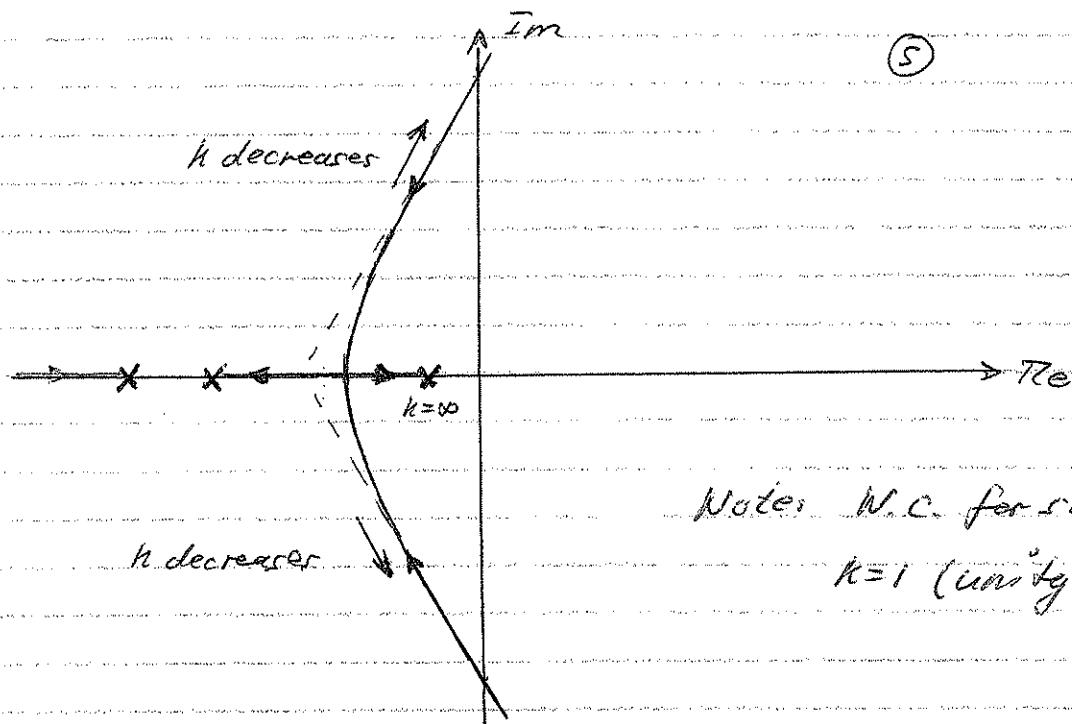
$$\left| \frac{V_o(s)}{V_i(s)} = \frac{A_0}{(1 + \frac{s}{p_1})(1 + \frac{s}{p_2})(1 + \frac{s}{p_3}) + \frac{A_0}{k}} \right|$$

- Find Root Locus plot for the 3 poles as a function of k
characteristic equation of closed-loop system

$$\left| D(s) = 1 + \frac{k}{A_0} (1 + \frac{s}{p_1})(1 + \frac{s}{p_2})(1 + \frac{s}{p_3}) \right| \quad 1 \leq k \leq A_0$$

$$\left| D(s) = 1 + \mu \cdot t(s) \right| \quad \mu = \frac{k}{A_0} \quad \frac{1}{A_0} \leq \mu \leq 1$$

Root Locus Plot for 3 neg. real poles in open-loop



Note: N.c. for stability
 $k=1$ (margin-gain)

Conclusion.

Depending on the initial position of the open-loop poles, the closed-loop system can become unstable as k approaches 1.

Notes:

If initial poles are closely spaced, the system is very likely to become unstable in a closed-loop configuration

Solution: Split poles apart

$$P_2 \approx 1.0 P_1 \quad P_3 > P_2$$

Opamp Stability Analysis

Pole/zero locations as a function of Phase Compensation circuitry

gm_s, source follower
transcond-

pole/zero	uncomp.	$C_c + R_o$	Source Follower
ω_z	∞	$-\frac{g_{m2}}{C_c f_{m2} R_o}$	$\frac{g_{m2}}{C_c}$
ω_{p1}	$-\frac{1}{R_o C_1}$	$-\frac{1}{\tau_{o1} C_c f_{m2} \tau_{z2}}$	$\frac{1}{\tau_{o1} C_c f_{m2} \tau_{z2}}$
ω_{p2}	$-\frac{1}{\tau_{o2} C_2}$	$-\frac{f_{m2}}{C_2 [1 + \frac{C_1}{C_o} + \frac{C_1}{C_2}]}$	$-\frac{g_{m2}}{C_2 [1 + \frac{C_1}{C_o} + \frac{C_1}{C_2}]}$

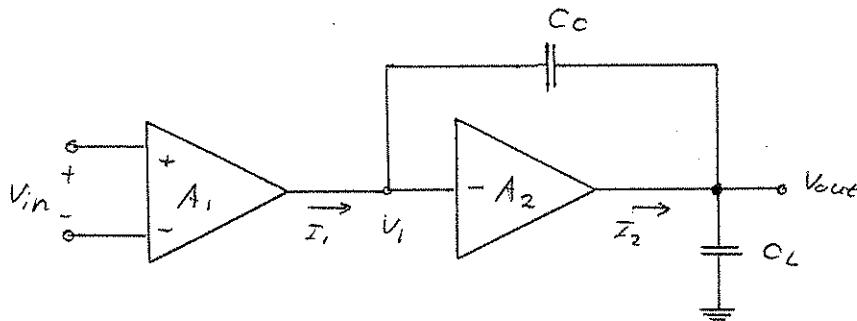
Numerical Example

$$f_{m2} = 5.00 \mu s \quad f_{m2} = 100 \mu s \quad R_o = 12 k\Omega \quad \tau_{o1} = 500 \mu s$$

$$\tau_{o2} = 200 \mu s \quad C_1 = 200 fF \quad C_2 = 5 pF \quad C_c = 2 pF$$

pole/zero	uncomp.	$C_c + R_o$	Source Follower
ω_z	∞	-50×10^6	-50×10^6
ω_{p1}	-10×10^6	-10×10^3	-10×10^3
ω_{p2}	-1.0×10^6	-87.7×10^6	-87.7×10^6
ω_{p3}	∞	-417×10^6	-500×10^6

4. Slew Rate



Note:

$$\text{since } |A_2| \gg 1 \\ \therefore V_1 \approx 0$$

Slew Rate limit:

$$SR = \left. \frac{dV_{out}}{dt} \right|_{\max} = \min \left\{ \frac{I_{1\max}}{C_C}; \frac{I_{2\max}}{(C_C + C_L)} \right\}$$

Notes: $I_{1\max} = I_{niff}$ $I_{2\max} = I_{out}$ (for class A output stage)

$$\text{If } SR_1 = SR_2 \quad \therefore \quad \frac{I_{niff}}{C_C} = \frac{I_{out}}{C_C + C_L} \quad \text{or } I_{out} = I_{niff} \left(1 + \frac{C_L}{C_C} \right)$$

If second pole of amplifier is at the unity-gain frequency,
then $|Q_B| \approx \frac{g_m}{C_C}$

$$\text{Furthermore if } SR = SR_c = \frac{I_{niff}}{C_C}$$

$$\text{then } SR = Q_B \frac{I_{niff}}{f_m}$$

$$\text{since } g_m = \sqrt{\mu C_o (\frac{w}{L})} = \frac{1}{2} I_{niff} \cdot 2$$

$$\text{we obtain } \left\| SR \approx Q_B \sqrt{\frac{I_{niff}}{\mu C_o (\frac{w}{L})}} \right\|$$

Performance Criteria and Design Guidelines

1. Gain

- low bias current
- long devices \rightarrow high v_o
- wide devices \rightarrow high f_m
- cascode configuration

2. Bandwidth ($BW \propto f_m$)

- high bias current
- short channels

3. Settling Rate

- high bias current

4. Offset Voltage

- large devices
- low bias current \rightarrow small V_{eff}

5. Noise

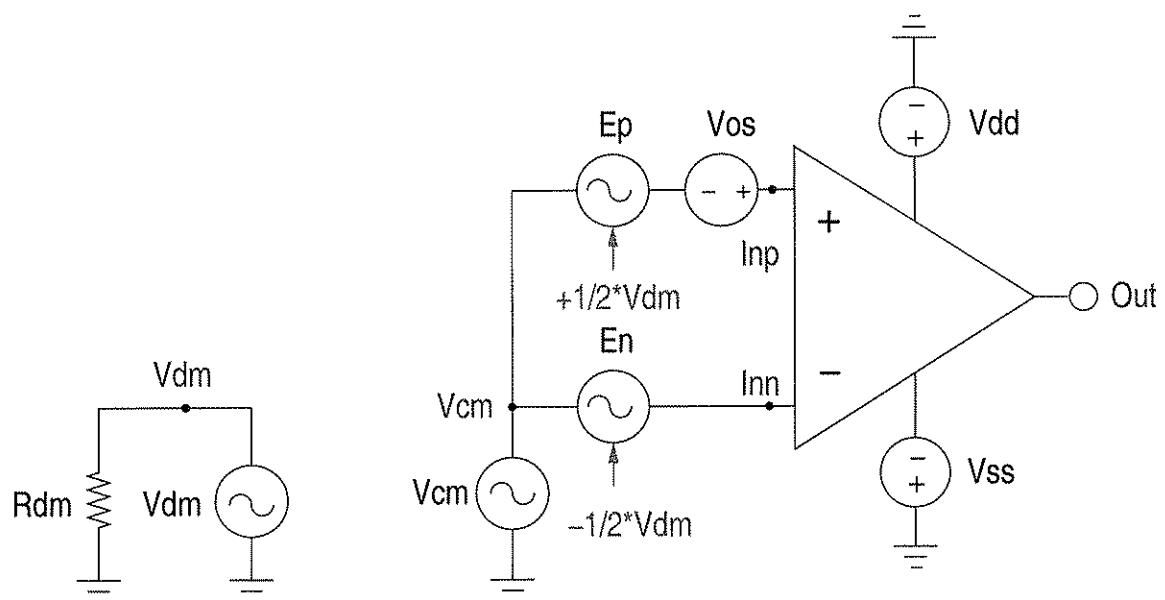
- large devices \rightarrow less "pp noise"
- high bias \rightarrow high f_m

Good Design = Good Compromise among
Conflicting criteria

IV - 18

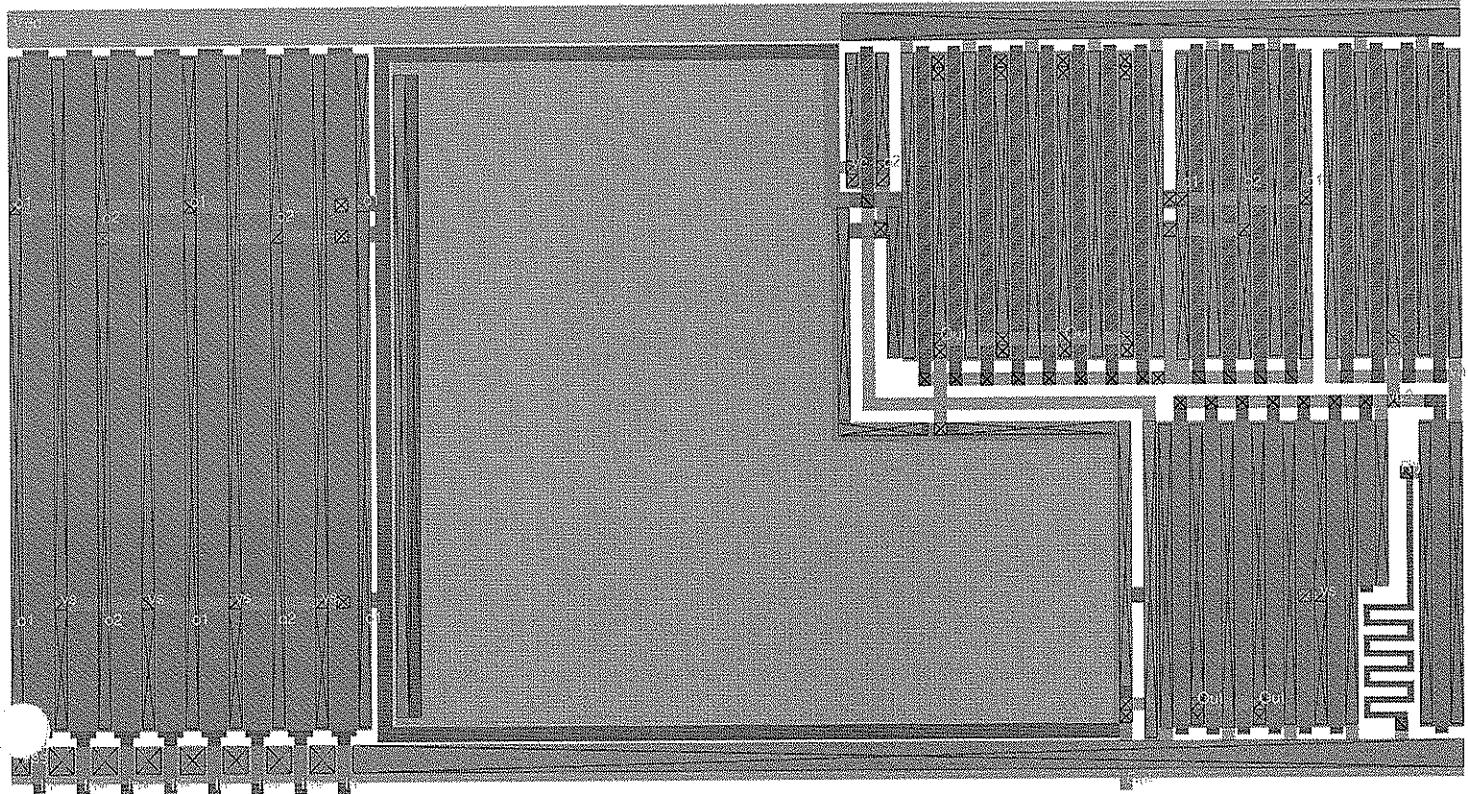
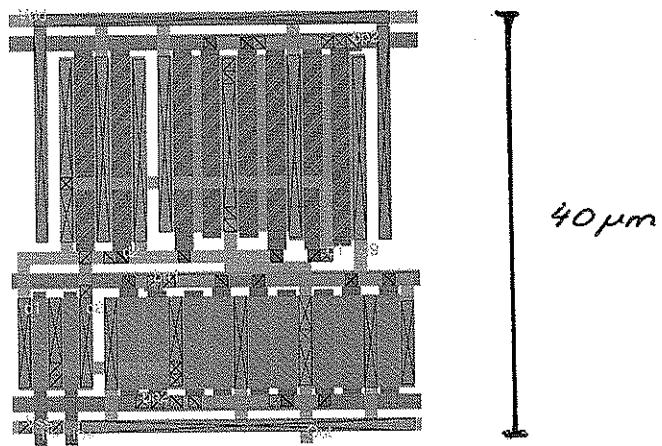
OpAmp Performance Evaluation

Test Circuit Set-up for Spice



IV-19

0.5 μ m CMOS OPAMPS

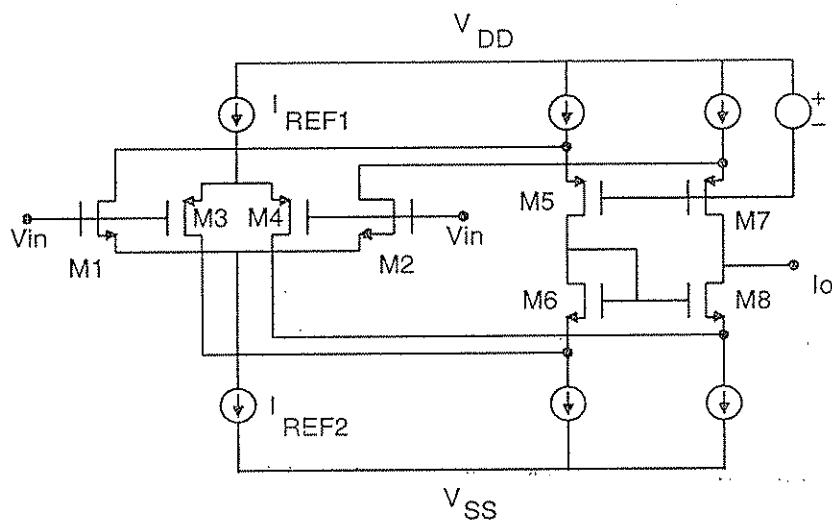


Low-Voltage Amplifiers

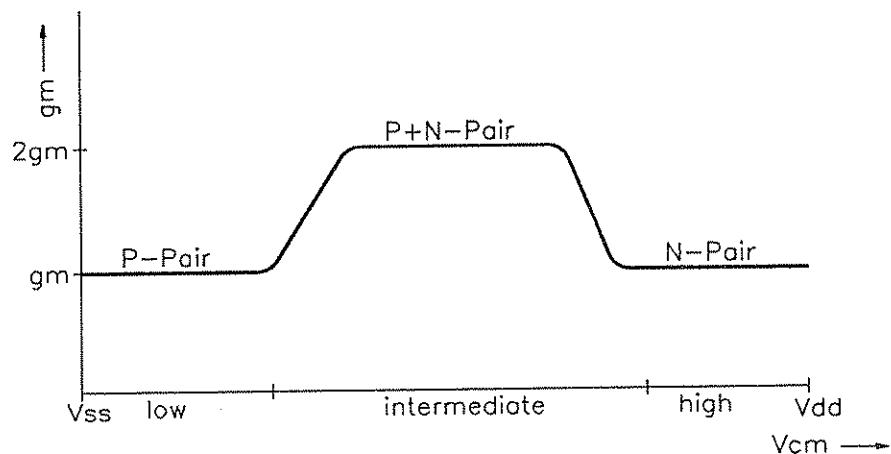
In order to keep the signal-to-offset, and signal-to-noise ratio as large as possible, circuits must be designed which have rail-to-rail input and output voltage swing.

A) Input stage with Rail-to-Rail swing

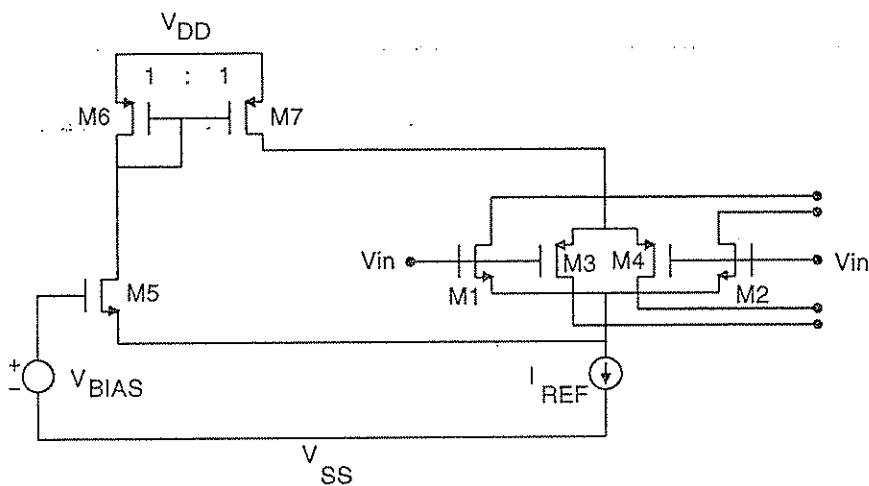
Basic configuration



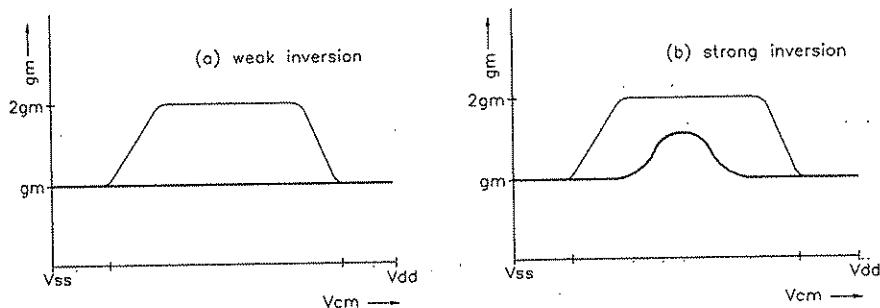
Problem: Transconductance of input stage varies as V_{cm} is changed



Solution: keep total diff. input current constant



Resulting input transconductance

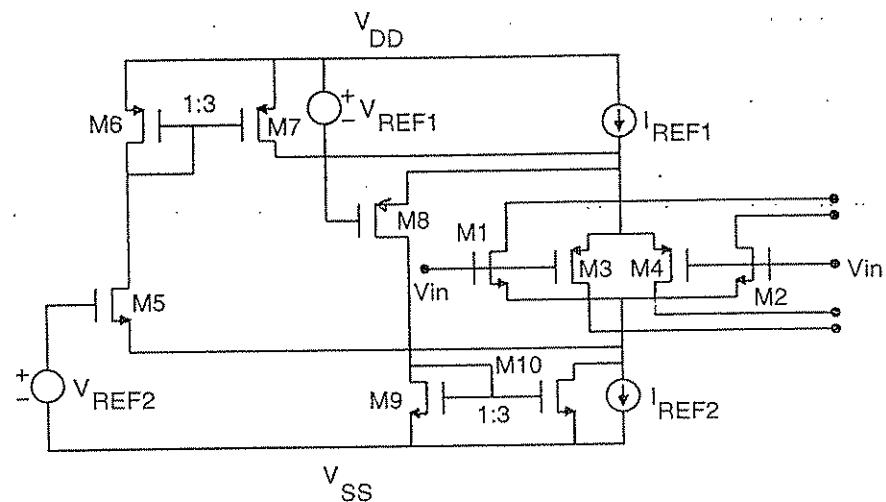


Comment: This solution works fine if the input stages are operated in weak inversion where the transconductance is proportional to the bias current. For the more typical case, however, where the input stages are operated in strong inversion (i.e. saturation), a 40% deviation results since

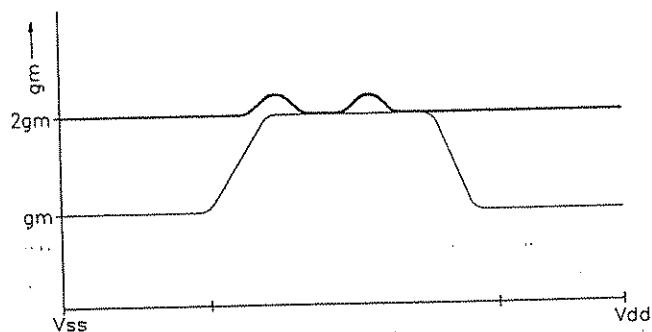
$$g_{m_{sat}} \propto [\sqrt{I_o \cdot x} + \sqrt{I_o(1-x)}]$$

$$\text{where } \text{Max} [\sqrt{I_o x} + \sqrt{I_o(1-x)}] = 1.41 \cdot \sqrt{I_o}$$

Improved Selection

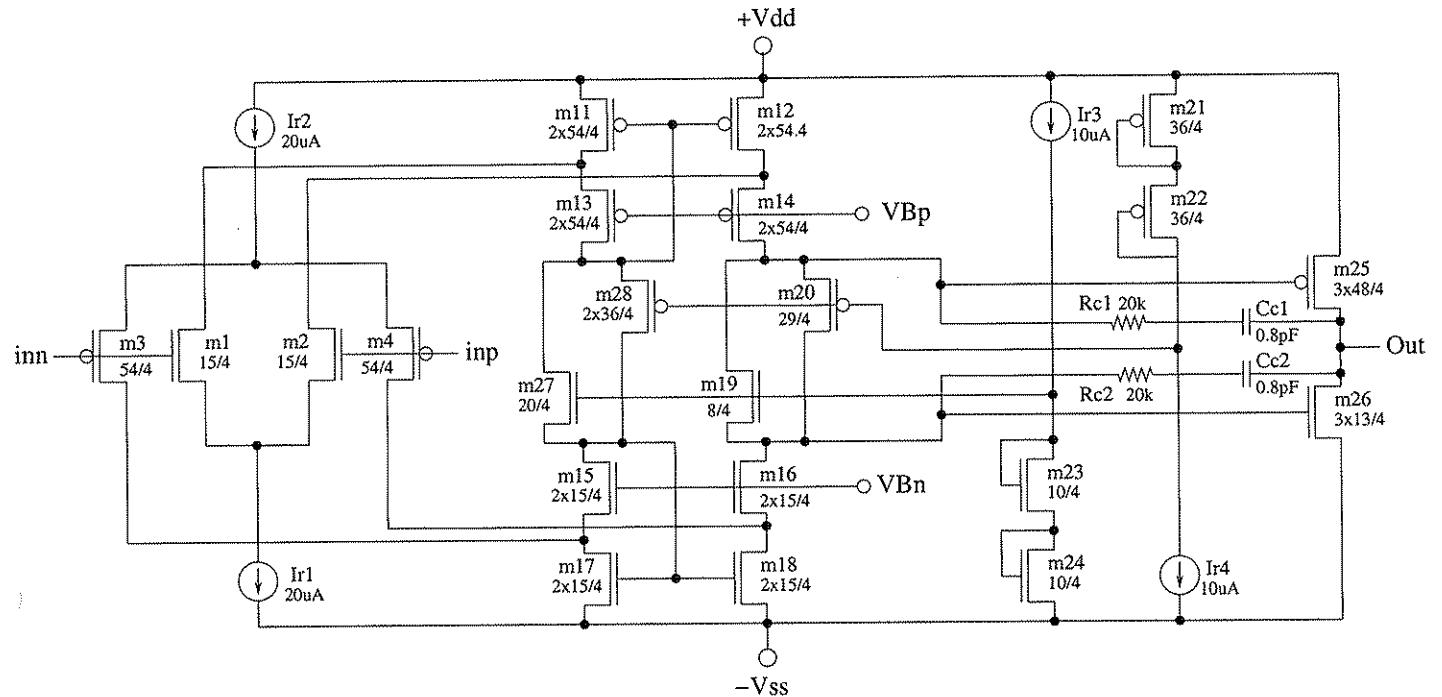


Resulting Transconductance (for saturation)

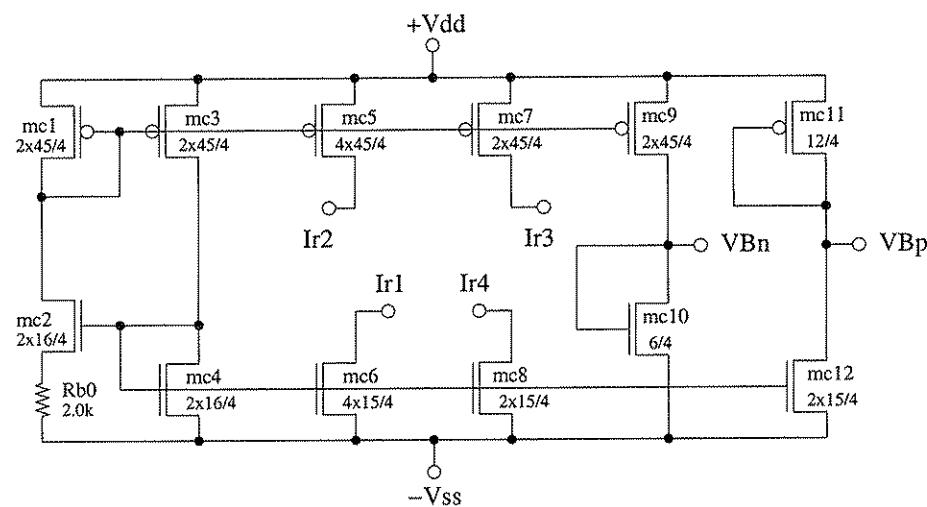


CMOS Amplifier with Rail-to-Rail Common-Mode Input Range

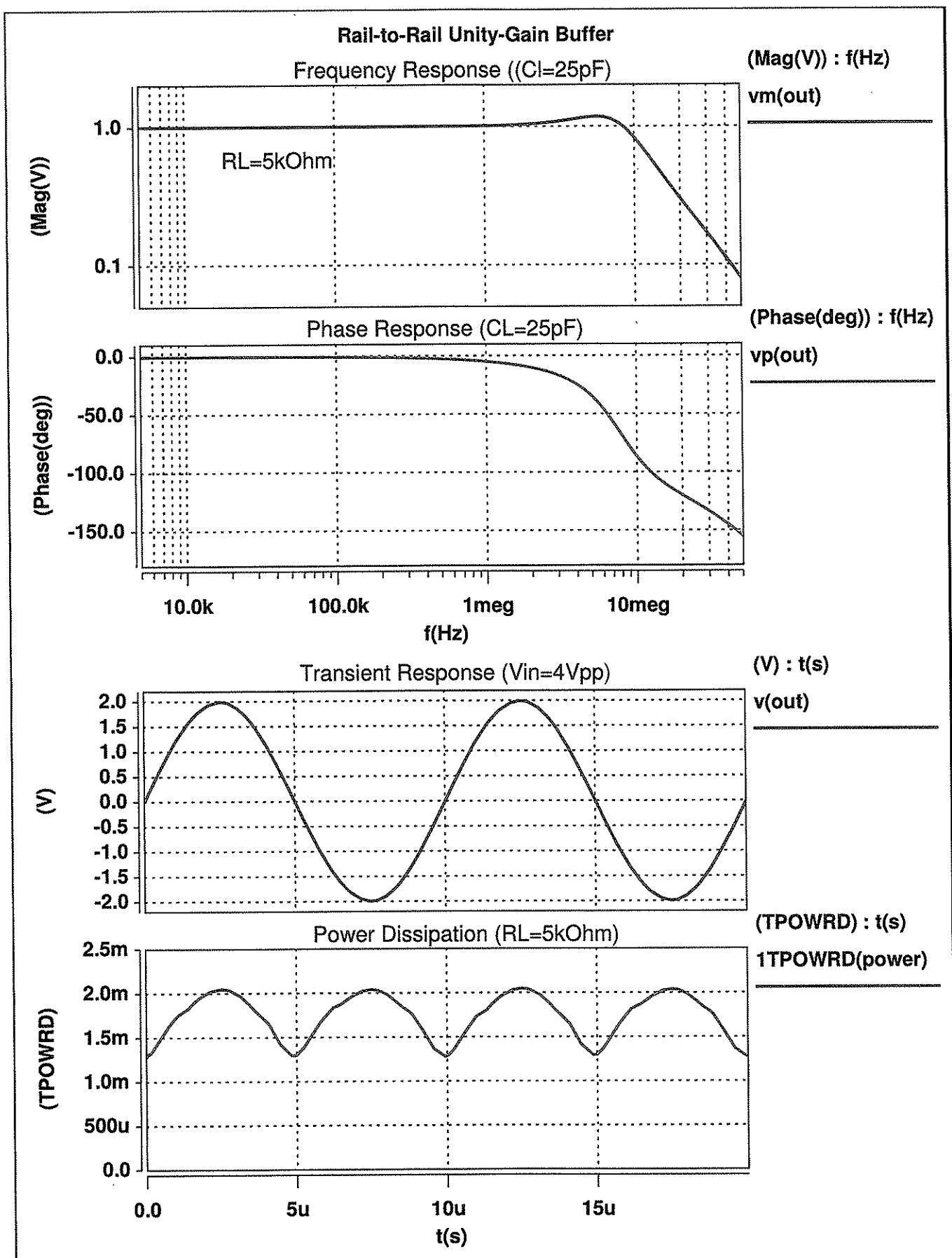
Core Amplifier Circuit



Bias Circuitry

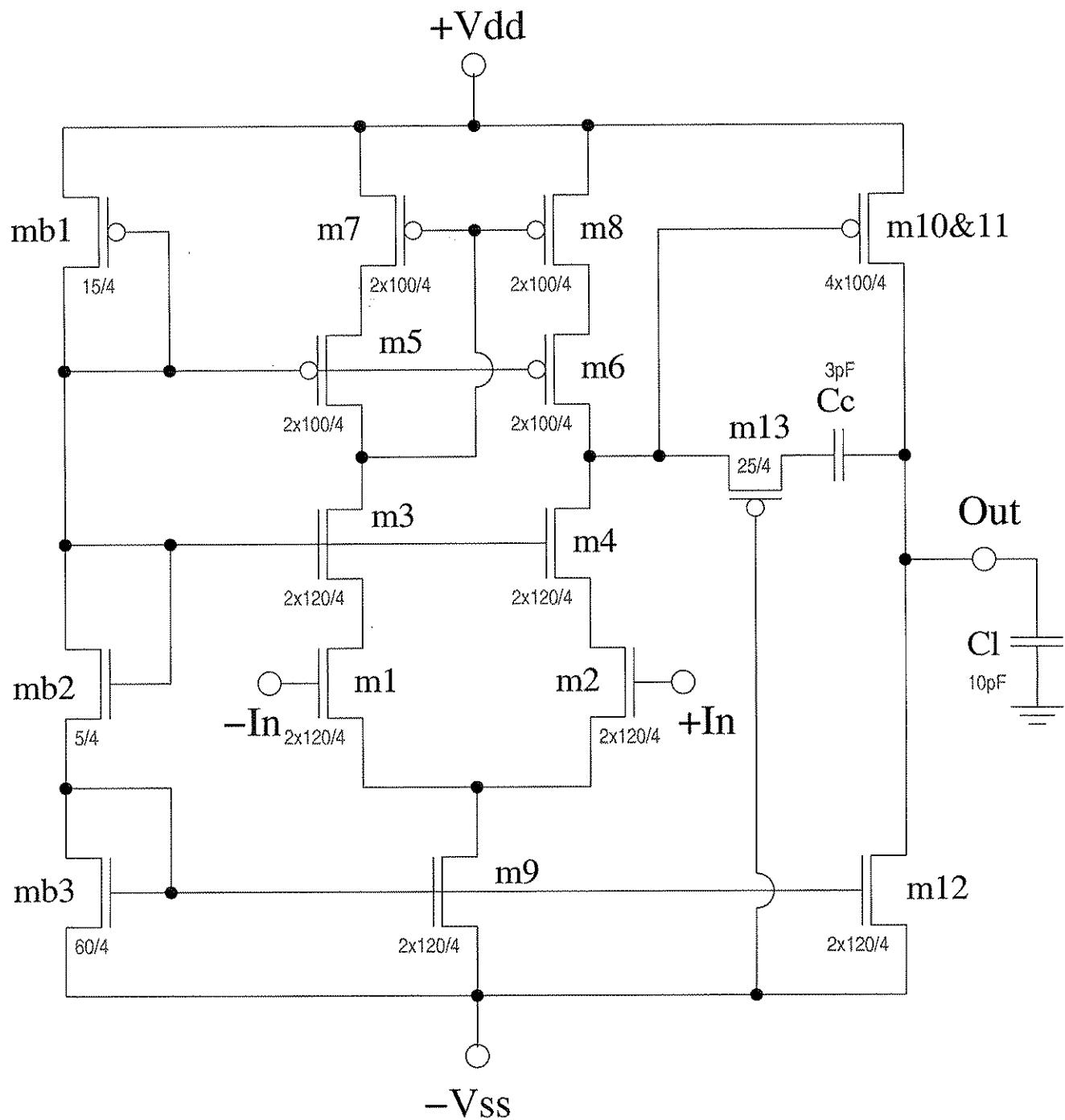


$$\sqrt{V} - 2 \beta \alpha$$



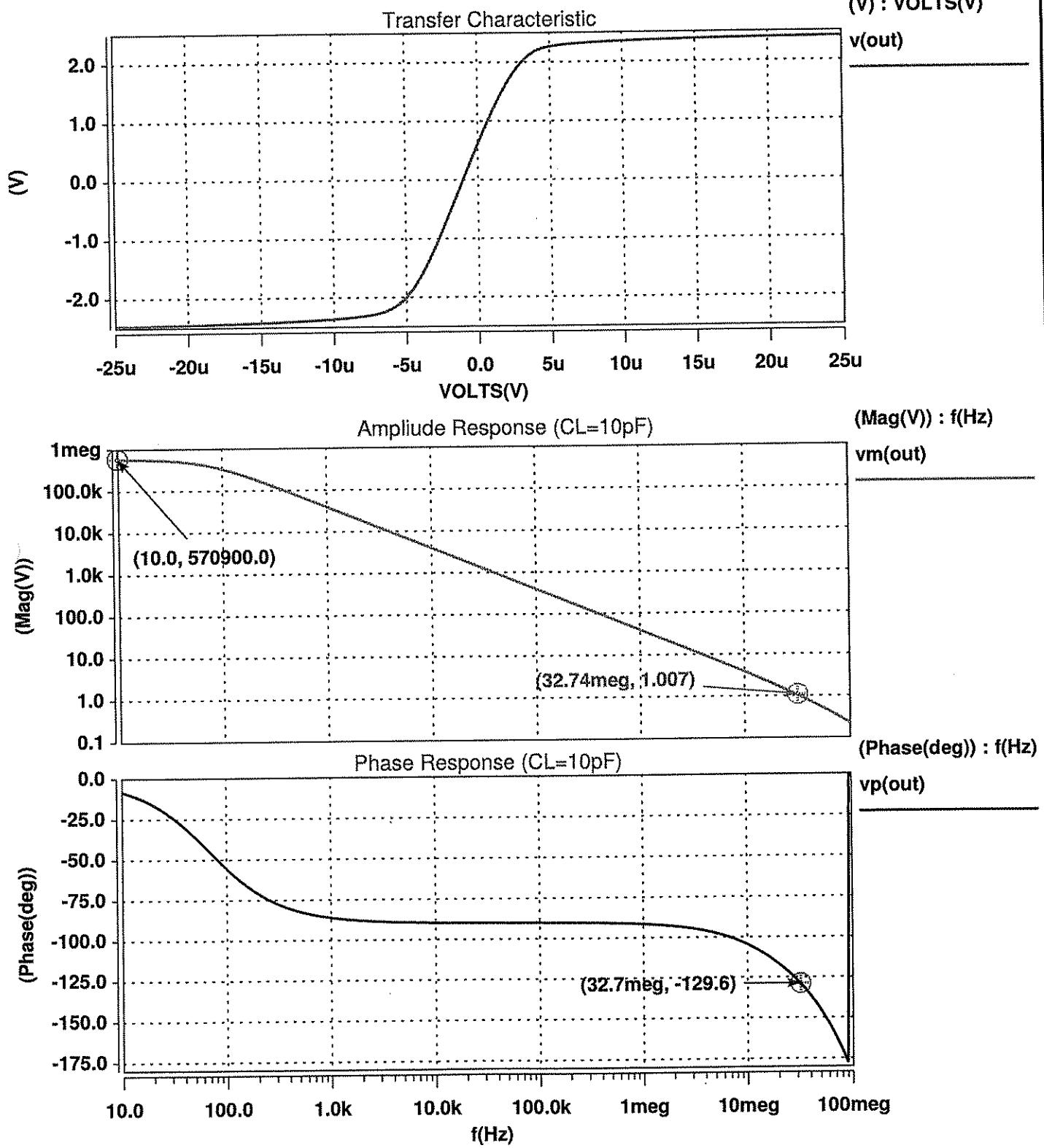
High Gain CMOS Opamp with Cascode Input Stage

Version 1 with n-channel Input Pair



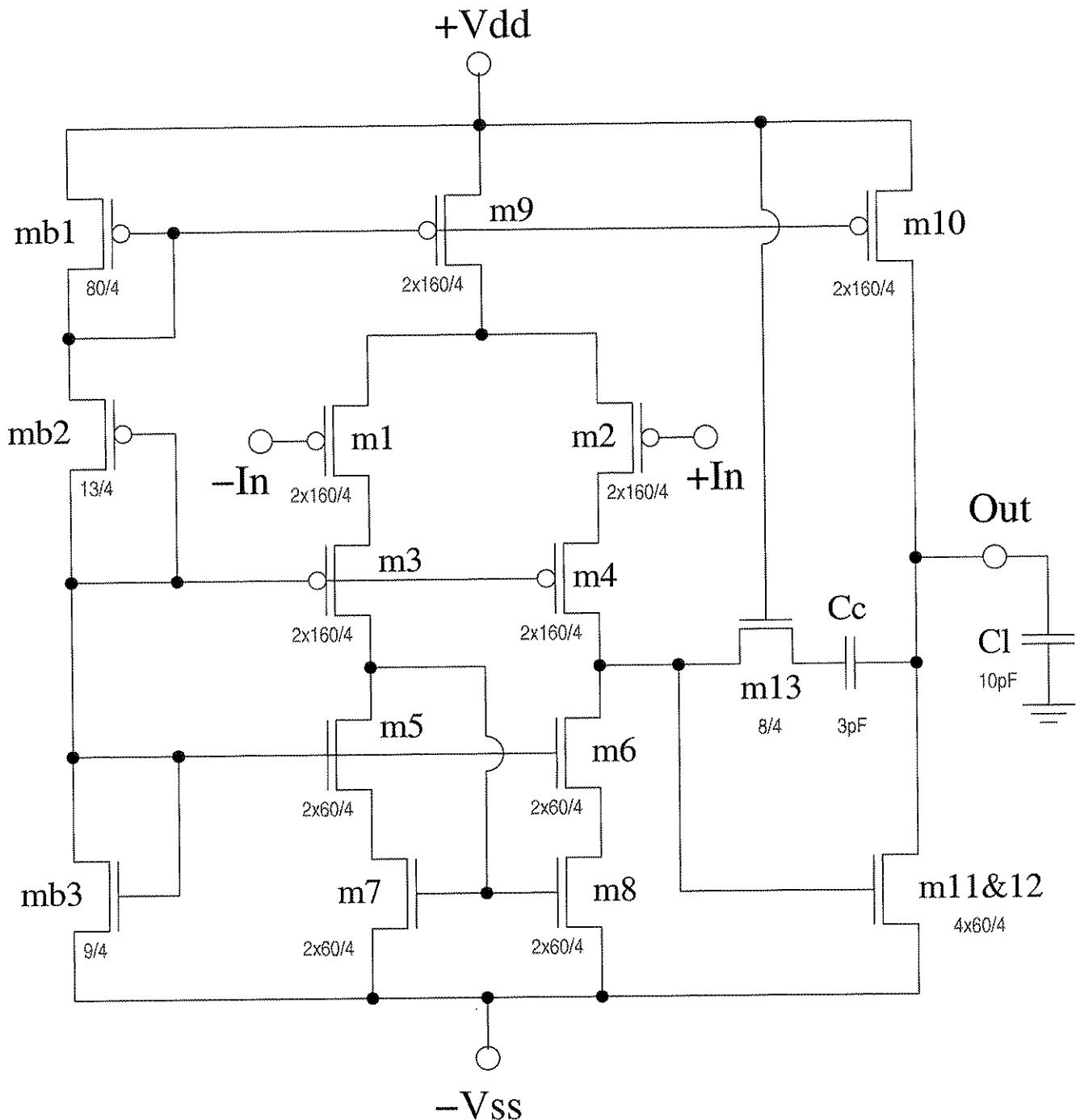
CMOS Amplifier with Cascode Differential Stage

Transfer Characteristic

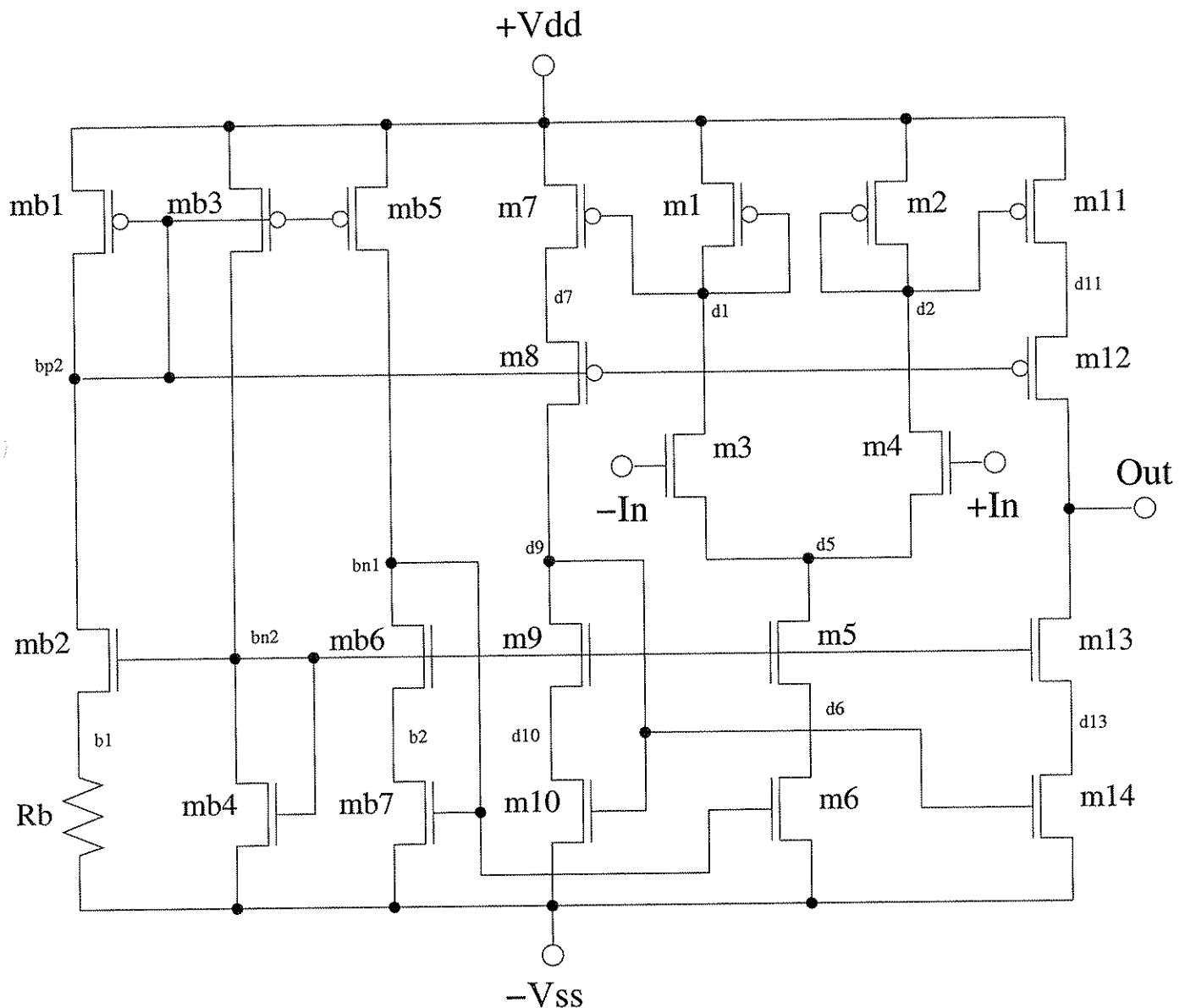


High Gain CMOS Opamp with Cascode Input Stage

Version2 with p-channel Input Pair

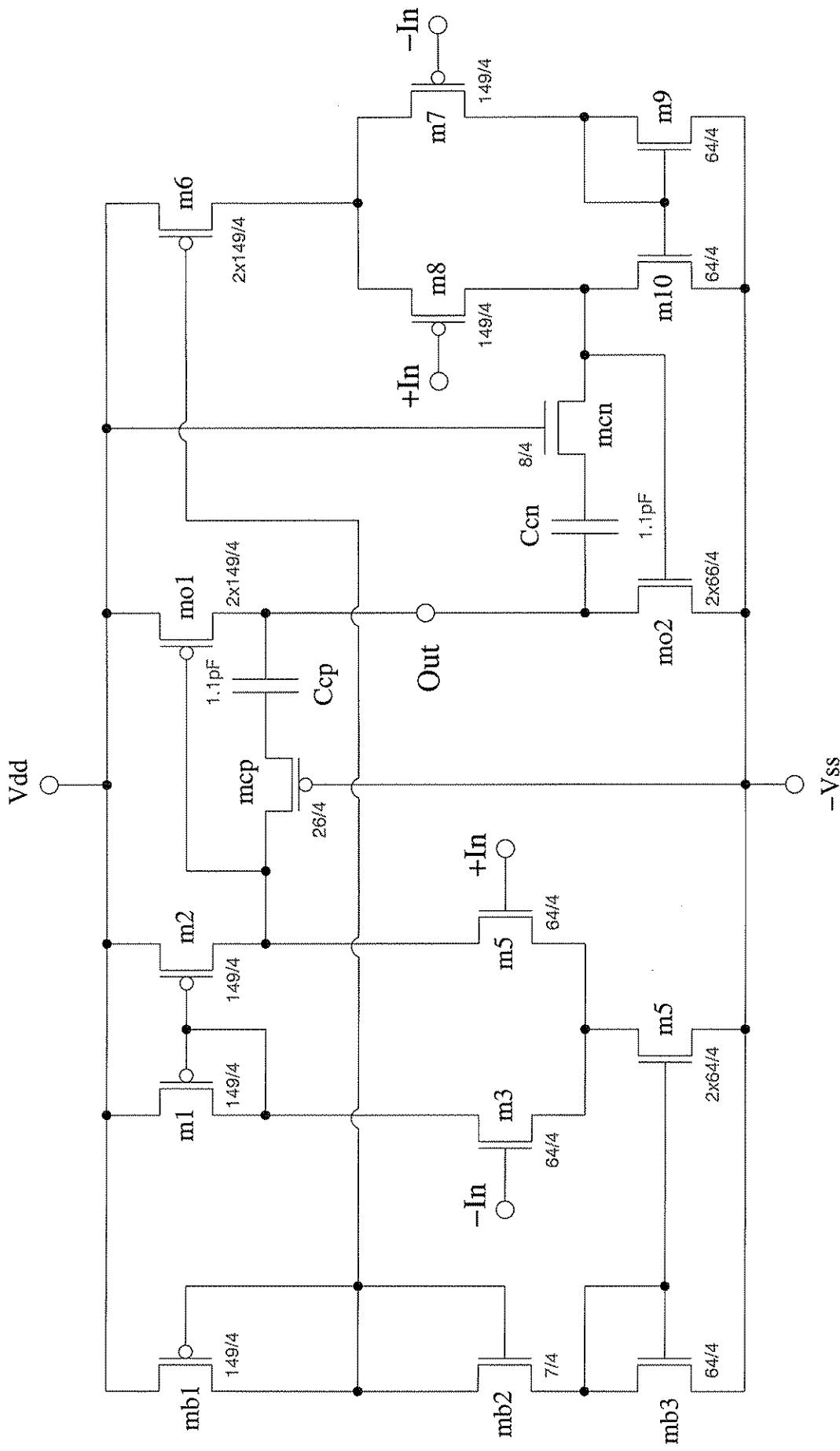


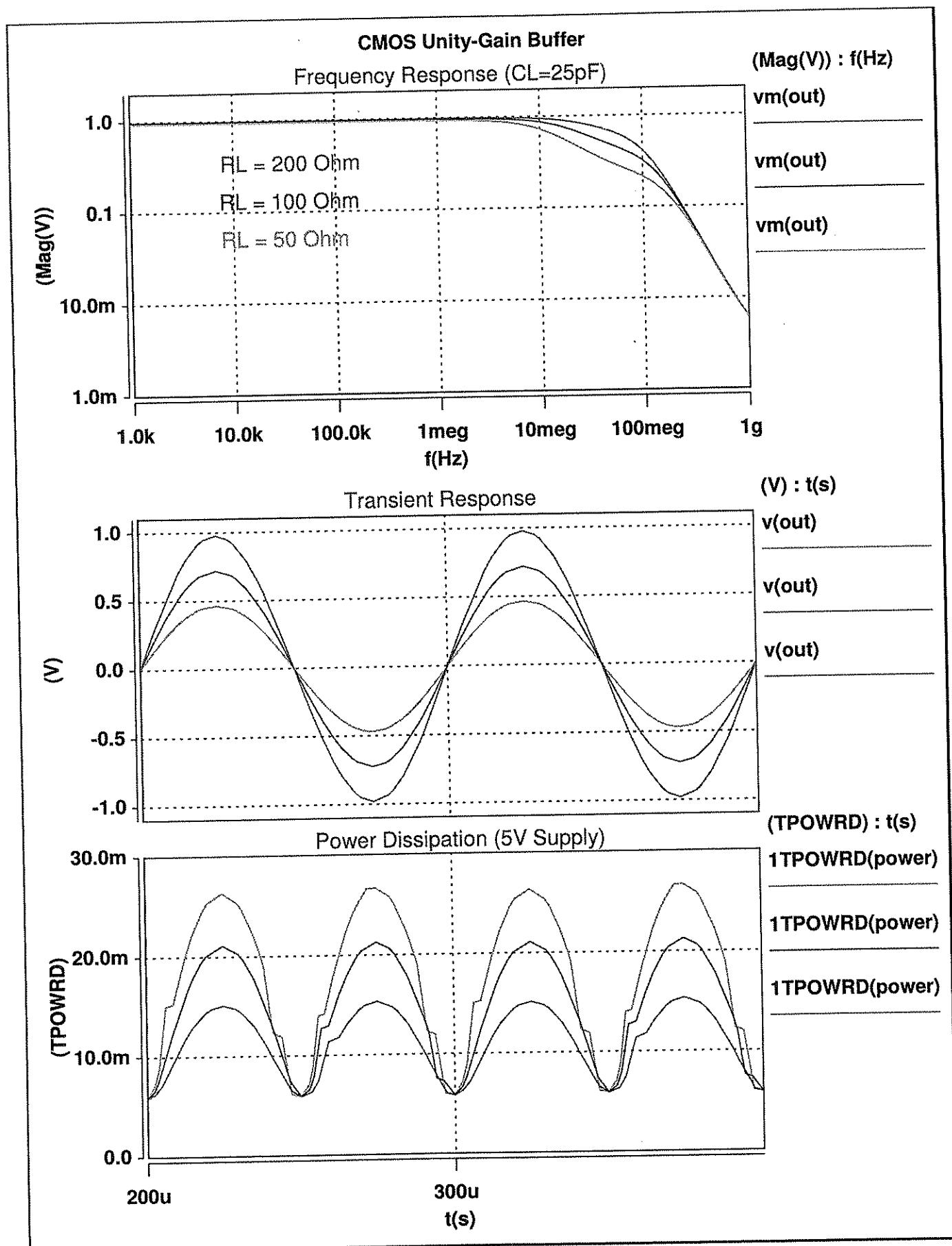
CMOS Transconductance Amplifier



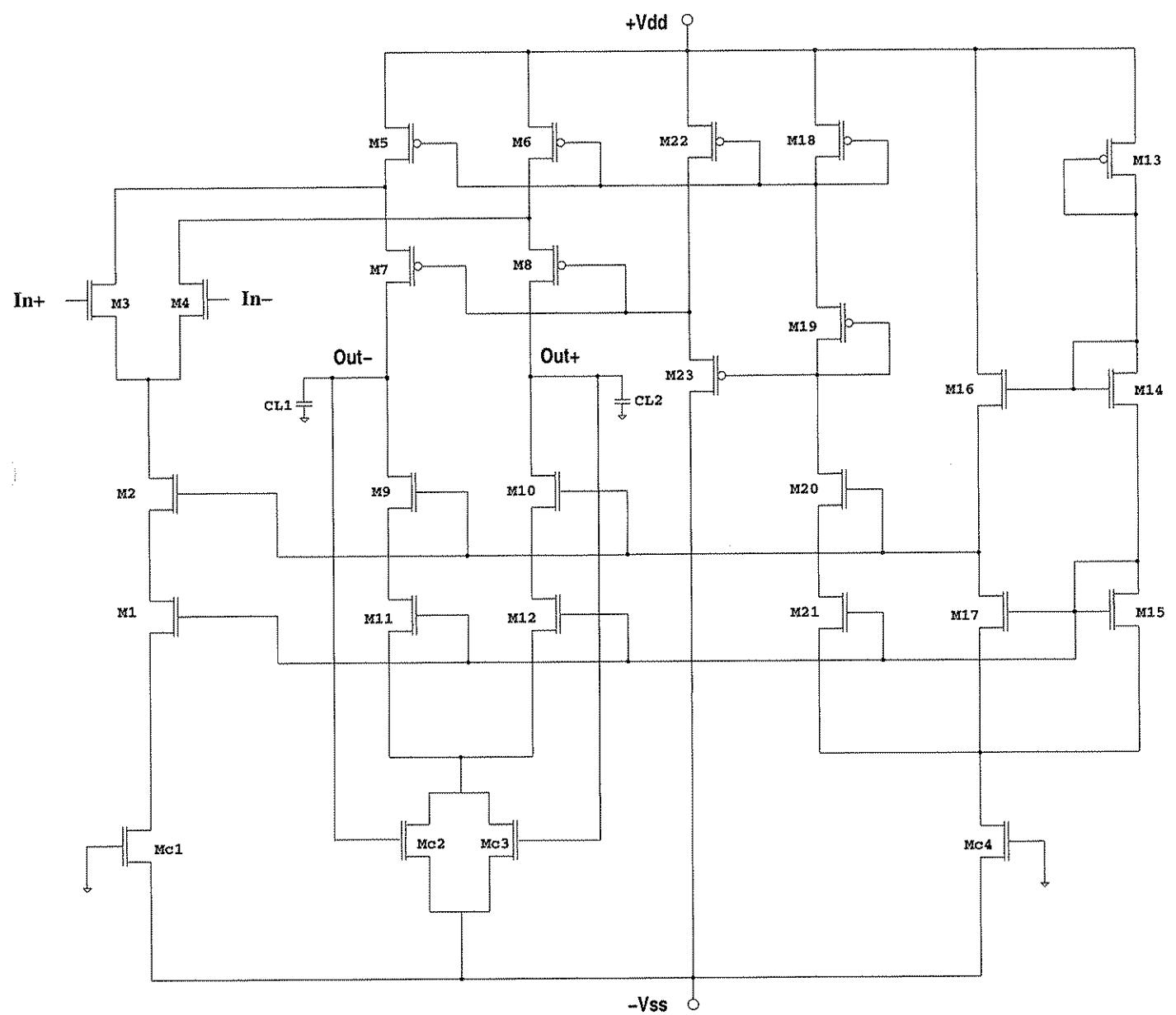
CMOS Push-Pull Amplifier with high Current Driving Capability

15 - 28



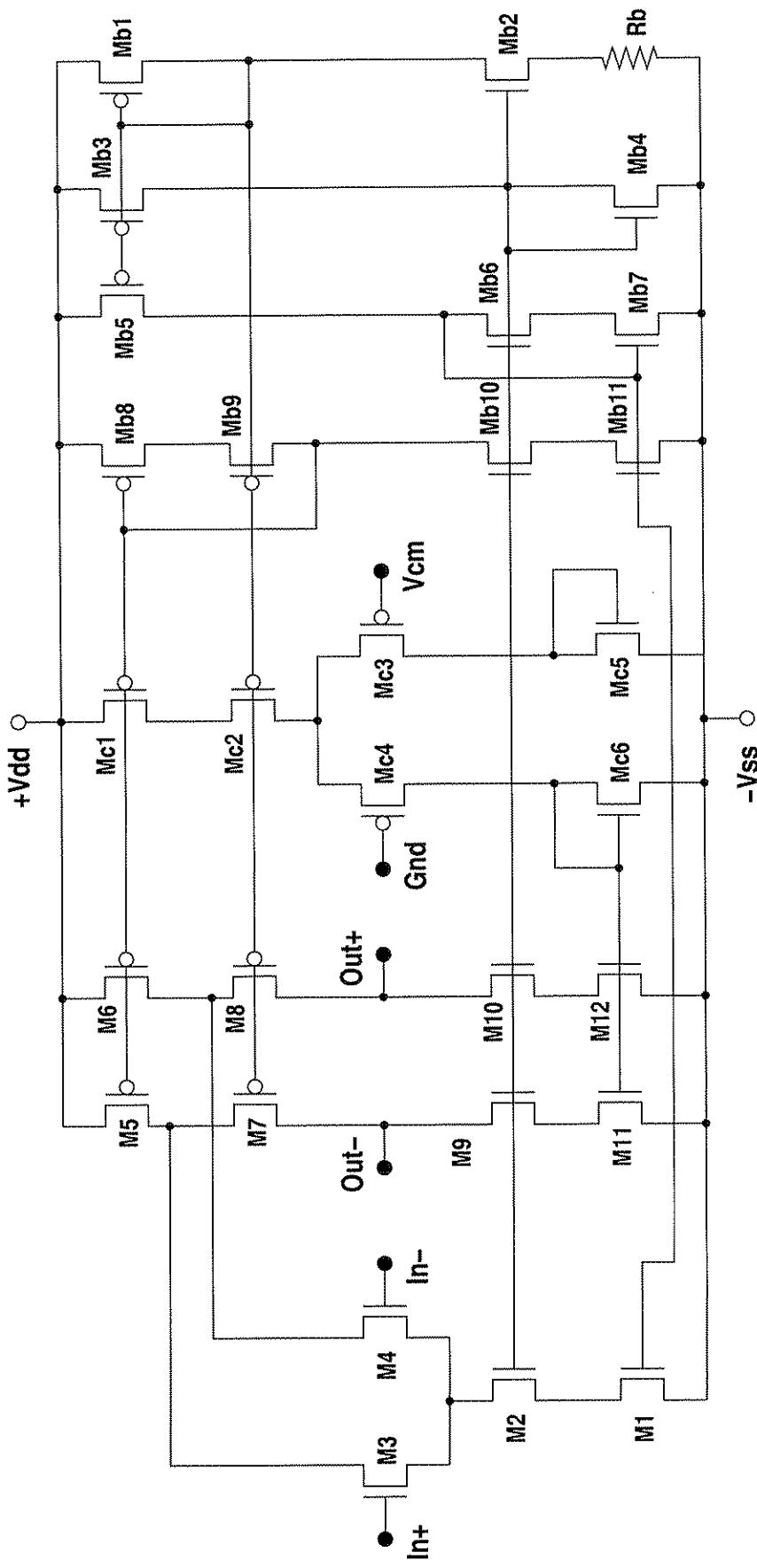


Folded Cascode Transconductance Amplifier with passive Common Mode Feedback



Folded Cascode Fully-Differential Transconductance Amplifier with active CMF

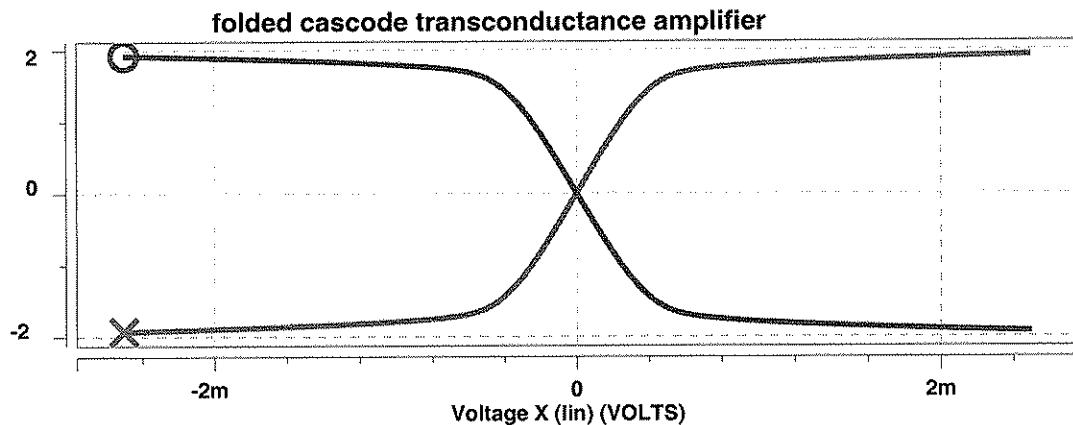
IV-31



IV-52

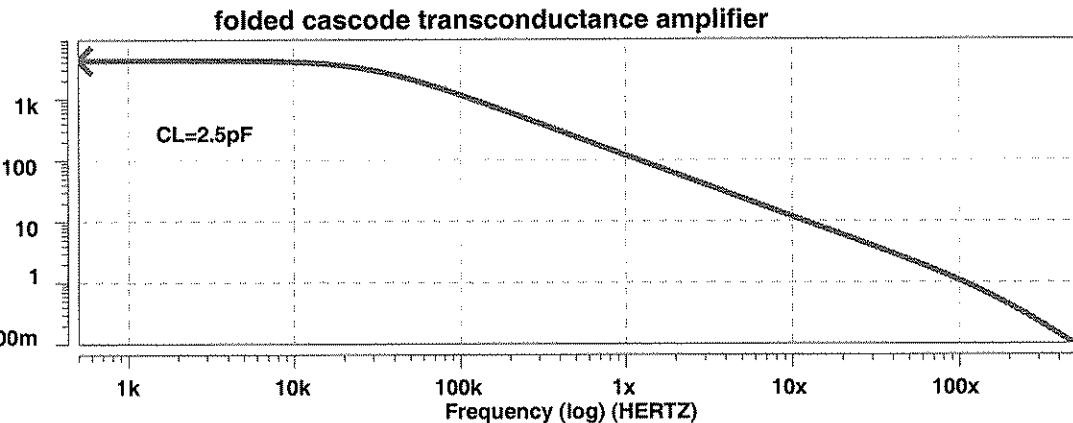
Wave	Symbol
J:A2:v(out+)	X
D0:A2:v(out-)	O

Voltages (lin)



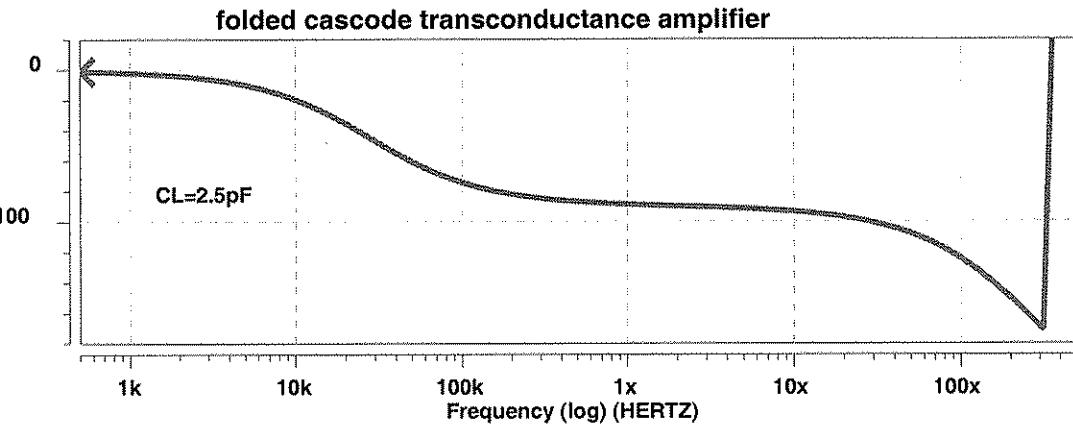
Wave	Symbol
D0:A1:vm(out+)	X

Volts Mag (log)



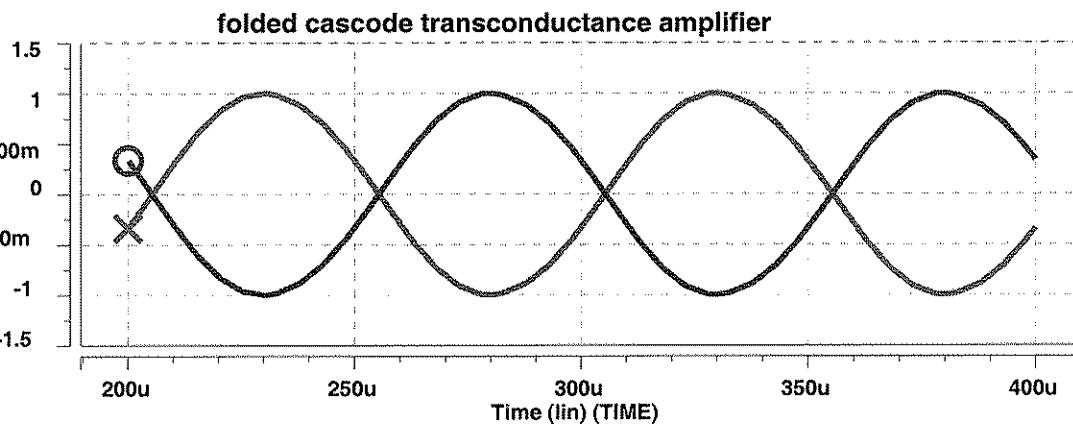
Wave	Symbol
D0:A1:pp(out+)	X

Volts Phase (lin)



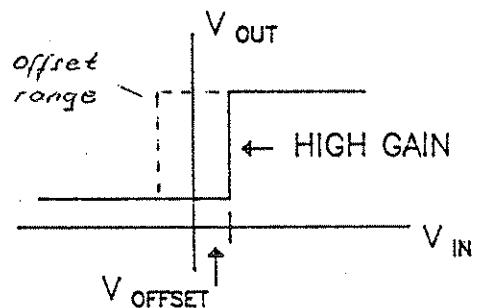
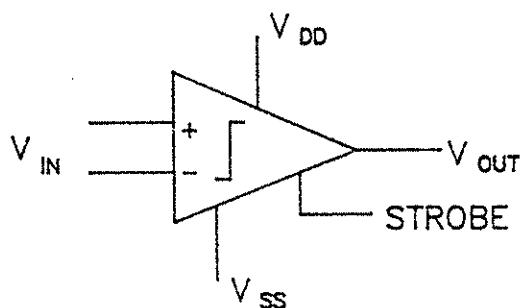
Wave	Symbol
D0:A0:v(out+)	X
D0:A0:v(out-)	O

Voltages (lin)



COMPARATORS

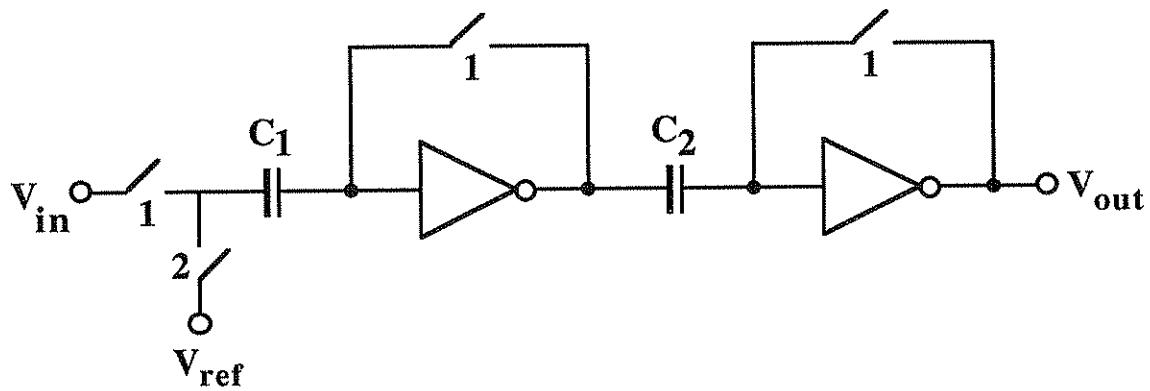
A. Definition and Features



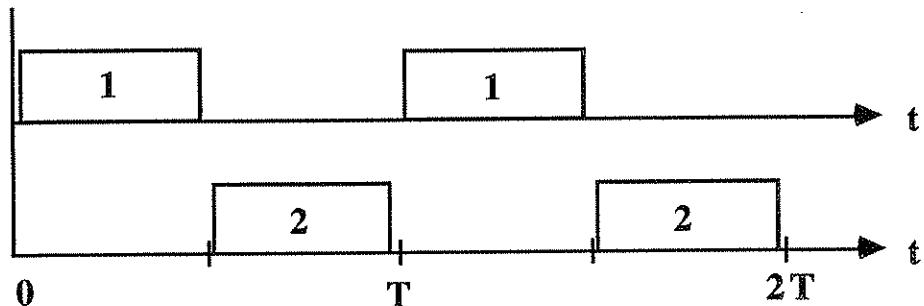
B. Requirements

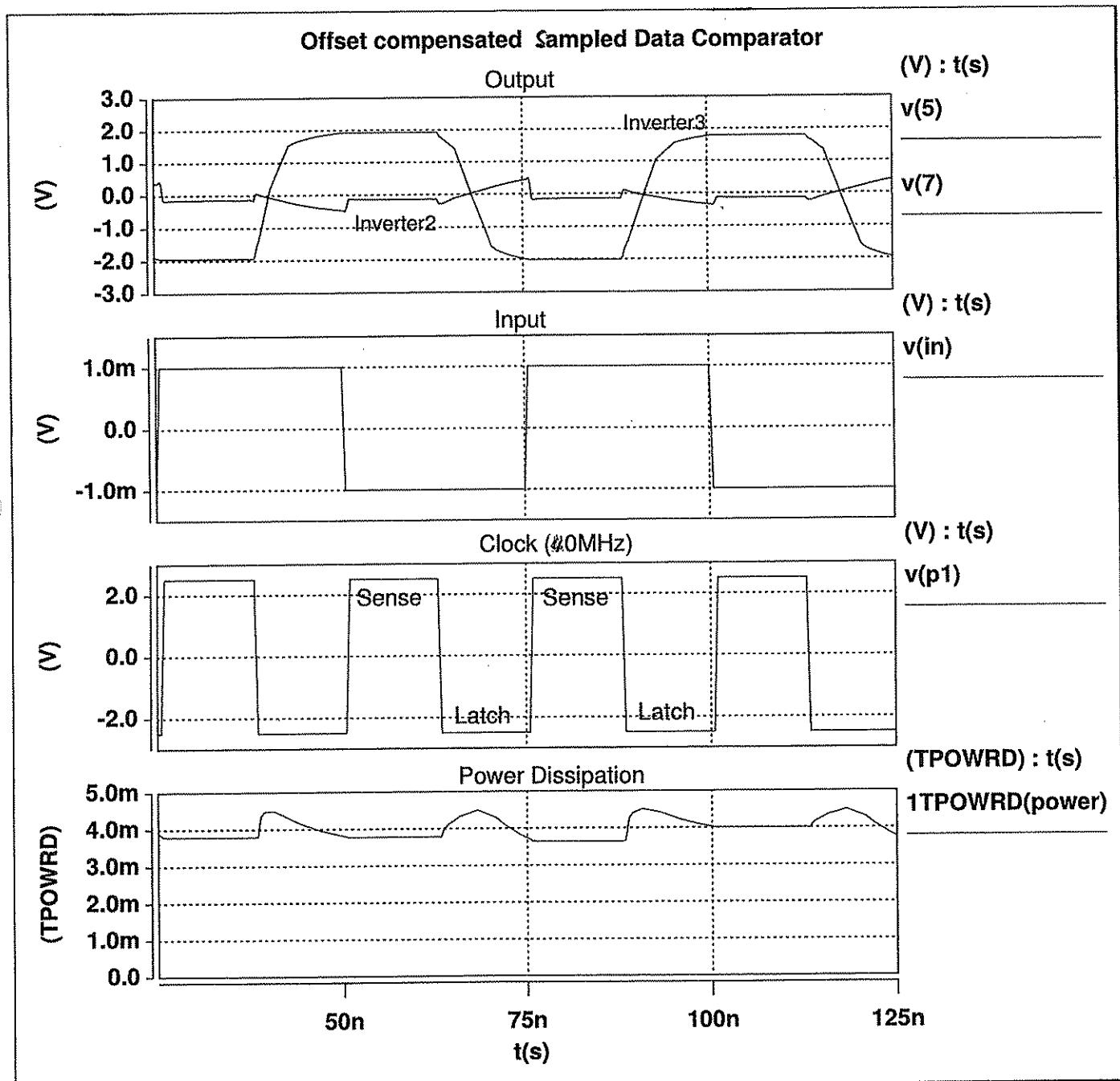
1. HIGH GAIN 10K Alternative: Employ positive feedback
2. LOW OFFSET VOLTAGE
3. COMMON MODE REJECTION ≈ 40 dB
4. FAST RESPONSE $< 1\mu\text{sec}$
5. LOW POWER; SMALL AREA
6. STROBE

Sampled-Data Comparator Version 1

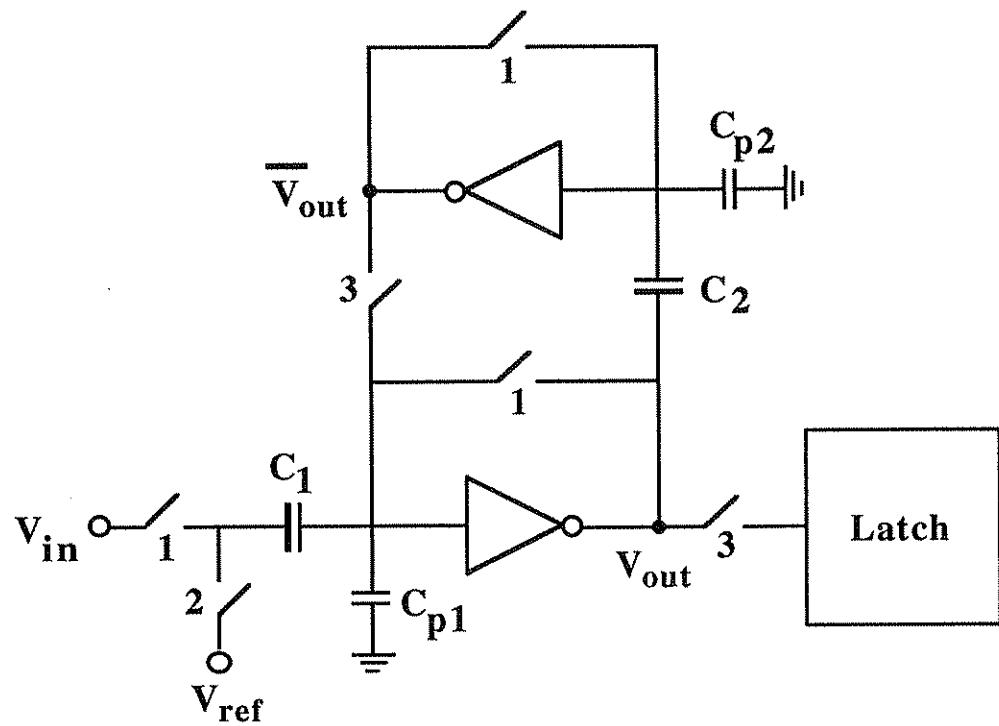


Clocking Scheme

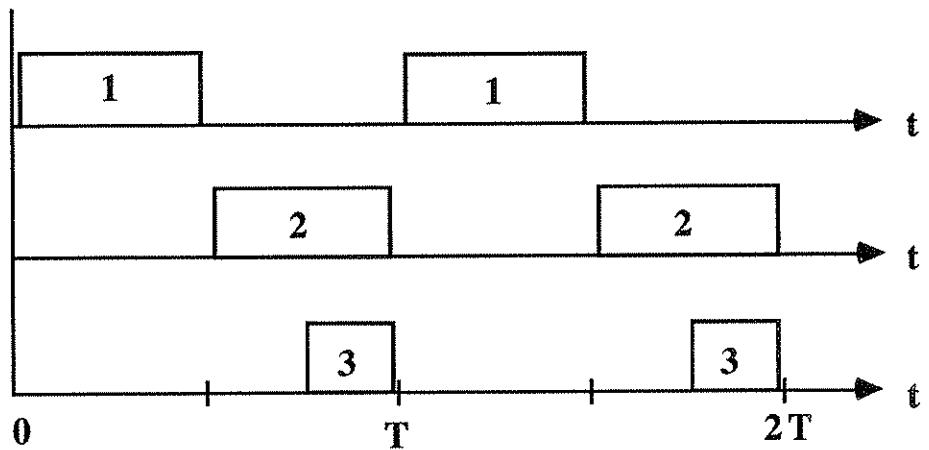


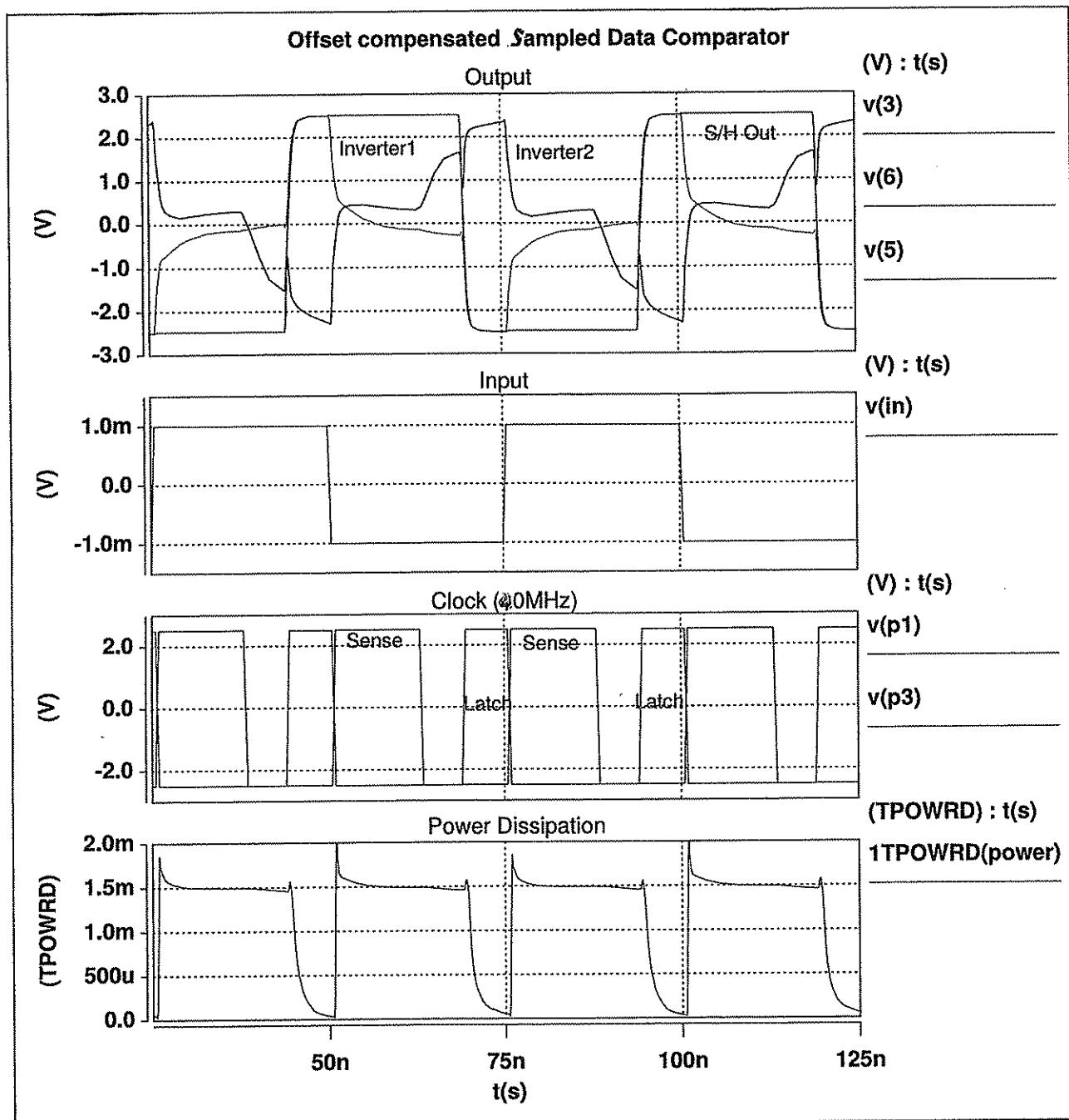


Sampled-Data Comparator Version 2

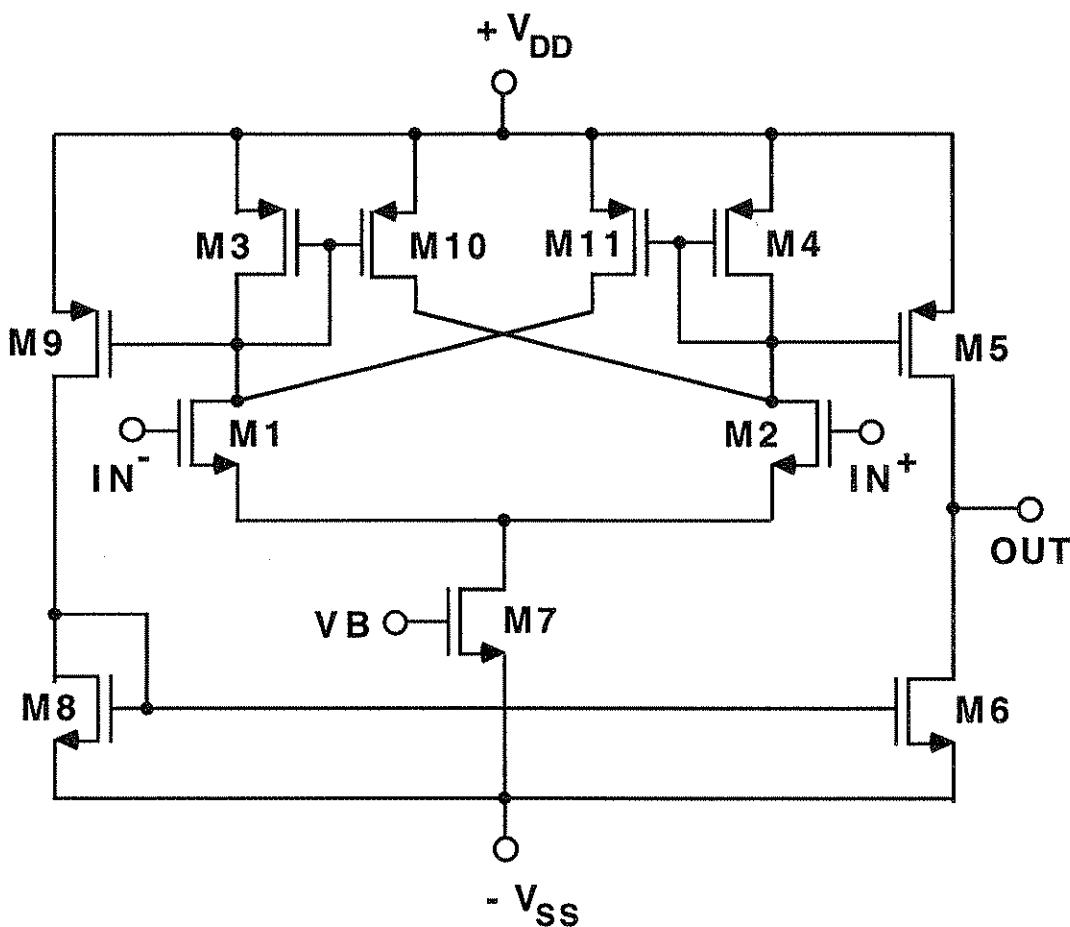


Clocking Scheme





CMOS Comparator with Hysteresis Version 4



$$\boxed{V_{Hyst} = \sqrt{\frac{2 I_7}{\mu C_{ox} (\frac{W}{L})_3 (1+h)}} [VK = 1]}$$

$$\text{where } h = \frac{(W/L)_10}{(W/L)_3} \quad K > 1$$

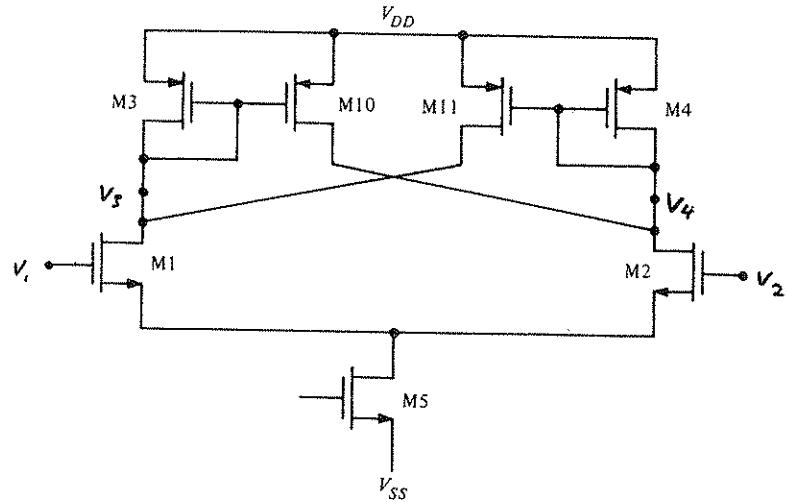
$V_{Hyst} = |V_{eff_1} - V_{eff_2}|$ @ trip point

pos. transition: $I_1 = I_7 \quad I_2 = I_{Q_0} = K \cdot I_3 \quad I_7 = I_1 + I_2 = I_3 (1+h)$
 (M4 & M11 are off)

$$\therefore V_{eff_1} = \sqrt{\frac{2 I_7}{(\frac{W}{L})_1 \mu C_{ox} (1+h)}}$$

$$V_{eff_2} = \sqrt{\frac{2 I_7 \cdot K}{\frac{W}{L} \mu C_{ox} (1+h)}}$$

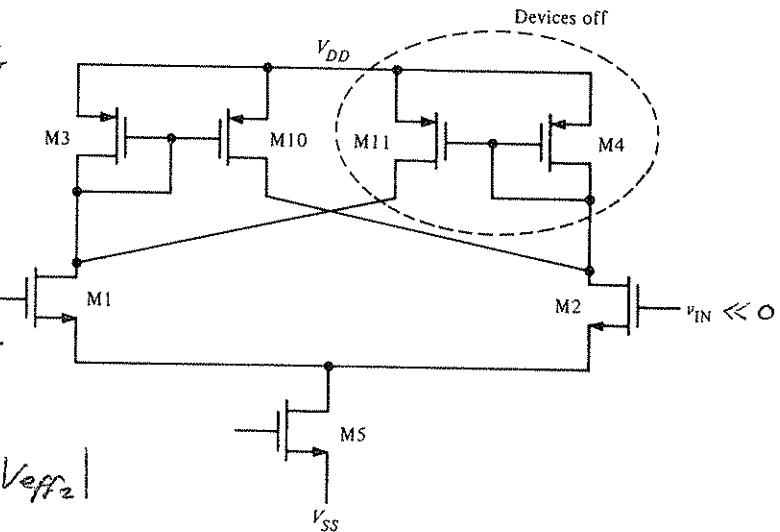
Comparator with Hysteresis



Pos. trip point:

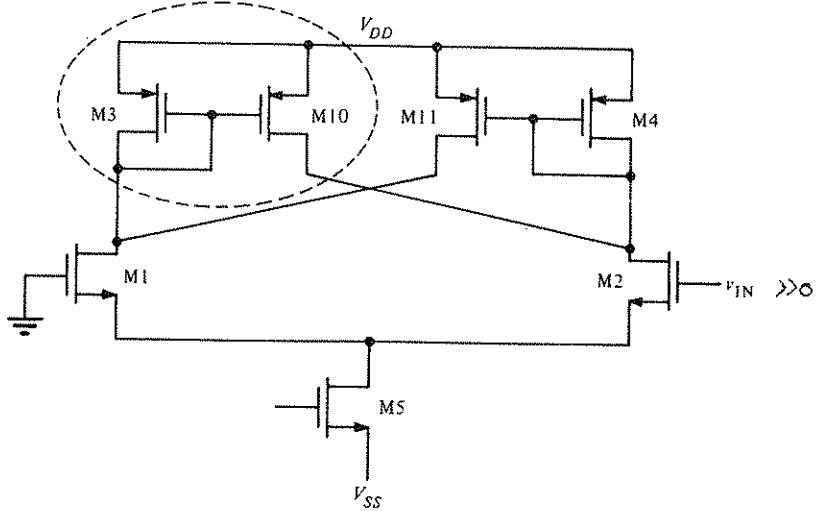
$$i_{r0} = \frac{(V_L)_0}{(W/L)_3}, i_3 = i_2$$

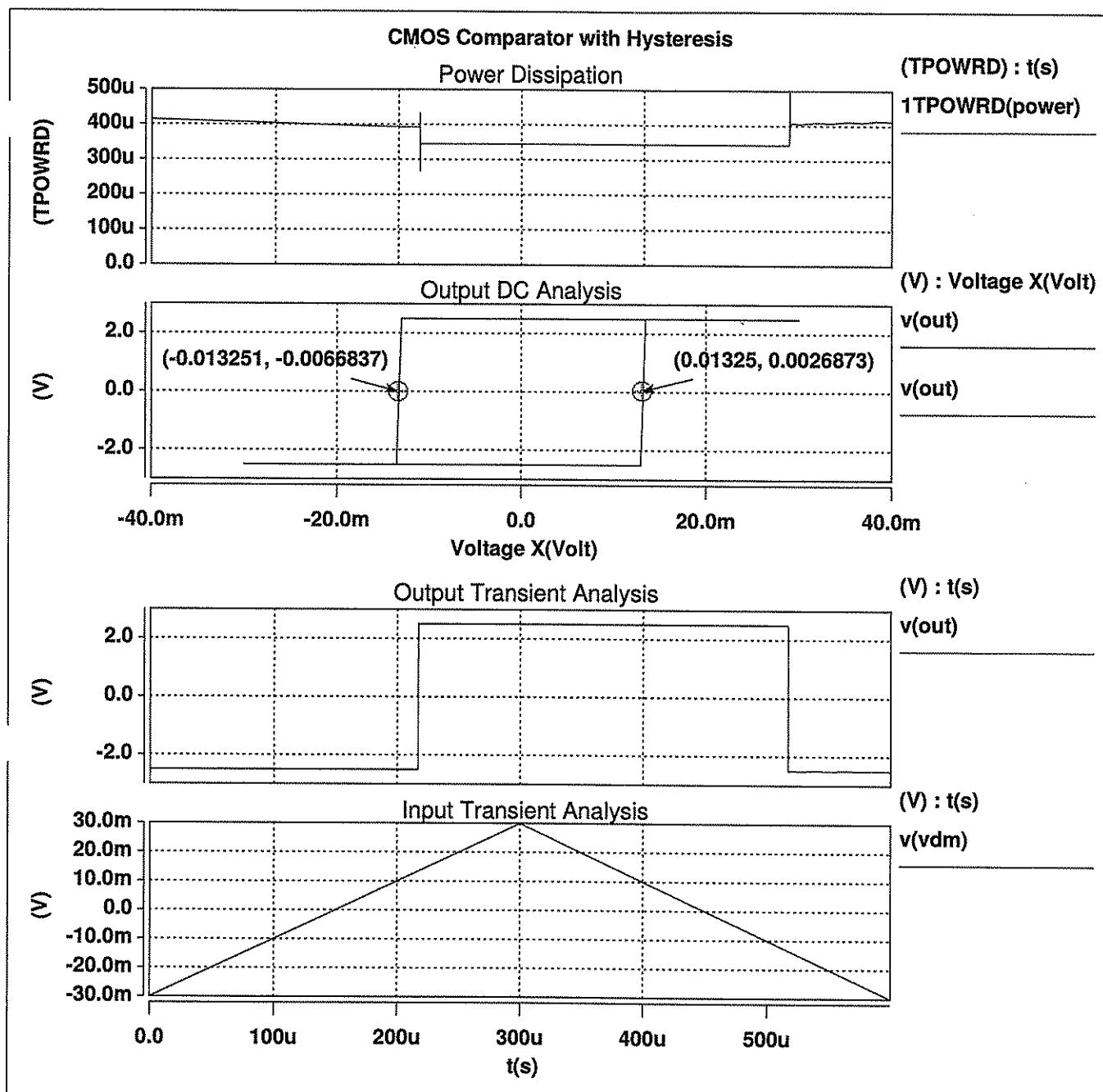
$$i_5^+ = i_1 + i_2 = i_3^+ + i_2^+$$



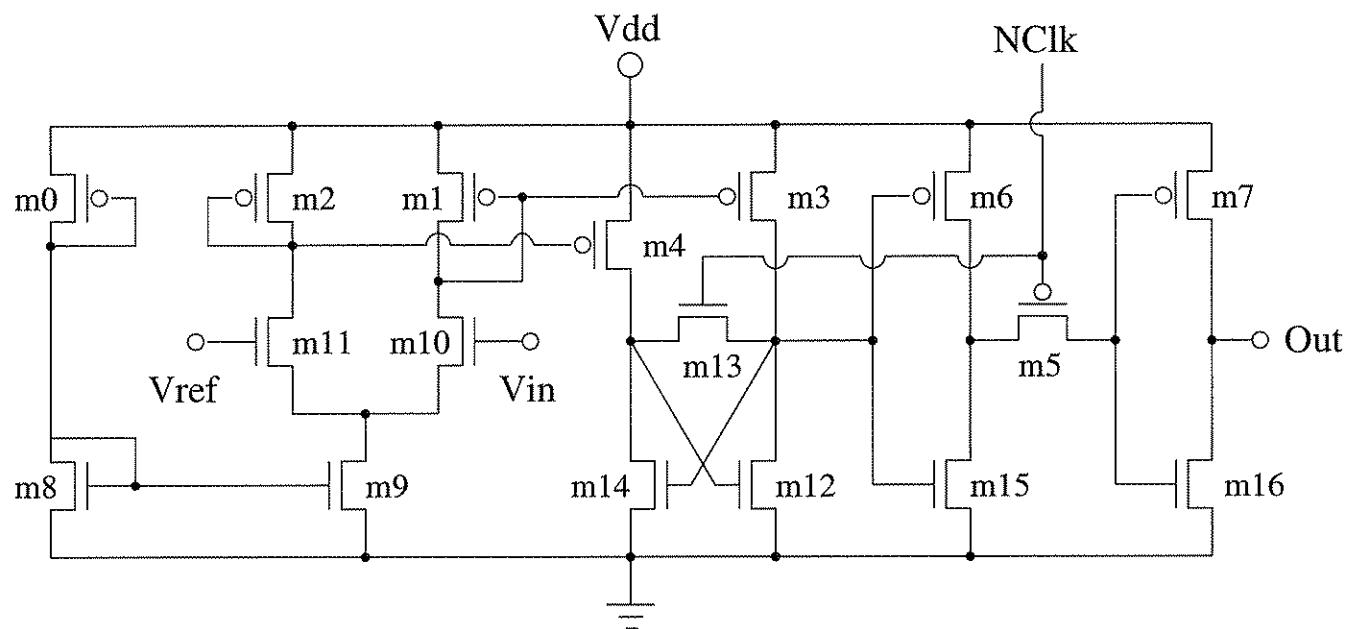
$$V_{hyst} = |V_{eff1} - V_{eff2}|$$

Devices off



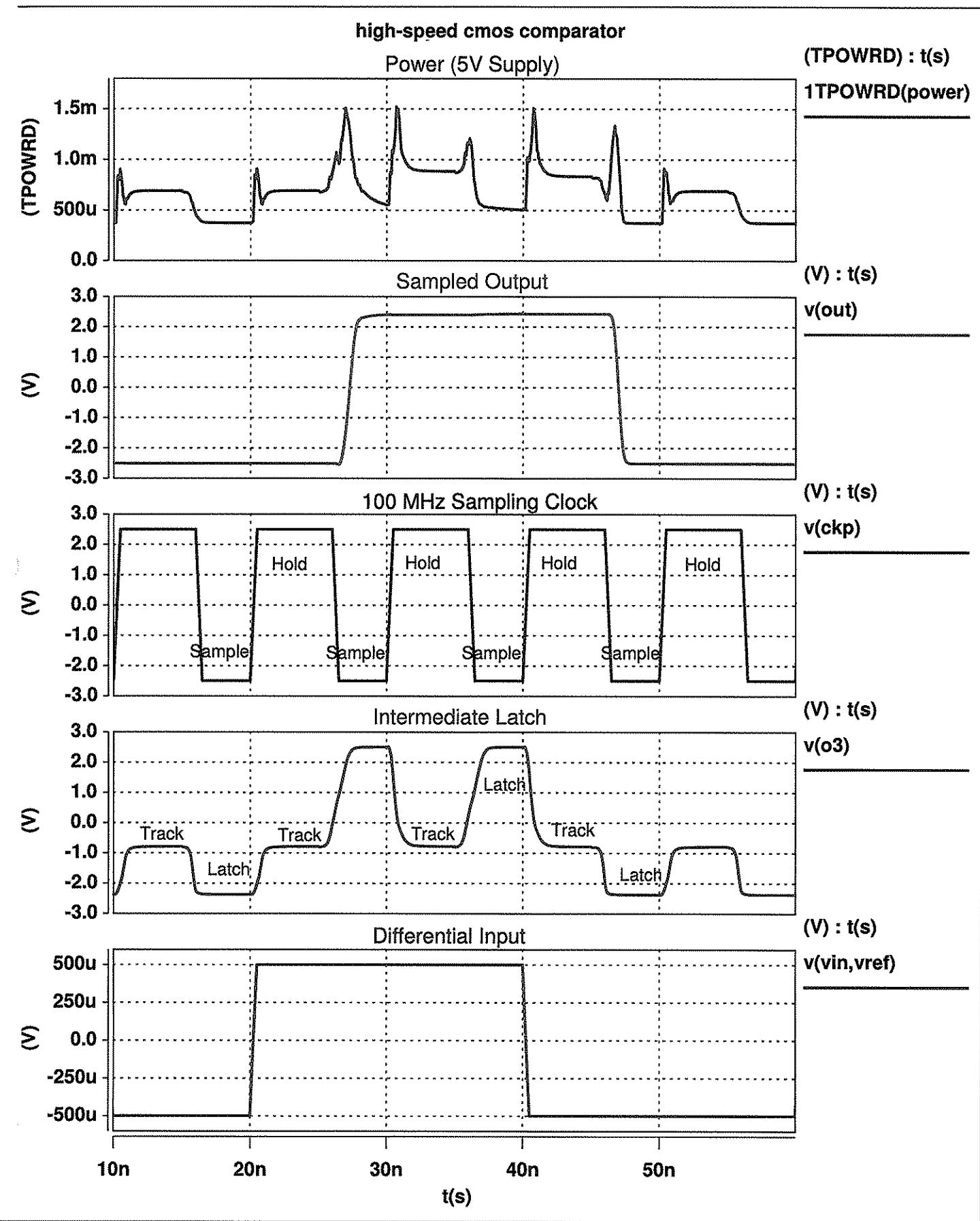


Latched CMOS Comparator



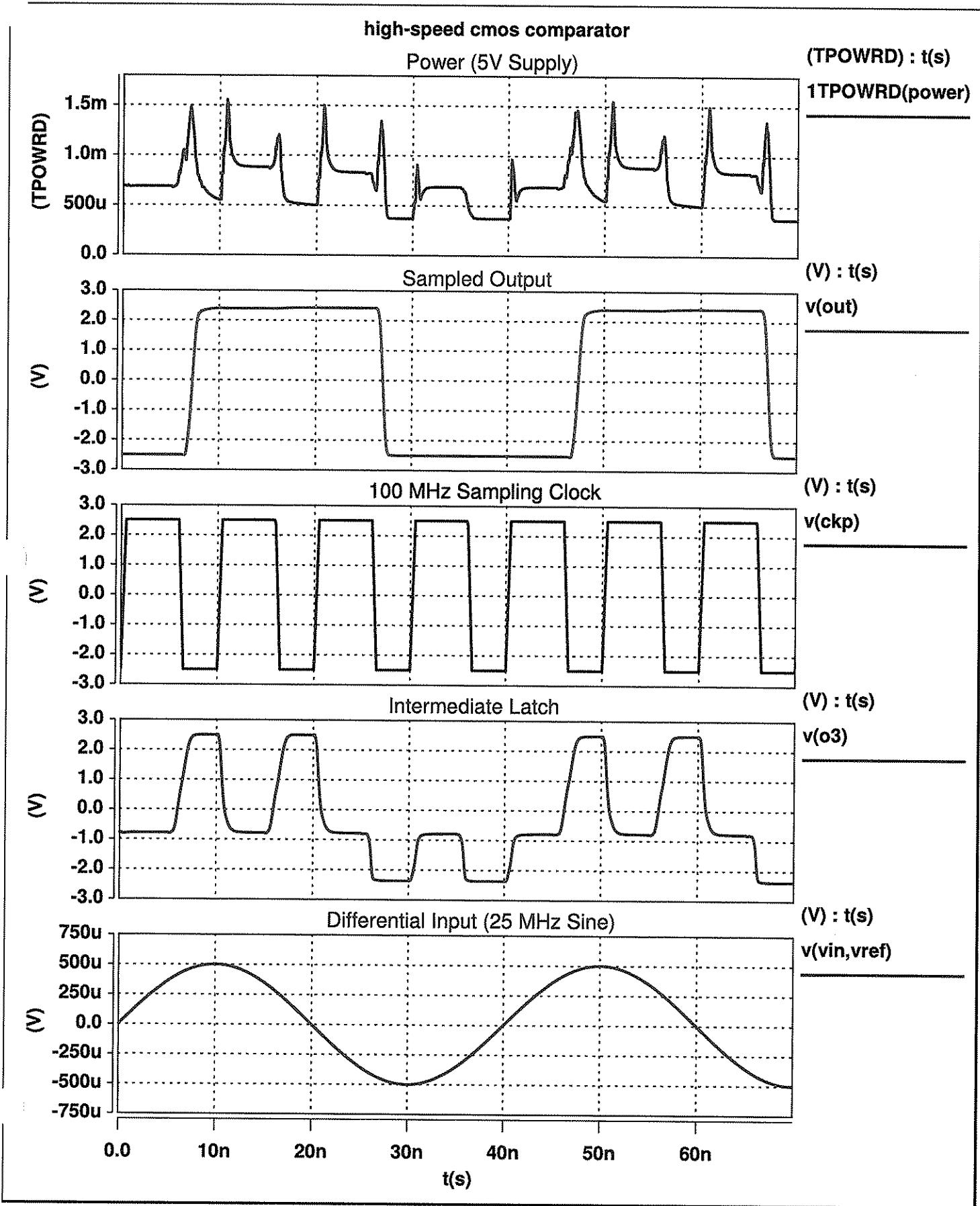
IV - 42

comp5-new



IV-43

comp5-new



IV-44

comp5-diff

