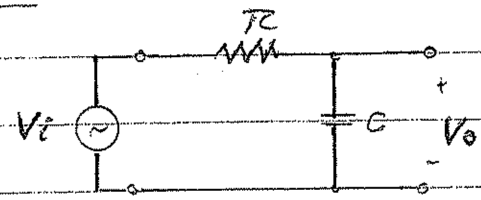
VII Switched-Capacitor Networks

1. Introduction

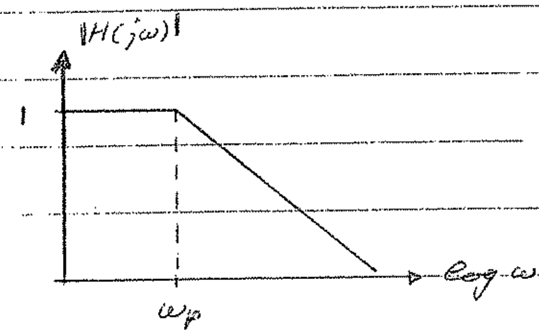
A) Problems in continuous time filters

Example 1st order LP



$$\frac{V_o}{V_i} = \frac{1}{(1 + sRC)} \hat{=} \frac{1}{(1 + s/\omega_p)}$$

$$\omega_p = \frac{1}{RC}$$



e.g. $\omega_p = 2\pi \cdot 10\,kHz$

$C = 1\,pF$

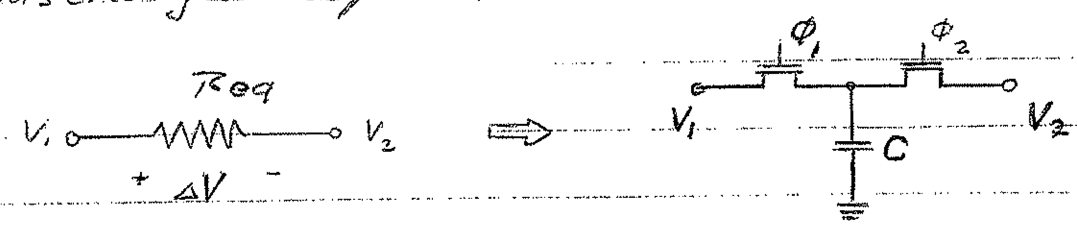
$\Rightarrow \underline{\underline{R = 16\,M\Omega}}$

Problems: - R too big

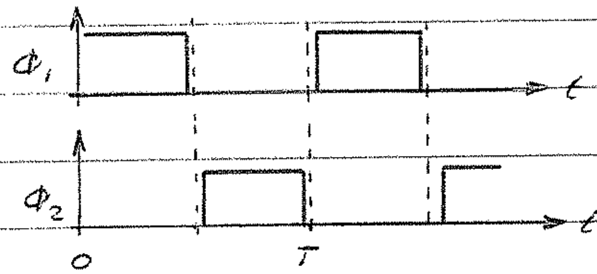
- Absolute control of R and C required

B) Solution

$\boxed{\text{Replace Resistors by periodically charged and discharged capacitor}}$



Φ_1, Φ_2 Nonoverlapping 2-phase Clock signal



<u>Resistor - Capacitor Relationship:</u> (ideal switches)

ΔQ: Transferred Charge per Cycle

$$\Delta Q = C\,(V_1 - V_2) \quad \Big| \; \frac{1}{T}$$

$$\underline{\frac{\Delta Q}{T} = \bar{I} = \frac{C}{T}\,\Delta V} \qquad \text{where } \Delta V = V_1 - V_2$$

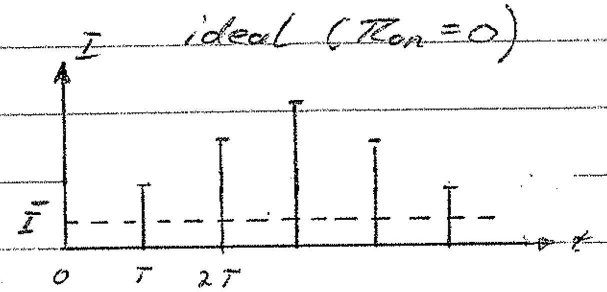
Compare with Ohm's law:

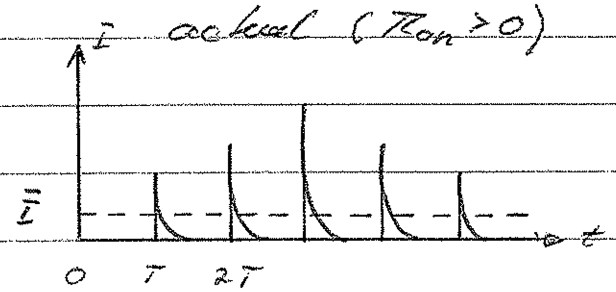
$$\underline{I = \frac{1}{R}\,\Delta V}$$

$$\Rightarrow \boxed{R_{eq} = \frac{T}{C} = \frac{1}{C \cdot f_c}}$$

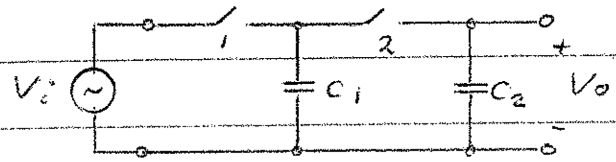
<u>Note:</u> The average current $\bar{I}$ is not a quantity that can physically be measured; it is the equivalent current that transfers the charge ΔQ in time T from node 1 to node 2 with $V_1 - V_2 = \Delta V$.

if we assume ideal switches ($R_{on} = 0$), the charge transfer is realized by a train of delta functions.
If R_{on} is included in the analysis, the charge transfer is realized by a train of exponentially damped functions.





Example: 1st order LP



$$H'(s) = \frac{1}{(1 + s R_{eq} C_2)} = \frac{1}{(1 + s \cdot T \frac{C_2}{C_1})} \hat{=} \frac{1}{(1 + s/\omega_p')}$$

$$\omega_p' = \frac{1}{T} \frac{C_1}{C_2} = f_c \frac{C_1}{C_2}$$

e.g. $\omega_p' = 2\pi \cdot 10\,kHz$

$C_2 = 1\,pF$

$f_c = 200\,kHz$

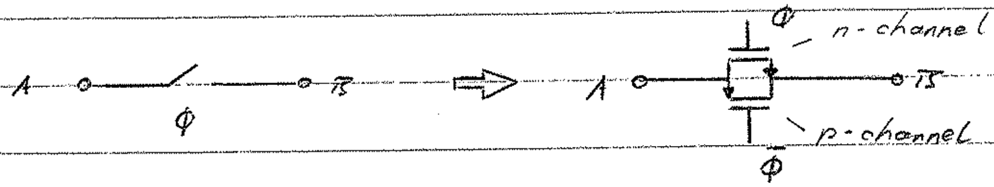
$$\Rightarrow \underline{C_1 = C_2 \frac{\omega_p'}{f_c} = C_2 \cdot \frac{2\pi}{20} = 0.314\,pF}$$

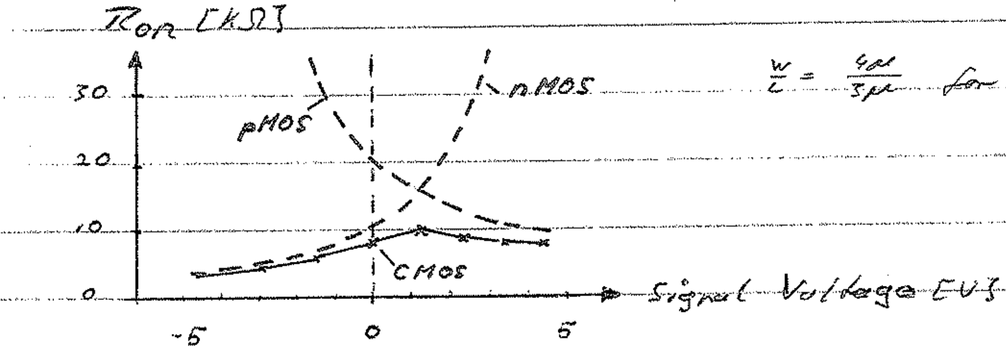
Advantages - Requires control of ratios of C only
- Small area (small C-spread)
- Frequency scaling is possible with clock

Disadvantages - Requires continuous time anti-aliasing filter
- Requires "quarz-stable" nonoverlapping 2-phase clock

2. CMOS Implementation

A) Switches: composite CMOS switch



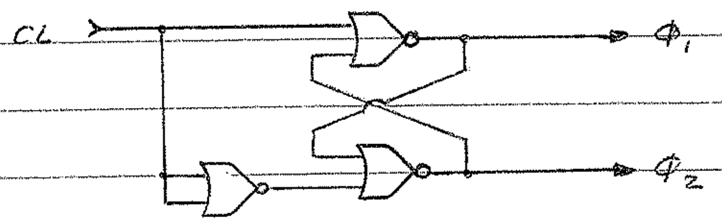


$\frac{W}{L} = \frac{4\mu}{3\mu}$ for p and n-channel

Advantages: - MOSFETS exhibit no DC offset
- extremely small leakage currents in off-state
- R_{on} variations acceptable over full signal swing
- 1st order elimination of clock-feedthrough

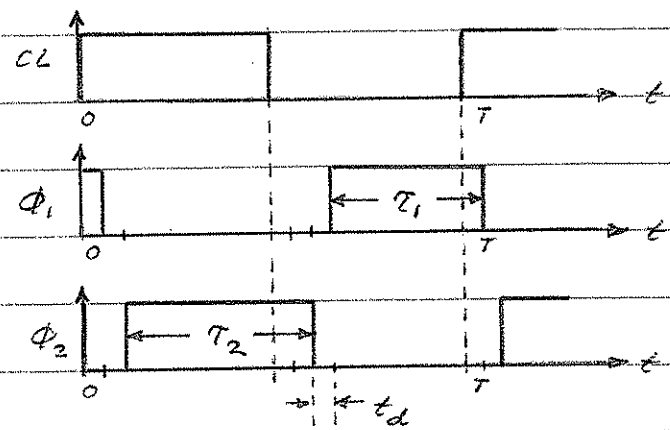
15) Nonoverlapping 2-phase Clock

Example with NOR Gates



$$\Phi_1 = \overline{CL + \Phi_2}$$

$$\Phi_2 = \overline{\overline{CL} + \Phi_1}$$



$$\tau_1 = \frac{1}{2}T - 2t_d$$

$$\tau_2 = \frac{1}{2}T$$

t_d : Gate delay of NOR

Notes: If the delay of a single gate is too small, the above scheme can be modified to yield a longer nonoverlapping time by including an even number of inverters in the crossed-over feedbacks.

$$t_{nonoverlap} = (1+N)\,t_d$$

$$\tau_1 = \frac{1}{2}T - (2+N)\,t_d$$

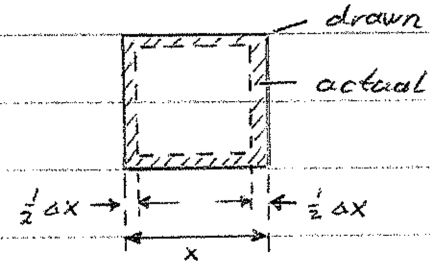
$$\tau_2 = \frac{1}{2}T - N\,t_d$$

N even $\{0, 2, 4, \ldots\}$

To reduce the clock-feedthrough effect, it is also important to realize symmetrical edges for Φ_1 and Φ_2

c) Capacitors

Undercut insensitive Structure

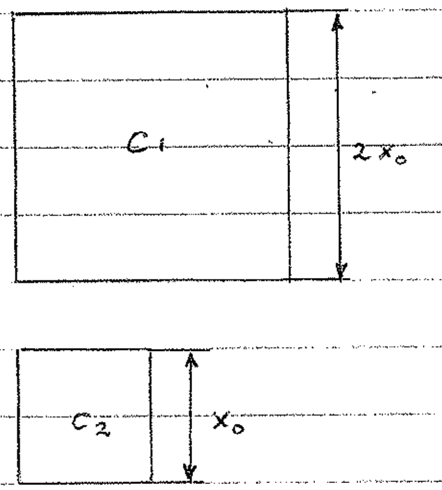


Uniform undercut (.2 – 0.5 μ)

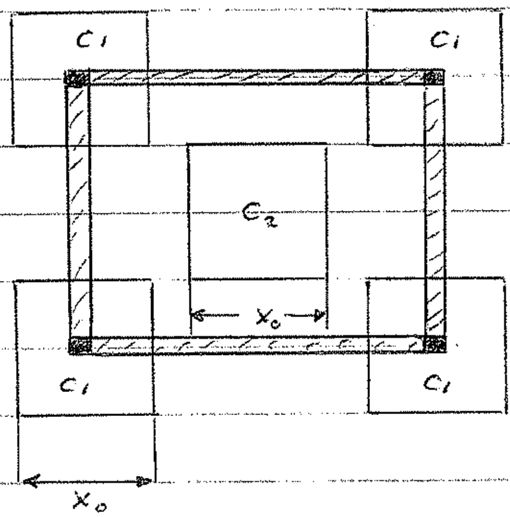
$$C_{ac} = C_{ox}\,(x - \Delta x)^2$$

Ex: Realize 2 Cap. with ratio 4

a) Minimum Area



b) Undercut insensitive
+Gradient insensitive



$$\frac{C_1}{C_2} = \frac{(2x_o - \Delta x)^2}{(x_o - \Delta x)^2} = 4\,\frac{\left(1 - \frac{\Delta x}{x_o} + \frac{1}{4}\frac{\Delta x^2}{x_o^2}\right)}{\left(1 - 2\frac{\Delta x}{x_o} + \frac{\Delta x^2}{x_o^2}\right)}$$

$$\frac{C_1}{C_2} = 4\,\frac{(x_o - \Delta x)^2}{(x_o - \Delta x)^2} = 4$$

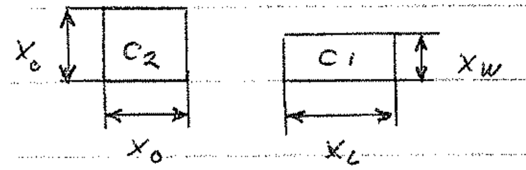
$$\cong 4\left(1 - \frac{\Delta x}{x_o}\right)\left(1 + 2\frac{\Delta x}{x_o}\right) \cong 4\left(1 + \frac{\Delta x}{x_o}\right)$$

In general:

$$\Rightarrow \varepsilon_r \cong \frac{\Delta x}{x_o} \cdot 2\left[1 - \sqrt{\frac{C_2}{C_1}}\right] \quad C_2 \le C_1 \qquad C_1 = C_2 \Rightarrow \quad \varepsilon_r = 0$$

Capacitors with fractional ratios

$$\frac{C_1}{C_2} = \pi \qquad 1 \leq \pi < 2$$



$$C_1 = C_{ox}(X_W - \Delta X)(X_L - \Delta X)$$

$$C_2 = C_{ox}(X_o - \Delta X)^2$$

$$\pi = \frac{C_1}{C_2} = \frac{(X_W - \Delta X)(X_L - \Delta X)}{(X_o - \Delta X)^2}$$

Def:

$W = \frac{X_W}{X_o}$	normalized width of C_1
$L = \frac{X_L}{X_o}$	normalized length of C_1
$\varepsilon_u = \frac{\Delta X}{X_o}$	undercut error of C_2

Thus:
$$\boxed{\pi = \frac{(W - \varepsilon_u)(L - \varepsilon_u)}{(1 - \varepsilon_u)^2}} \qquad (1)$$

condition for zero undercut error: undercut area C_1

$$\pi\left(4X_o \tfrac{1}{2}\Delta X - 4\tfrac{1}{4}\Delta X^2\right) = \left(X_W \tfrac{1}{2}\Delta X + X_L \tfrac{1}{2}\Delta X\right)2 - 4\tfrac{1}{4}\Delta X^2 \;\Big|\; \frac{1}{X_o^2}$$

$$\pi(2\varepsilon_u - \varepsilon_u^2) = (W\varepsilon_u + L\varepsilon_u) - \varepsilon_u^2$$

$$\boxed{\pi(2 - \varepsilon_u) + \varepsilon_u = W + L} \qquad (2)$$

Solve for W and L in (1) + (2)

Solution:

$$W \cong \pi\left(1 - \sqrt{1 - \tfrac{1}{\pi}}\right) \qquad L \cong \pi\left(1 + \sqrt{1 - \tfrac{1}{\pi}}\right)$$

more precise:

$$W = \pi(1 - \varepsilon_u)\left(1 + \tfrac{\varepsilon_u}{\pi} - \sqrt{1 - \tfrac{1}{\pi}}\right) \qquad L = \pi(1 - \varepsilon_u)\left(1 + \tfrac{\varepsilon_u}{\pi} + \sqrt{1 - \tfrac{1}{\pi}}\right)$$

e.g. $\pi = 1.5$

$$W_{1.5} \cong 0.634 \qquad L_{1.5} \cong 2.366 \qquad \underline{X_o = 25\mu m} \Rightarrow \; X_W = 15.85\mu m \quad X_L = 59.15\mu m$$

5. SC Filter Design

A) The z - Transform

Def $$\boxed{X(z) = \mathcal{Z}[x(t)] = \mathcal{Z}[x(n)] = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}}$$

Note: Very often $x(t) = 0$ for $t < 0$

Then

$$X(z) = \sum_{n=0}^{\infty} x(n)\, z^{-n}$$ one-sided z-Transf.

Recall: Laplace transf. of a discrete function

$$X(s) = \mathcal{L}[x(t)] = \mathcal{L}[x(n)] = \sum_{n=-\infty}^{\infty} x(n)\, e^{-snT}$$

Relationship between s- and z-domain:

$$\boxed{z = e^{sT}}$$

Inverse function:

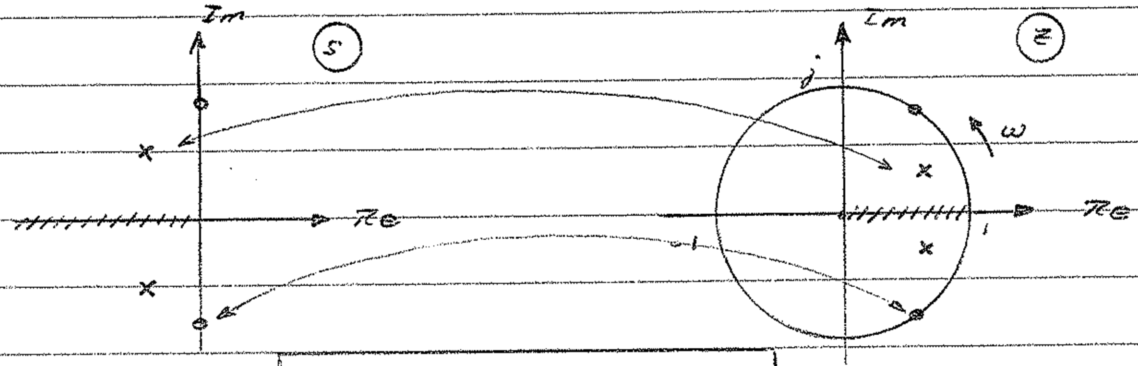
$$s = \frac{1}{T} \ln[z]$$ nonlinear funct.

i.e. polynomial in s does not transform into polynomial in z

Mapping between s- and z-domain

a) Matched z-Transform (MZT)

Poles and zeros in s are exactly mapped into poles and zeros in z.



$$\boxed{(s+p) \rightarrow \frac{1}{T}(1-e^{-pT}z^{-1})}$$

b) Bilinear z-Transform (BLT)

The nonlinear function $s = \frac{1}{T}\ln[z]$ is approximated by a bilinear function:

Recall: $\ln x = 2\left[\left(\frac{x-1}{x+1}\right) + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \dots\right]$ $x > 0$

$$\boxed{s \longrightarrow \frac{2}{T}\,\frac{(1-z^{-1})}{(1+z^{-1})}}$$

Frequency warping:

$$s = j\omega \rightarrow j\frac{2}{T}\,tg\left[\frac{T}{2}\omega'\right]$$

c) Lossless Discrete Int. z-Transform (LDI)

The nonlinear function $s = \frac{1}{T}\ln[z]$ is approximated by a linear function

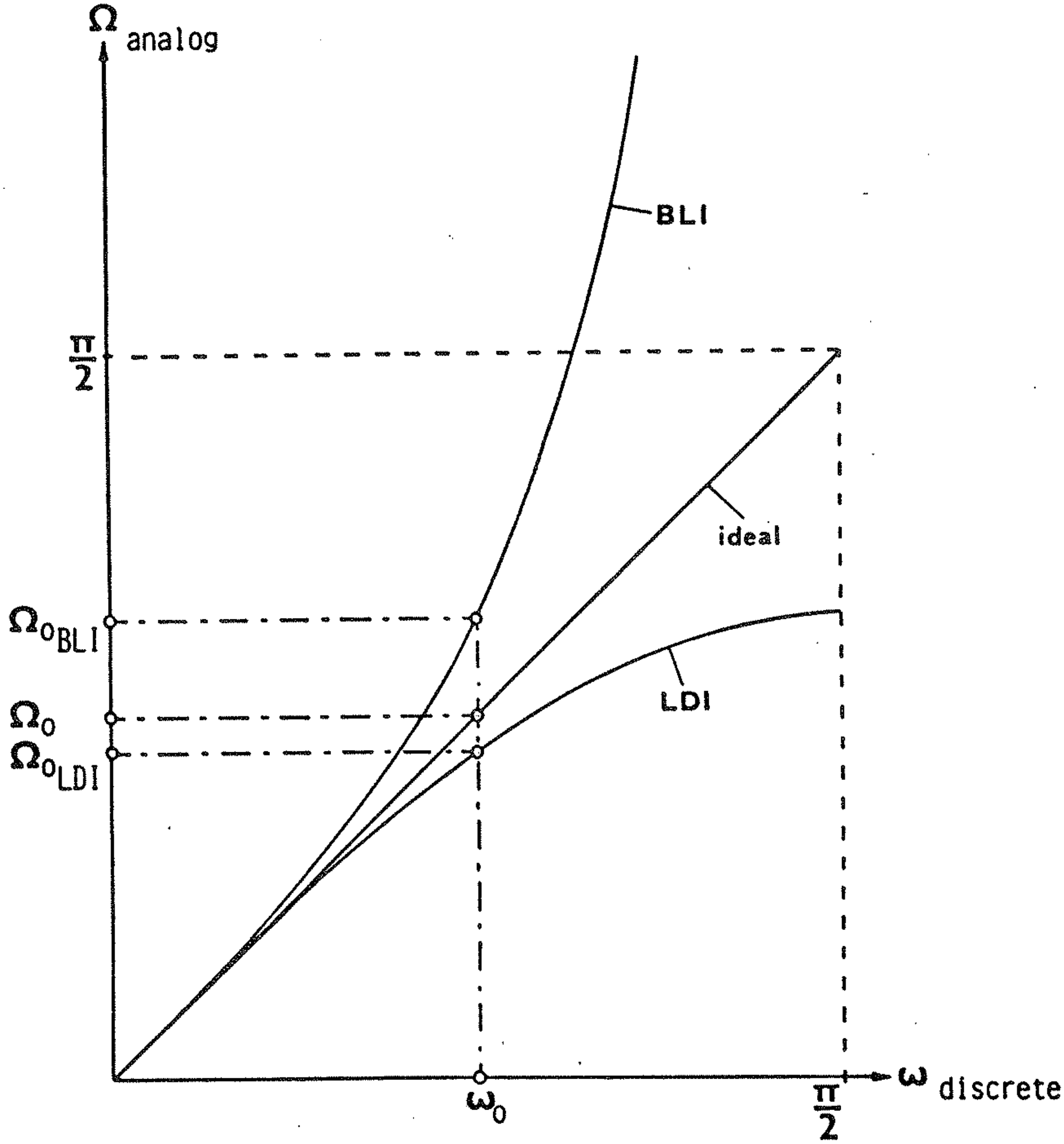
Recall: $\ln x = \left(\frac{x-1}{x}\right) + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \dots$ $x \geq \frac{1}{2}$

$$\boxed{s \longrightarrow \frac{1}{T}\,\frac{(1-z^{-1})}{z^{-1/2}}}$$

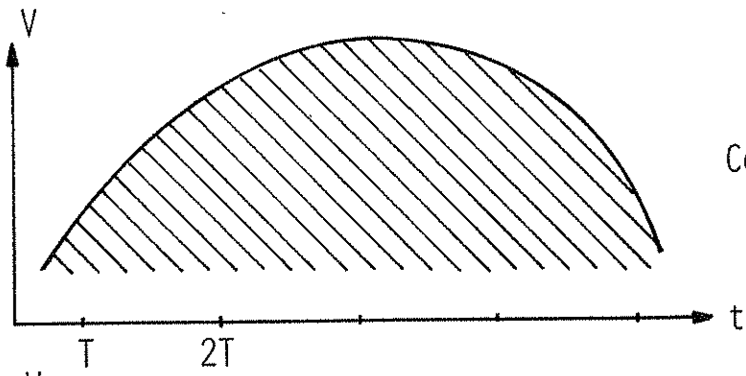
Frequency warping:

$$s = j\omega \rightarrow j\frac{2}{T}\sin\left[\frac{T}{2}\omega'\right]$$

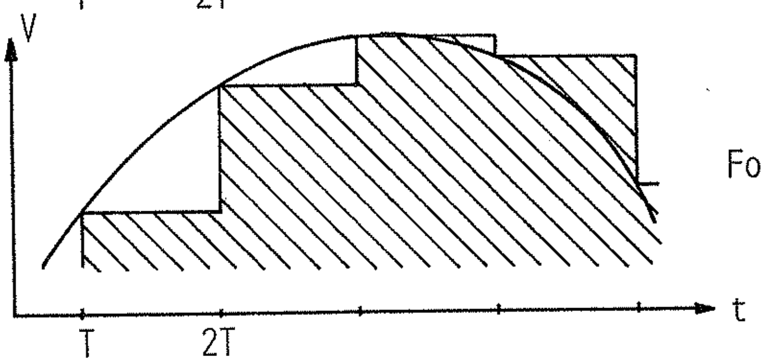
Mapping of the Frequency Axis



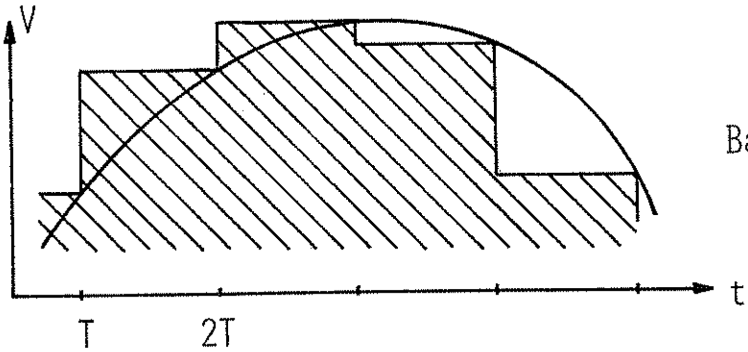
$$\omega_{LDI} = \left[\frac{T}{2}\right]^{-1} \sin \omega \frac{T}{2}$$



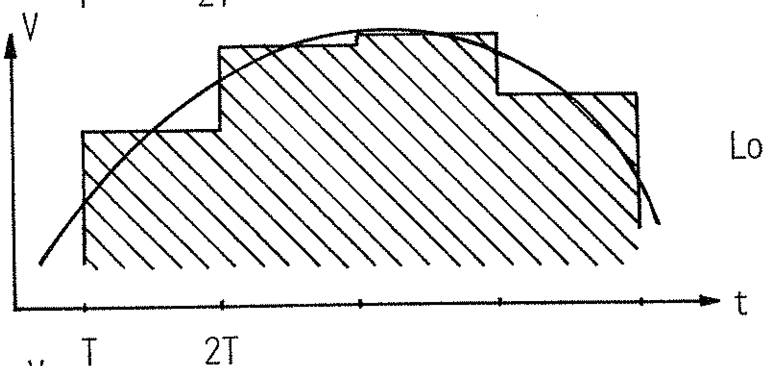
Continuos Int.



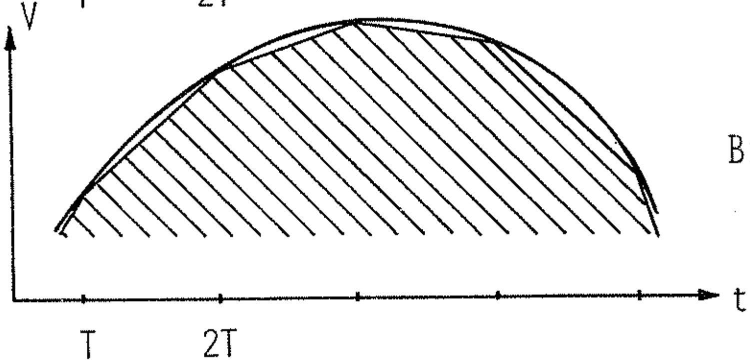
Forward-Diff. Int.



Backward-Diff. Int.



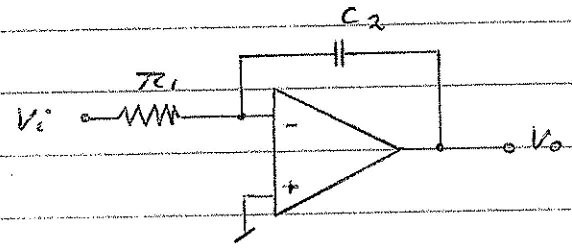
Lossles Discr. Int.



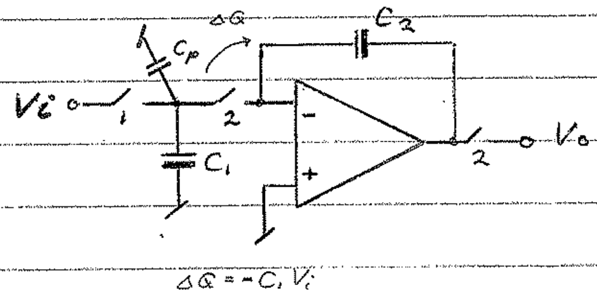
Bilinear Int.

B) The time-discrete Integrator

continuous-time



discrete-time



$$\frac{d}{dt} V_o(t) = -\frac{1}{R_1 C_2} V_i(t)$$

$$V_o(t) = -\frac{1}{R_1 C_2} \int V_i(t)\, dt$$

$$\Big\updownarrow \mathcal{L}$$

$$V_o(s) = -\frac{1}{R_1 C_2} \frac{1}{s} V_i(s)$$

$$V_o(n) - V_o(n-1) = -\frac{C_1}{C_2} V_i(n-\tfrac{1}{2})$$

$$\sum_n [V_o(n) - V_o(n-1)] = -\frac{C_1}{C_2} \sum_n V_i(n-\tfrac{1}{2})$$

$$\mathcal{Z} \updownarrow$$

$$V_o(z) = -\frac{C_1}{C_2} \frac{z^{-1/2}}{(1-z^{-1})} V_i(z)$$

Realized $s \leftrightarrow z$ domain mapping

$$\boxed{s \longrightarrow \frac{1}{T} \frac{(1-z^{-1})}{z^{-1/2}}}$$ LDI z-Transform

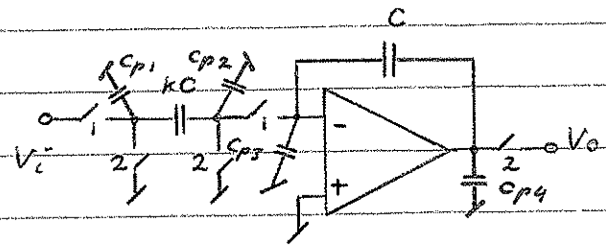
Integrator constant

$$\underline{\frac{1}{R_1 C_2} \longrightarrow \frac{1}{T} \frac{C_1}{C_2}}$$

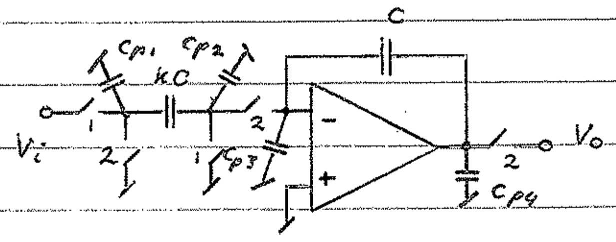
$$\omega_I \longrightarrow f_c \frac{C_1}{C_2}$$

The parasitic insensitive Integrator

inverting



noninverting



we assume amplifier gain $A_0 \gg 1$

Influence of the different parasitics:

c_{p1}: does not contribute to charge balance at ampl. input node

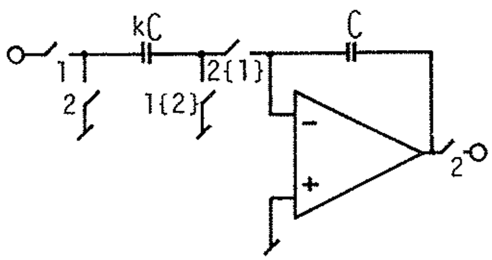
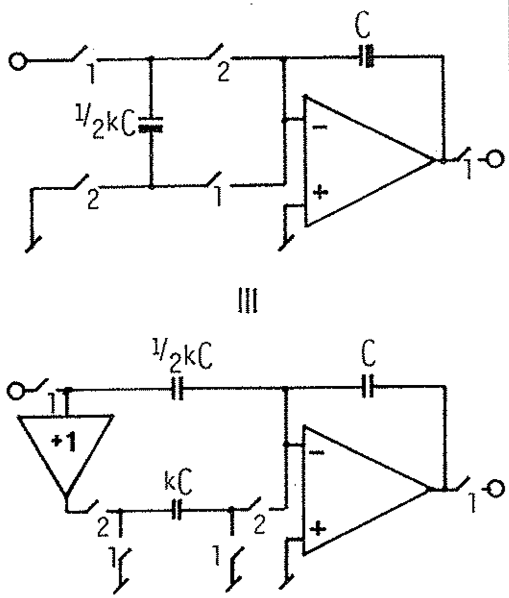
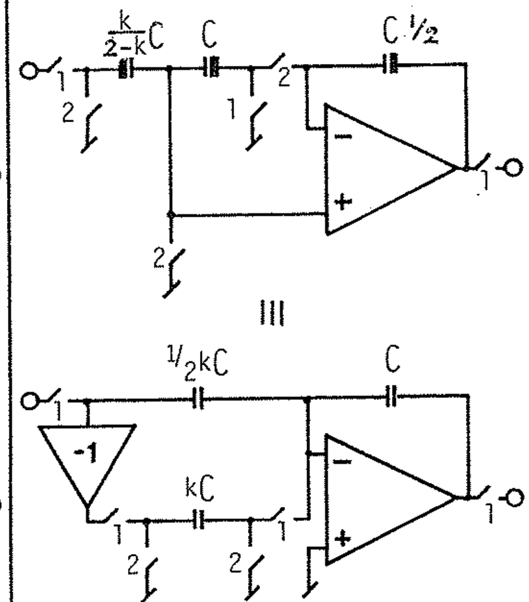
c_{p2}: switched between ground and virtual ground
$\Rightarrow$ does not accumulate charge

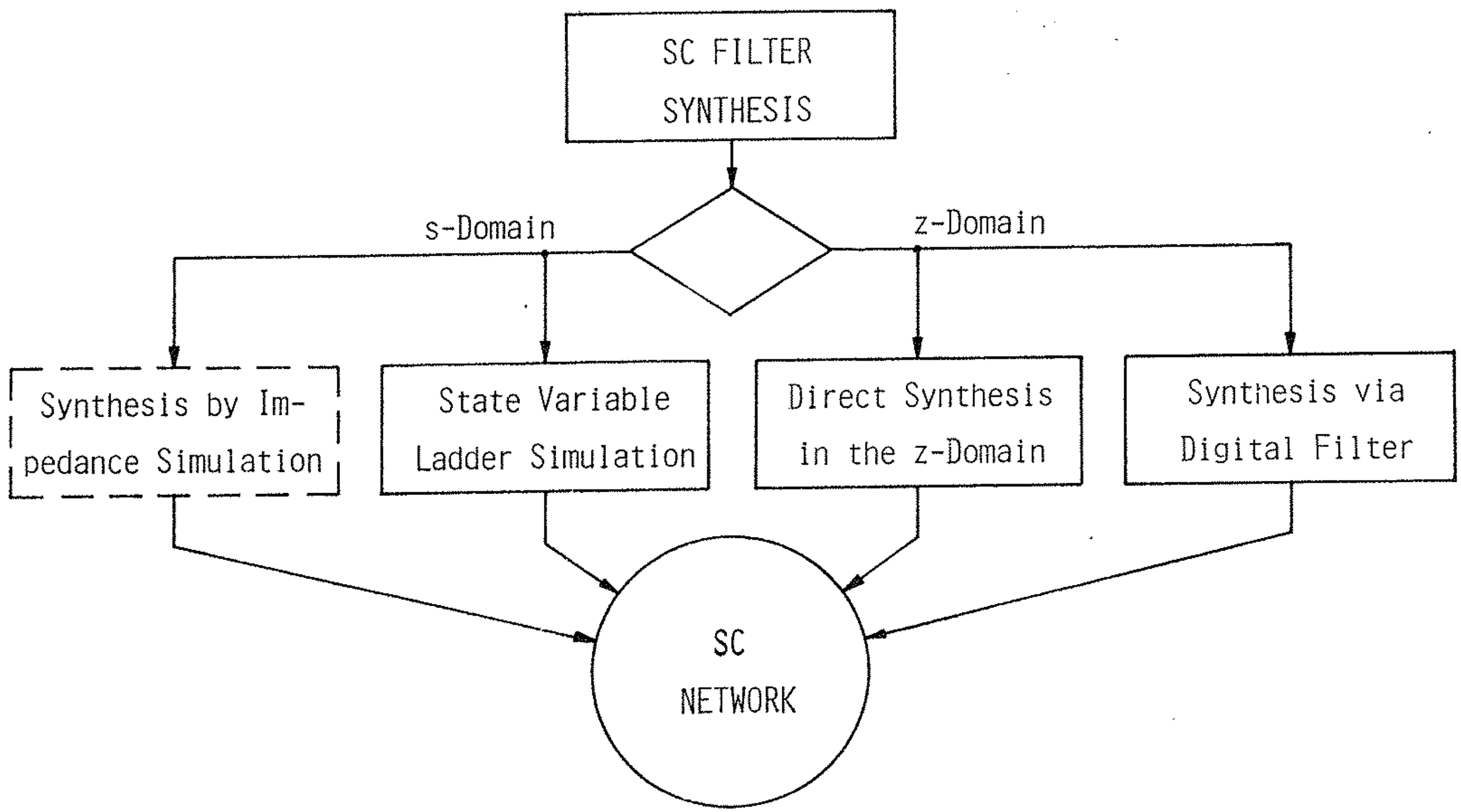
c_{p3}: connected to virtual ground (amp. neg. input)
$\Rightarrow$ does not accumulate charge

c_{p4}: Charged by ampl. to V_0 but does not contribute to charge balance at amp input

$$\Rightarrow \quad \underline{H^-(z) = -k\frac{z^{-1/2}}{(1-z^{-1})}} \qquad\qquad \underline{H^+(z) = k\frac{z^{-1/2}}{(1-z^{-1})}}$$

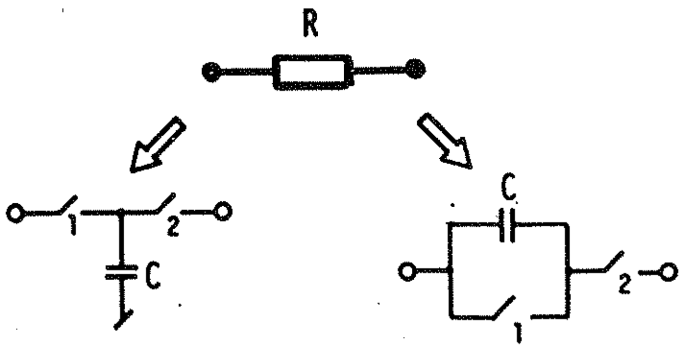
Rule to obtain parasitic insensitive circuits:
All effective capacitors in a SC network must be connected to nodes which are
grounded or virtually grounded (amplifier input)
or connected to a voltage source (amplifier output)

LOSSLESS DISCRETE INTEGRATOR	BI-LINEAR INTEGRATOR	
		
$H(z) = \pm k \dfrac{z^{-1/2}}{1-z^{-1}}$	$H(z) = -k \dfrac{1}{2} \dfrac{1+z^{-1}}{1-z^{-1}}$	$H(z) = +k \dfrac{1}{2} \dfrac{1+z^{-1}}{1-z^{-1}}$


SC FILTER
SYNTHESIS
s-Domain
z-Domain
Synthesis by Impedance Simulation
State Variable Ladder Simulation
Direct Synthesis in the z-Domain
Synthesis via Digital Filter
SC
NETWORK

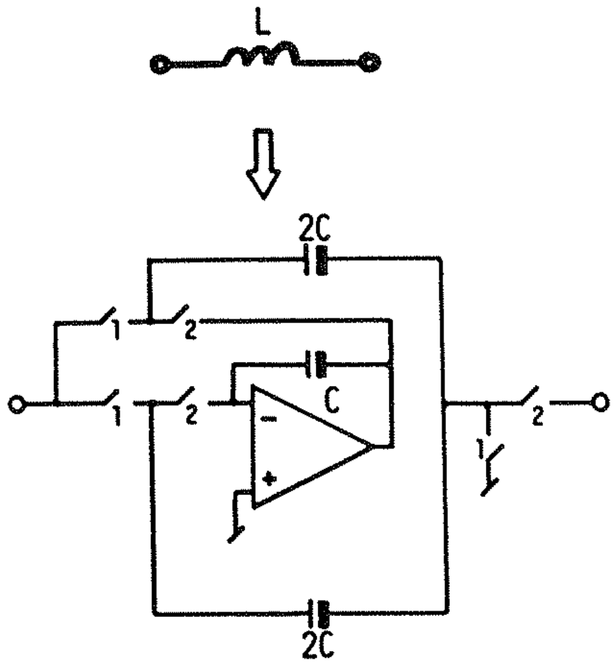
1. Synthesis by Impedance Simulation

1.1 Resistance



$$\boxed{R = \frac{T}{C} = \frac{1}{C f_c}}$$

1.2 Inductance



$$\boxed{L = \frac{T^2}{16C}}$$

Synthesis by Impedance Simulation

Advantages:

- simple Synthesis Procedure
- requires few active Elements (Amplifiers)

Disadvantages:

- SC Circuits are adversly affected by Stray-Capacitances (top- and bottom-plate Parasitics).
- requires always a Predistortion Procedure (even for very high rates of Sampling- to Signal Frequency).
- may yield unstable SC Circuits.

2. Synthesis of SC Ladder Filters

General Procedure

i) Start with general doubly terminated LC-R ladder filter.

ii) Reduce given network to minimum reactance structure (by introducing controlled voltage-sources).

iii) Realize remaining reactive elements by SC integrators and controlled sources by SC summers.

iv) Calculate capacitor ratios according to the relationship between time continuous integrator and chosen SC realization.

Simulation of LC-π Ladder Filters

Example 3^{rd} order Lowpass Filter

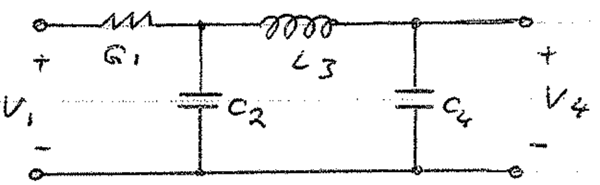
cut-off frequency: $f_c = 1\,kHz$

sampling rate: $f_s = 16\,kHz$

1. Prewarping of cut-off frequency for LDI z-transform

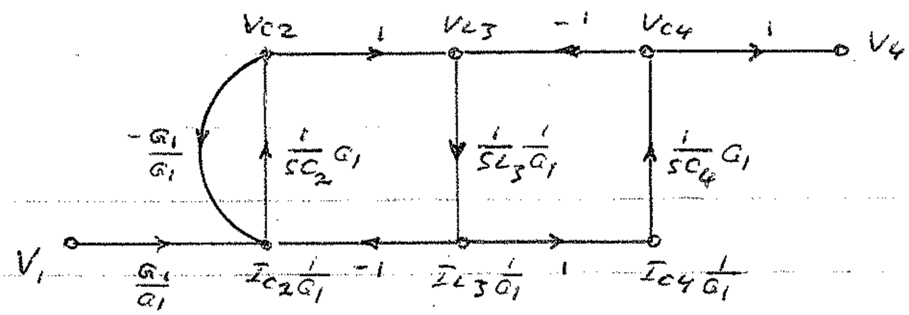
$$\boxed{f_c' = \frac{f_s}{\pi} \sin\left[\pi \frac{f_c}{f_s}\right] = 0.994\,kHz}$$

2. LC-π Prototype Filter



$G_1 = 1\Omega$
$C_2 = 1.621 \times 10^{-4}\,F$
$L_3 = 2.136 \times 10^{-4}\,H$
$C_4 = 2.417 \times 10^{-4}\,F$

3. SFG in s-domain



Impedance scaling by G_1

NW equations:

$$V_{C2} = I_{C2}\frac{1}{sC_2}$$

$$I_{C2} = (V_1 - V_{C2})G_1 - I_{L3}$$

$$I_{L3} = V_{L3}\frac{1}{sL_3}$$

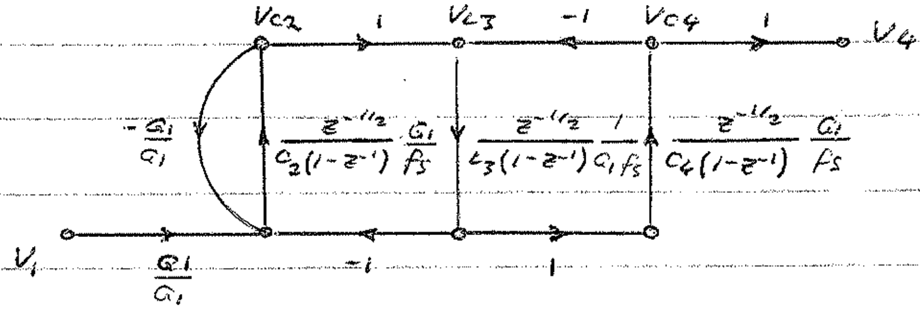
$$V_{L3} = V_{C2} - V_{C4}$$

$$V_{C4} = I_{C4}\frac{1}{sC_4}$$

$$I_{C4} = I_{L3}$$

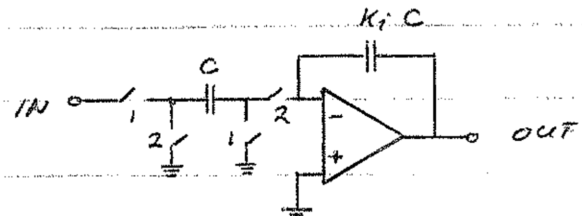
$$V_4 = V_{C4}$$

4. SFG in z-domain



Frequency scaling by f_s

5. Implementation of discrete-time integrator (LDI equivalent)



equivalent integrator constant:

$$K_i = \begin{cases} \frac{f_s}{G_1} C_i & i \text{ even} \\ f_s G_1 L_i & i \text{ odd} \end{cases}$$

6. equivalent element values:

passive	active SC
$G_1 = 1\Omega^{-1}$	$G_{in} = G_1 / G_1 = 1$
$C_2 = 1.621 \times 10^{-4}$ F	$K_2 = \frac{f_s}{G_1} C_2 = 2.593$
$L_3 = 2.156 \times 10^{-4}$ F	$K_3 = f_s G_1 L_3 = 3.417$
$C_4 = 2.417 \times 10^{-4}$ F	$K_4 = \frac{f_s}{G_1} C_4 = 3.867$

7. Implementation of resistive termination (G_1)

Problem: Input loop in z-domain SFG comprising G_1 and C_2 is not realizable as LDI equivalent circuit, since each loop delay must be an integer multiple of the sampling period.

Solution ①: Multiply G_1 by $z^{+1/2}$

$$G_1 \to G_1 z^{1/2} = G_1 (\cos\omega T/2 + j\sin\omega T/2)$$

Solution ②: Multiply G_1 by $z^{-1/2}$

$$G_1 \to G_1 z^{-1/2} = G_1 (\cos\omega T/2 - j\sin\omega T/2)$$

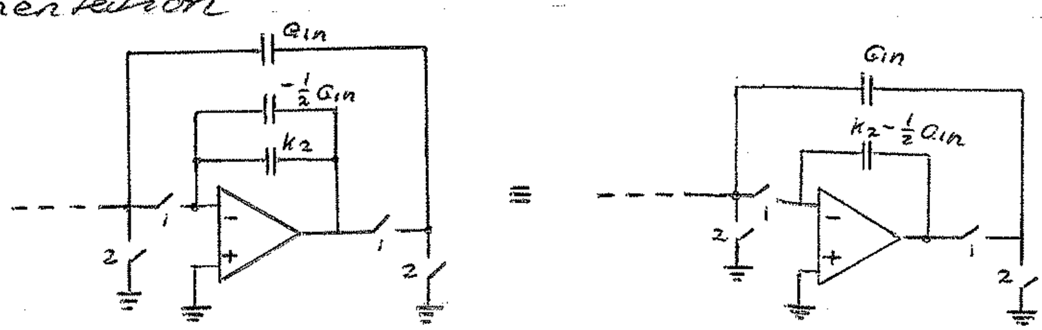
Note: In both cases the passive real termination G_1 becomes complex.

Solution ③: Combine solution ① and ② to eliminate a complex termination by multiplying G_1 by $\frac{1}{2}(z^{1/2} + z^{-1/2})$

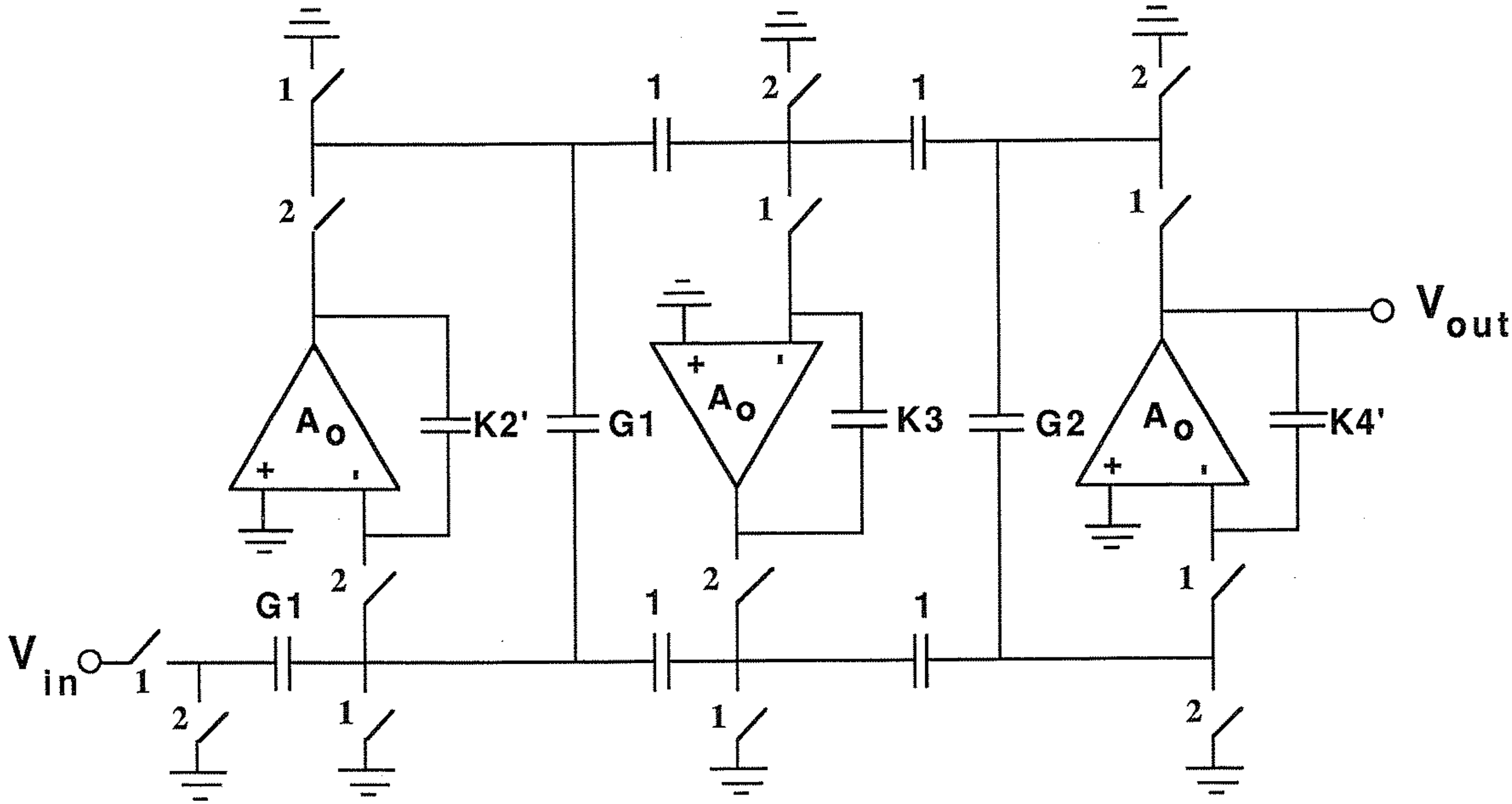
$$G_1 \to G_1 \frac{1}{2}(z^{1/2} + z^{-1/2}) = G_1 \cos\omega T/2$$

(bilinear termination)

SC Implementation



3rd Order SC Ladder Filter



$K2' = K2 - G1/2$

$K4' = K4 - G2/2$

Example ②: 3rd order lowpass with finite zero

Filter specs: Passband edge: $f_1 = 1kHz$

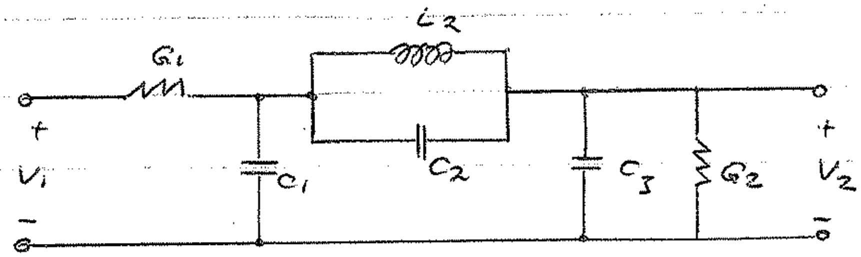
Passband ripple: $\leq 0.5 dB$

Stopband edge: $f_2 = 3kHz$

Stopband loss: $\geq 40 dB$

Sampling rate: $f_S = 20kHz$

1. R-LC Prototype Filter



normalized element values: ($G_n = G_1$; $f_n = f_S$)

$$\left| \begin{array}{l} G_1 = G_2 = 1 \\ C_1 = 4.8848 \\ L_2 = 3.2807 \\ C_2 = 0.2611 \\ C_3 = 4.8848 \end{array} \right|$$

2. Elimination of serial capacitor C_2

NW equations: $I_{C2} = (V_{C1} - V_{C3})\, sC_2$ (1)

C1 branch: C3 branch

$$\left| \begin{array}{l} I_{C1} = I_{G1} - I_{L2} - I_{C2} \\ V_{C1} = I_{C1} \frac{1}{sC_1} \end{array} \right| \qquad \left| \begin{array}{l} I_{C3} = I_{L2} + I_{C2} - I_{G2} \\ V_{C3} = I_{C3} \frac{1}{sC_3} \end{array} \right| \qquad \begin{array}{r} (2) \\ (3) \end{array}$$

Eliminating I_{C2} and plugging eq. (2) into (3) yields:

for C1 branch: for C_3 branch:

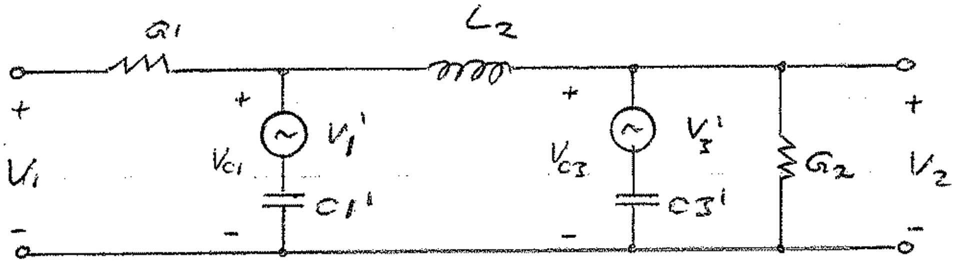
$$V_{C1} = (I_{a1} - I_{L2})\frac{1}{sC_1} - (V_{C1} - V_{C3})\frac{C_2}{C_1} \qquad V_{C3} = (I_{L2} - I_{a2})\frac{1}{sC_3} - (V_{C3} - V_{C1})\frac{C_2}{C_3}$$

$$V_{C1} = (I_{a1} - I_{L2})\frac{1}{s(C1+C2)} + V_{C3}\frac{C2}{C1+C2} \qquad V_{C3} = (I_{L2} - I_{a2})\frac{1}{s(C_2+C_3)} + V_{C1}\frac{C2}{(C2+C3)}$$

Finally, we substitute $(C1+C2)$ by C_1' and (C_2+C_3) by C_3'. We can then write

$$V_{C1} = (I_{a1} - I_{L2})\frac{1}{sC_1'} + V_{C3}\frac{C2}{C_1'} \qquad V_{C3} = (I_{L2} - I_{a2})\frac{1}{sC_3'} + V_{C1}\frac{C2}{C_3'}$$

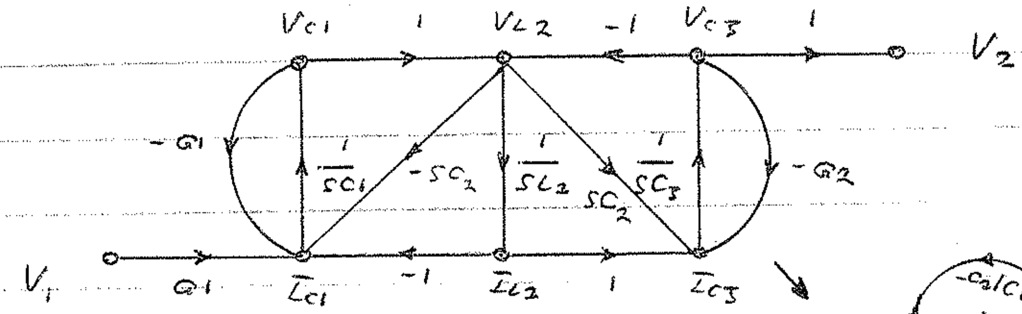
The equivalent circuit of this modified system is shown below

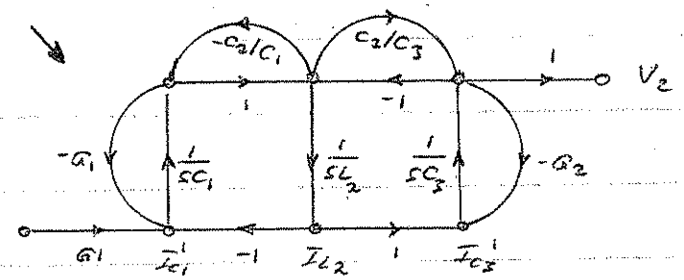


where: $C_1' = C1 + C2$ $C3' = C2 + C3$ $V_1' = V_{C3}\frac{C2}{C1'}$ $V_3' = V_{C1}\frac{C2}{C_3'}$

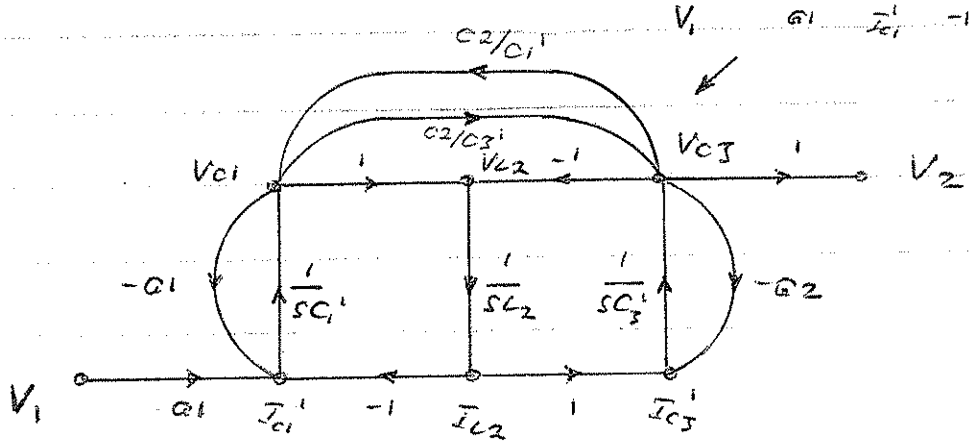
Note: This modified circuit contains only 3 reactive elements and can thus be realized by a 3 integrator circuit

3. Modified SFG





|||



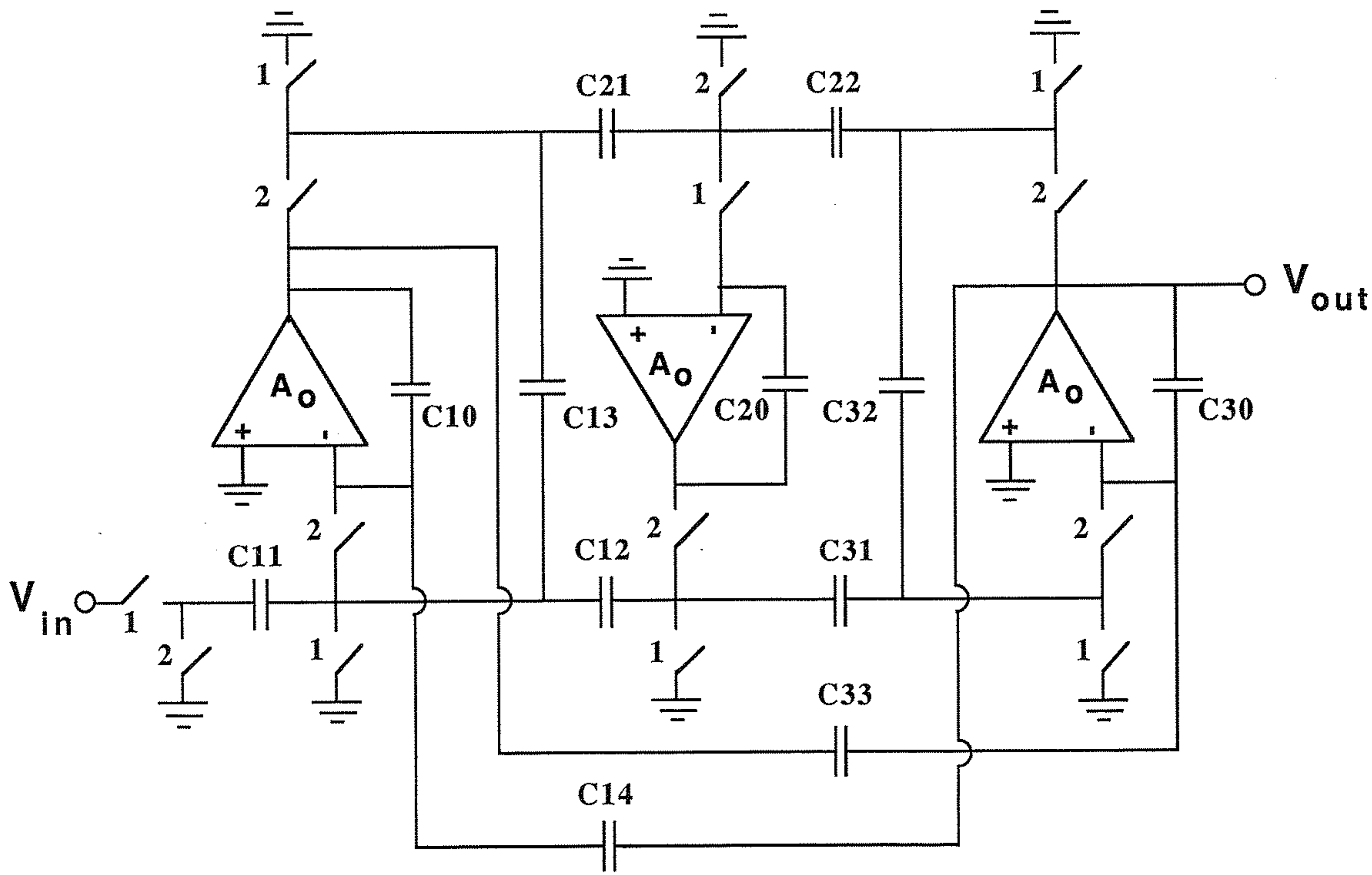
$$C_1' = C_1 + C_2$$
$$C_3' = C_3 + C_2$$

4. SC Implementation (LDI equivalent solution)

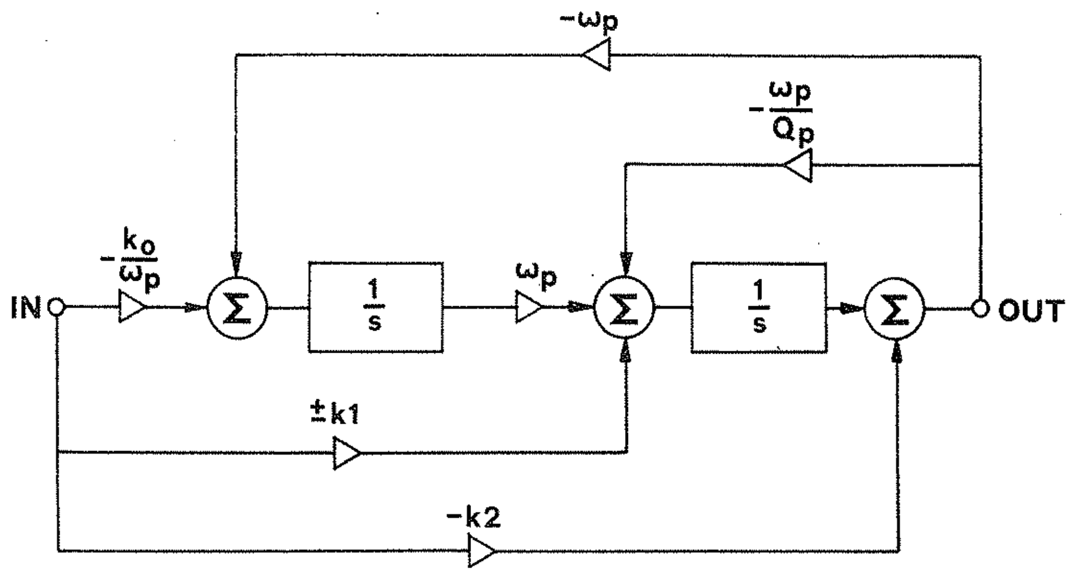
Element correspondences: (see Fig. next page)

$C10 = C_1' - \frac{1}{2}G_1$	$C20 = L_2$	$C30 = C_3' - \frac{1}{2}G_2$
$C11 = G1$	$C21 = 1$	$C31 = 1$
$C12 = 1$	$C22 = 1$	$C32 = G2$
$C13 = G1$		$C33 = C2$
$C14 = C2$		

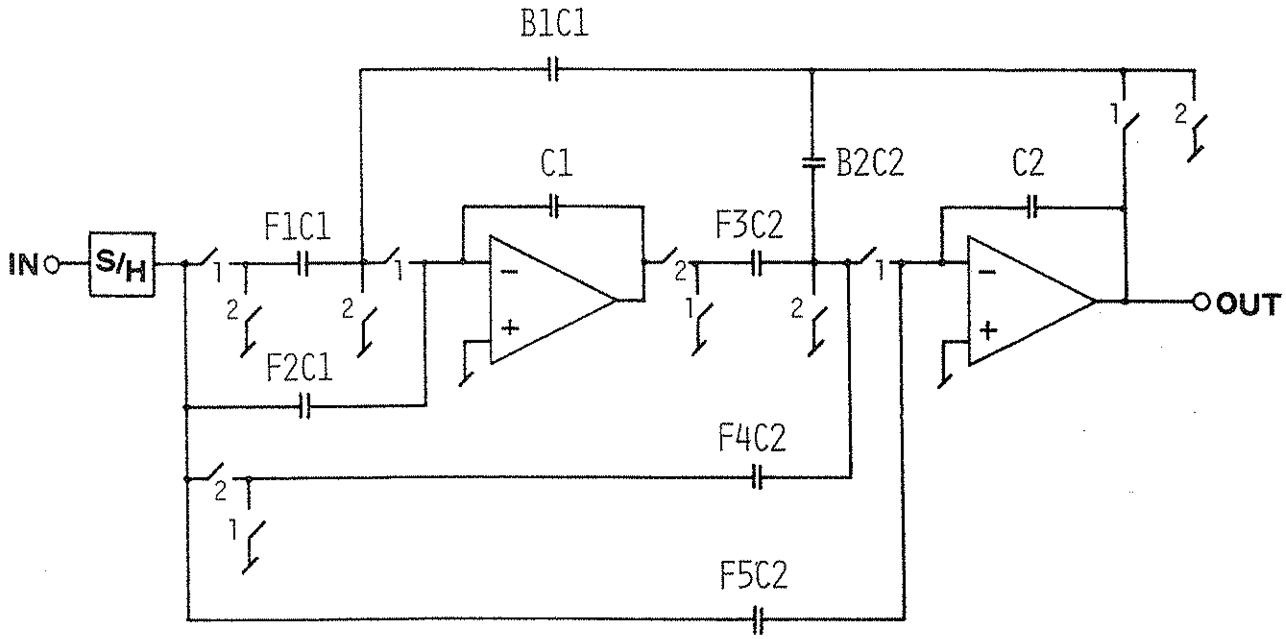
3rd Order SC Ladder Filter with Finite Zero



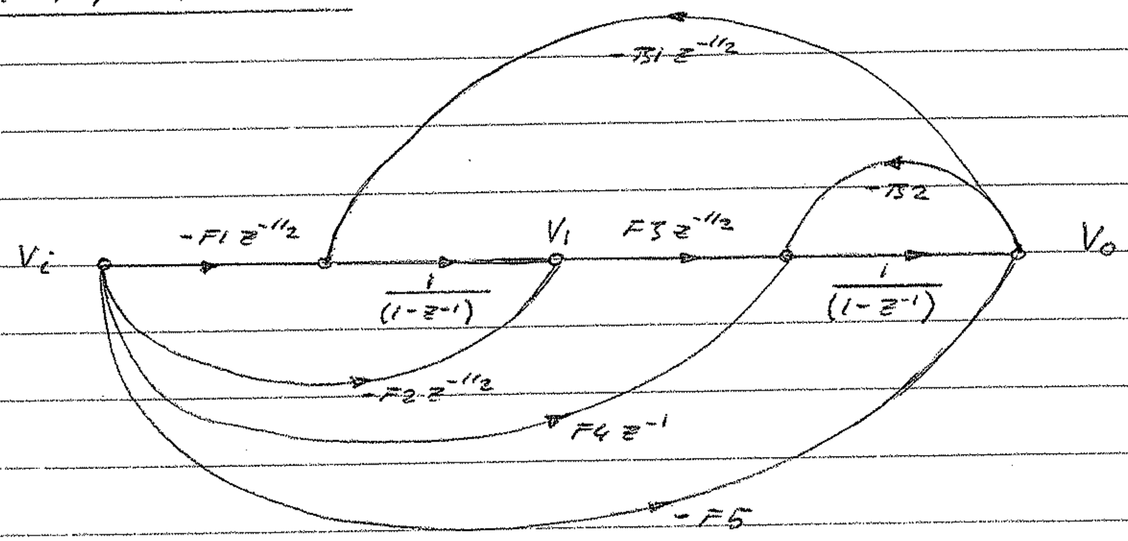
5. Direct Synthesis in the z-domain



SC Biquad



SFG representation:



$$\frac{V_o}{V_i} = \frac{-F1F3z^{-1} - F2F3z^{-1}(1-z^{-1}) + F4z^{-1}(1-z^{-1}) - F5(1-z^{-1})^2}{(1-z^{-1})^2 + B1F3z^{-1} + B2(1-z^{-1})}$$

$$\frac{V_o}{V_i} = \frac{F5}{(1+B2)} \frac{\left(1 - z^{-1}\left[2 - \frac{F1F3+F2F3-F4}{F5}\right] + z^{-2}\left[1 - \frac{F2F3-F4}{F5}\right]\right)}{\left(1 - z^{-1}\left[2 - \frac{B1F3+B2}{1+B2}\right] + z^{-2}\frac{1}{(1+B2)}\right)} \qquad (1)$$

Biquadratic Function in s

$$\frac{V_o}{V_i} = -K_0 \frac{\left(\omega_z^2 \pm s\frac{\omega_z}{Q_z} + s^2\right)}{\left(\omega_p^2 + s\frac{\omega_p}{Q_p} + s^2\right)}$$

$\Big\downarrow$ MZT

$$\frac{V_o}{V_i} = -K_0 \frac{\left(1 - z^{-1} 2e^{\mp\sigma_z T}\cos\tilde{\omega}_z T + z^{-2}e^{\mp 2\sigma_z T}\right)}{\left(1 - z^{-1} 2e^{-\sigma_p T}\cos\tilde{\omega}_p T + z^{-2}e^{-2\sigma_p T}\right)} \qquad (2)$$

where $\sigma_p = \frac{\omega_p}{2Q_p}$ $\quad \tilde{\omega}_p = \omega_p\sqrt{1-\frac{1}{4Q_p^2}}$ $\quad T$: clock cycle

$\sigma_z = \frac{\omega_z}{2Q_z}$ $\quad \tilde{\omega}_z = \omega_z\sqrt{1-\frac{1}{4Q_z^2}}$

compare coefficients of equal powers in (1) and (2)

$$\boxed{\begin{aligned} \frac{F5}{1+B2} &= K_0 \\ 1-\frac{F1F3+F2F3-F4}{2F5} &= e^{\mp\sigma_z T}\cos\tilde{\omega}_z T \\ 1-\frac{F2F3-F4}{F5} &= e^{\mp 2\sigma_z T} \\ 1-\frac{B1F3+B2}{2(1+B2)} &= e^{-\sigma_p T}\cos\tilde{\omega}_p T \\ \frac{1}{1+B2} &= e^{-2\sigma_p T} \end{aligned}}$$

5 equations
7 unknowns

$\Rightarrow$ 2 free parameters

For optimum dynamic (both amps exhibit equal max. swing)
$\underline{B1 \cong F3}$ (better: $B1 = F3\frac{Q_p}{1+Q_p}$) for lowpass only

Minimum phase network
(zeros in left half plane)
$\sigma_z \le 0 \Rightarrow \underline{F4 = 0}$

$$\boxed{\begin{aligned} B2 &= e^{2\sigma_p T}-1 \\ F3^2 = B1^2 &= 2e^{2\sigma_p T}-2e^{\sigma_p T}\cos\tilde{\omega}_p T - B2 \\ F5 &= k_0 e^{2\sigma_p T} \\ F2 &= \frac{F5}{F3}[1-e^{-2\sigma_z T}] \\ F1 &= \frac{2F5}{F3}[1-e^{-\sigma_z T}\cos\tilde{\omega}_z T]-F2 \end{aligned}}$$

Nonminimum phase network
(zeros in right half plane)
$\sigma_z > 0 \Rightarrow \underline{F2 = 0}$

$$\boxed{\begin{aligned} B2 &= e^{2\sigma_p T}-1 \\ F3^2 = B1^2 &= 2e^{2\sigma_p T}-2e^{\sigma_p T}\cos\tilde{\omega}_p T - B2 \\ F5 &= k_0 e^{2\sigma_p T} \\ F4 &= F5[e^{2\sigma_z T}-1] \\ F1 &= \frac{2F5}{F3}[1-e^{\sigma_z T}\cos\tilde{\omega}_z T]+F4 \end{aligned}}$$

Example ① Design a 2nd order lowpass with

$f_p = 1\,kHz$; $Q_p = 1$; Unity Gain at $\omega = 0$

$f_s = 16\,kHz$

a) Transfer function in s and in z:

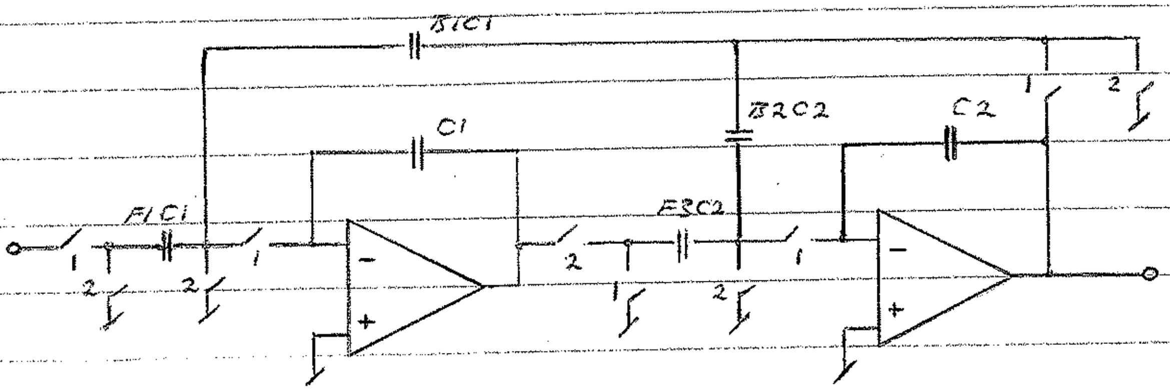
$$H(s) = -\frac{\omega_p^2}{\omega_p^2 + s\frac{\omega_p}{Q_p} + s^2}$$

MZT

$$H(z) = -\frac{1 + e^{-\frac{\omega_p T}{Q_p}} - 2e^{-\frac{\omega_p T}{2Q_p}}\cos\left[\omega_p T\sqrt{1-\frac{1}{4Q_p^2}}\right]}{1 - z^{-1}\,2e^{-\frac{\omega_p T}{2Q_p}}\cos\left[\omega_p T\sqrt{1-\frac{1}{4Q_p^2}}\right] + z^{-2}e^{-\frac{\omega_p T}{Q_p}}}$$

$$= \frac{0.12591}{1 - z^{-1}\,1.54932 + z^{-2}\,0.67523}$$

b) Circuit Topology:



$$\boxed{H(z) = -\frac{F1F3\,z^{-1}}{(1+B2)\left(1 - z^{-1}\left[2 - \frac{B1F3+B2}{1+B2}\right] + z^{-2}\frac{1}{1+B2}\right)}}$$

c) Coefficient comparison:

Note that the additional factor z^{-1} in the z-domain transfer function numerator has no influence on the filter amplitude response, since it merely adds an excess linear phase

$$z^{-1} \longrightarrow e^{-j\omega T}$$

Therefore:

$$\boxed{\begin{aligned} \frac{F1F3}{1+B2} &= 0.12591 \\ 2-\frac{B1F3+B2}{1+B2} &= 1.54932 \\ \frac{1}{1+B2} &= 0.67523 \\ B1 &\tilde{=} F3 \end{aligned}}$$

~~(optimum dynamic)~~
not good for low-Q filters
better $\underline{B1 = \frac{Q_p}{1+Q_p} F3}$ (lowpass only)

Solution:

$$\boxed{\begin{aligned} F3 = B1 &= 0.43182 \\ F1 &= 0.43182 \\ B2 &= 0.48098 \end{aligned}}$$

If $B1 = \frac{Q_p}{1+Q_p} F3$

$B1 = F1 = 0.30531$
$B2 = 0.48098$
$F3 = 0.61063$

Capacitors:

$C_{B1} = 1$ (1)	$C_{B2} = 1.114$ (1)
$C_{F1} = 1$ (1)	$C_{F3} = 1$ (1.268)
$C_1 = 2.516$ (3.275)	$C_2 = 2.516$ (2.079)

( ) dynamically optimized

Example ② Design the same lowpass filter by means of the bilinear z-transform

a) Frequency prewarping

$$f_p' = \frac{f_s}{\pi}\, tg\left[\pi \frac{f_g}{f_s}\right] = \underline{\underline{1.013\ kHz}}$$

b) Transfer function in s and z-domain:

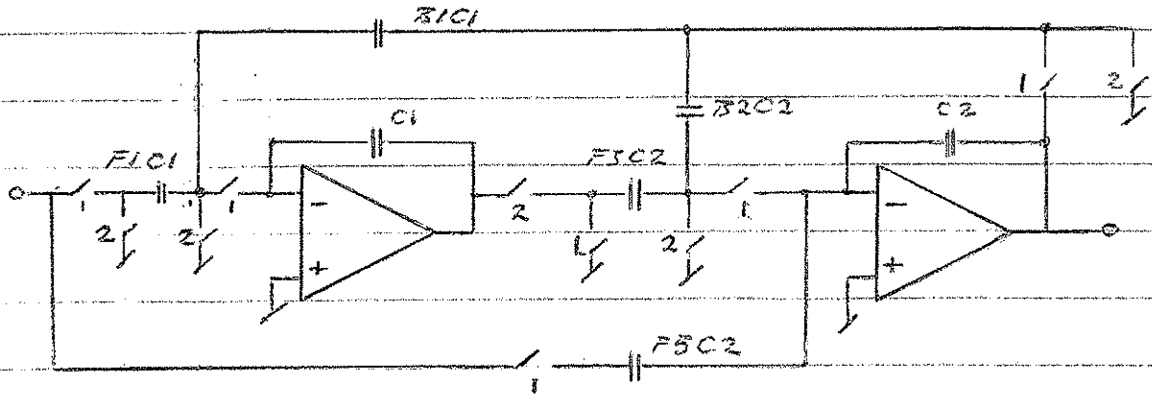
$$H(s) = -\frac{\omega_p'^2}{\omega_p'^2 + s\frac{\omega_p'}{a_p} + s^2}$$

$$\Big\downarrow BZT$$

$$H(z) = -\frac{\dfrac{\omega_p'^2 T^2}{4(1+a_pT)+\omega_p'^2T^2}\,(1+z^{-1})^2}{1 - z^{-1}\,2\dfrac{(4-\omega_p'^2T^2)}{4(1+a_pT)+\omega_p'^2T^2} + z^{-2}\,\dfrac{4(1-a_pT)+\omega_p'^2T^2}{4(1+a_pT)+\omega_p'^2T^2}}$$

$$= -\frac{0.031944\,(1+z^{-1})^2}{1 - z^{-1}\,1.55102 + z^{-2}\,0.67879}$$

c) Circuit Topology



$$H(z) = -\frac{F5\left(1 - z^{-1}\left[2 - \frac{F1F3}{F5}\right] + z^{-2}\right)}{(1+B2)\left(1 - z^{-1}\left[2 - \frac{B1F3+B2}{1+B2}\right] + z^{-2}\frac{1}{1+B2}\right)}$$

Thus:

$$\frac{F5}{1+B2} = 0.031944$$

$$\frac{F1F3}{F5} = 4$$

$$2 - \frac{B1F3+B2}{1+B2} = 1.55102$$

$$\frac{1}{1+B2} = 0.67879$$

$$B1 \cong F3$$

Solution:

$$B1 = F3 = 0.43387$$

$$B2 = 0.47321$$

$$F1 = 0.43387$$

$$F5 = 0.047060$$

Capacitors:

$C_{B1} = 1$	$C_{B2} = 10.055$
$C_{F1} = 1$	$C_{F3} = 9.220$
$C_1 = 2.505$	$C_{F5} = 1$
	$C_2 = 21.250$

Design of a 2nd order SC Bandpass Filter

Filter Specs:

$$f_p = 50\,\text{kHz} \qquad Q_p = 5$$
$$f_s = 1\,\text{MHz}$$

Transfer Function:

$$\boxed{H(z) = -\frac{F2\,F3}{(1+B2)}\;\frac{z^{-1}(1-z^{-1})}{\left(1 - z^{-1}\left[2 - \frac{B1F3 + B2}{1+B2}\right] + z^{-2}\frac{1}{1+B2}\right)}}$$

The above specs yields (MZT)

$$\boxed{H(z) = -0.06283\;\frac{z^{-1}(1-z^{-1})}{(1 - 1.84422\,z^{-1} + 0.93910\,z^{-2})}}$$

To balance the 2 amplifier output swings, we set $B1 = F3$

By comparing the coefficients of equal power in the above 2 z-domain transfer functions, we obtain:

$$\boxed{\begin{aligned} B1 &= F3 = 0.31785 \\ B2 &= 0.06485 \\ F2 &= 0.19768 \end{aligned}}$$

Cap Values:

$$\boxed{\begin{array}{ll} C1 = 5.059 & C2 = 15.420 \\ B1\,C1 = 1.608 & B2\,C2 = 1 \\ F2\,C1 = 1 & F3\,C2 = 4.901 \end{array}}$$

Example ③ Design a 2^{nd} order Bandpass with

$f_p = 1kHz$; $Q_p = 5$; Unity Gain in Passband

$f_S = 16kHz$

a) z-domain transfer function (MZT equivalent)

$$\boxed{H_{BP}(z) = -\frac{0.078540\,(1-z^{-1})}{1 - z^{-1}\,1.77660 + z^{-2}\,0.924465}}$$

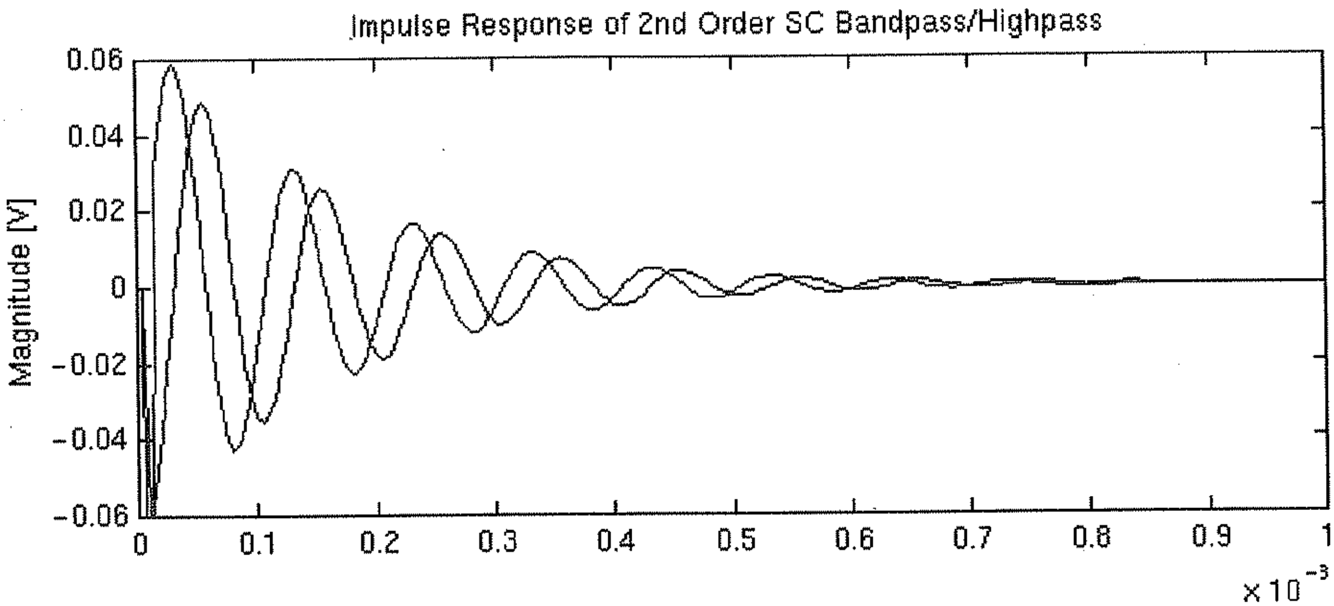
b) SC implementation

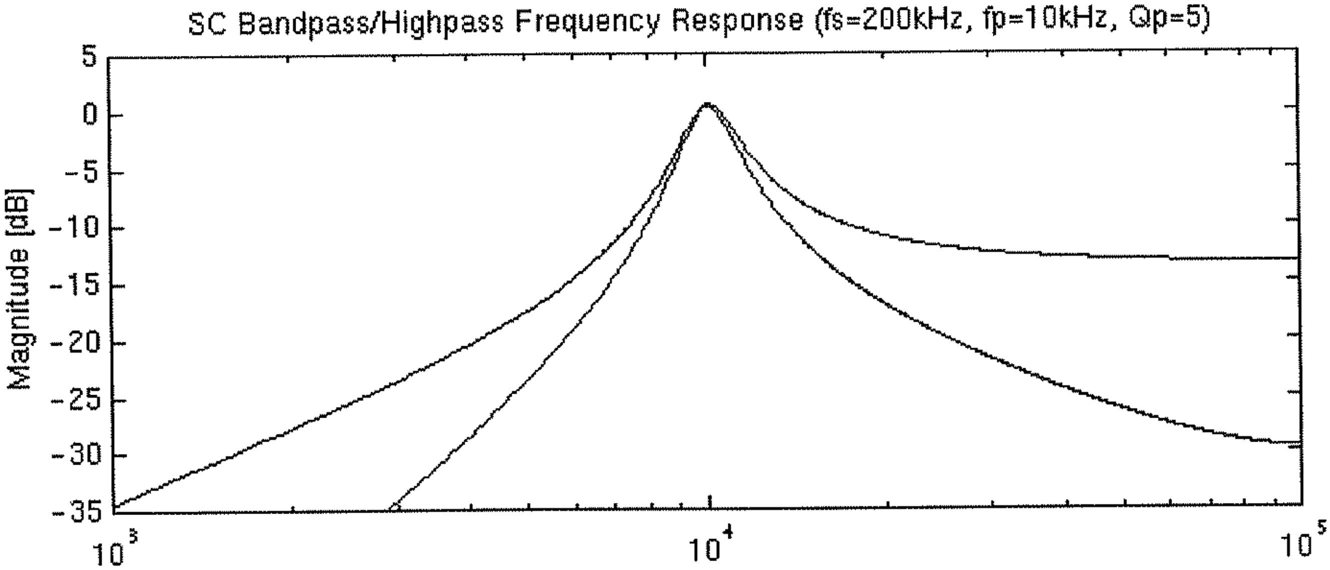
Select $F1 = F4 = F5 = 0$

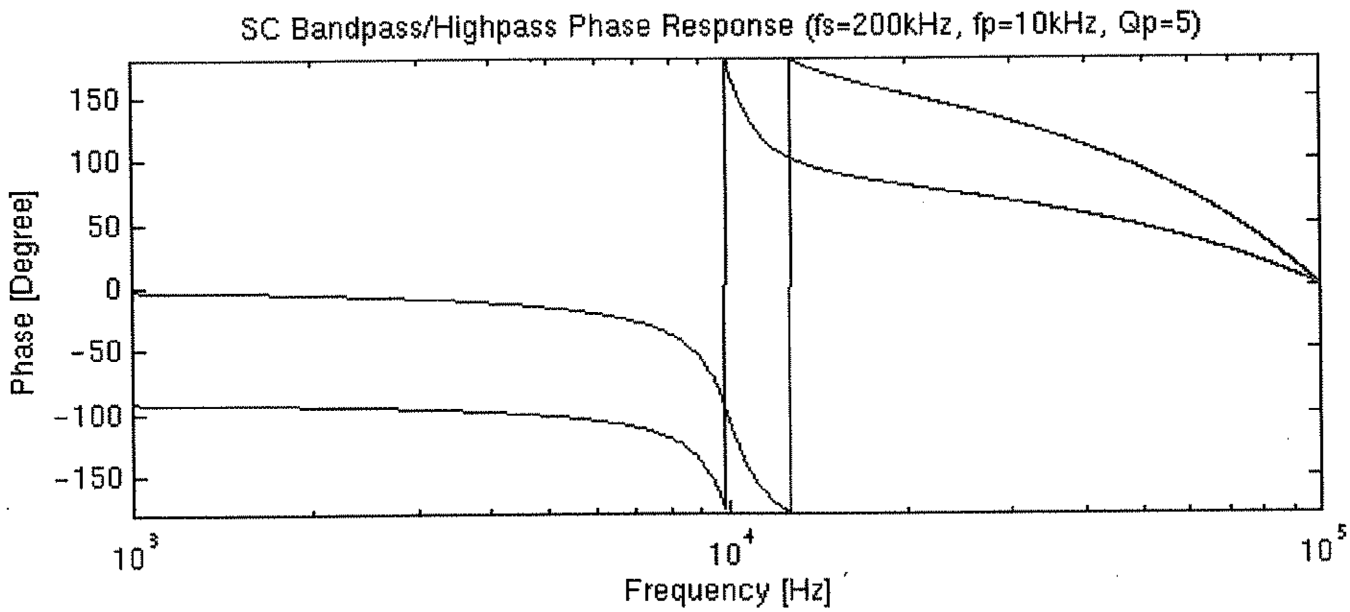
$$\boxed{H_{BP}(z) = -\frac{F2\,F3\,z^{-1/2}\,(1-z^{-1})}{(1+B2)\left(1 - z^{-1}\left[2 - \frac{B1F3 + B2}{(1+B2)}\right] + z^{-2}\frac{1}{(1+B2)}\right)}}$$

c) Result from coefficient comparison:

$B1 = 0.39993$	$\Rightarrow$	$C1 = 5.092$		$C2 = 12.239$
$B2 = 0.081707$		$CB1 = 2.036$		$CB2 = 1$
$F2 = 0.19638$		$CF2 = 1$		$CF3 = 4.895$
$F3 = 0.39993$				

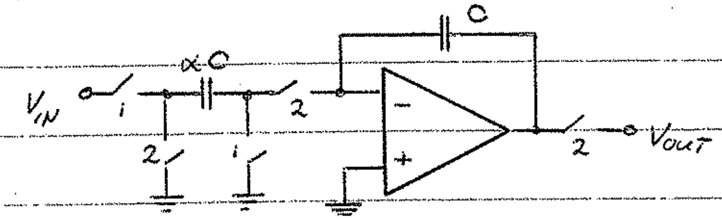

Impulse Response of 2nd Order SC Bandpass/Highpass
Magnitude [V]
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
$\times 10^{-3}$


SC Bandpass/Highpass Frequency Response (fs=200kHz, fp=10kHz, Qp=5)
Magnitude [dB]
5
0
-5
-10
-15
-20
-25
-30
-35
10^3
10^4
10^5


SC Bandpass/Highpass Phase Response (fs=200kHz, fp=10kHz, Qp=5)
Phase [Degree]
150
100
50
0
-50
-100
-150
10^3
10^4
10^5
Frequency [Hz]

C-Spread Reduction Techniques

Example Noninverting Integrator with $\alpha \ll 1$

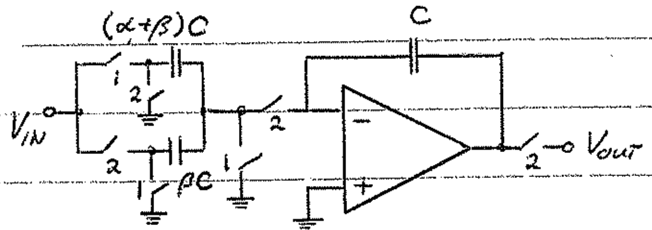


$$\boxed{\frac{V_{OUT}}{V_{IN}} = \frac{\alpha z^{-1/2}}{(1-z^{-1})}}$$

Solution A

Solution B

Difference Integrator



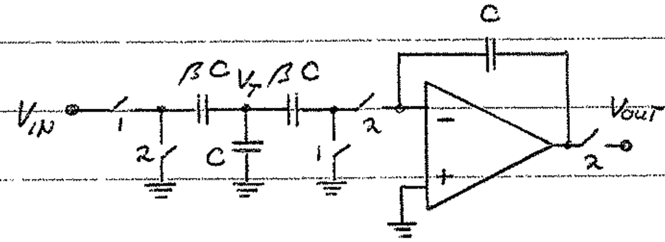
assumption: V_{IN} only changes during phase 1

$$\boxed{\begin{aligned}\frac{V_{OUT}}{V_{IN}} &= \frac{[(\alpha+\beta)-\beta]\, z^{-1/2}}{(1-z^{-1})} \\ &= \frac{\alpha z^{-1/2}}{(1-z^{-1})}\end{aligned}}$$

C-Spread $= \beta^{-1}$

e.g. $\alpha = 0.01$
select $\beta = 0.1$
$\therefore$ Spread $= 10$

Integrator with T-Section



$$\frac{V_{OUT}}{V_T} = \frac{\beta z^{-1/2}}{(1-z^{-1})}$$

$$\frac{V_T}{V_{IN}} = \frac{\beta}{1+2\beta}$$

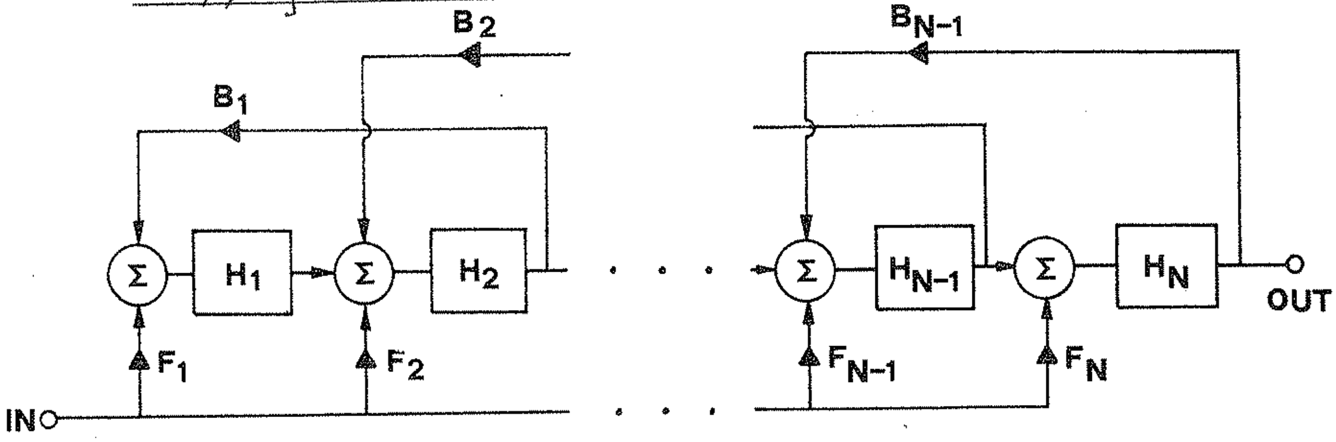
$$\therefore \boxed{\frac{V_{OUT}}{V_{IN}} = \frac{\beta^2}{(1+2\beta)} \frac{z^{-1/2}}{(1-z^{-1})}}$$

where $\boxed{\beta = \alpha\left[1+\sqrt{1+\frac{1}{\alpha}}\right]}$

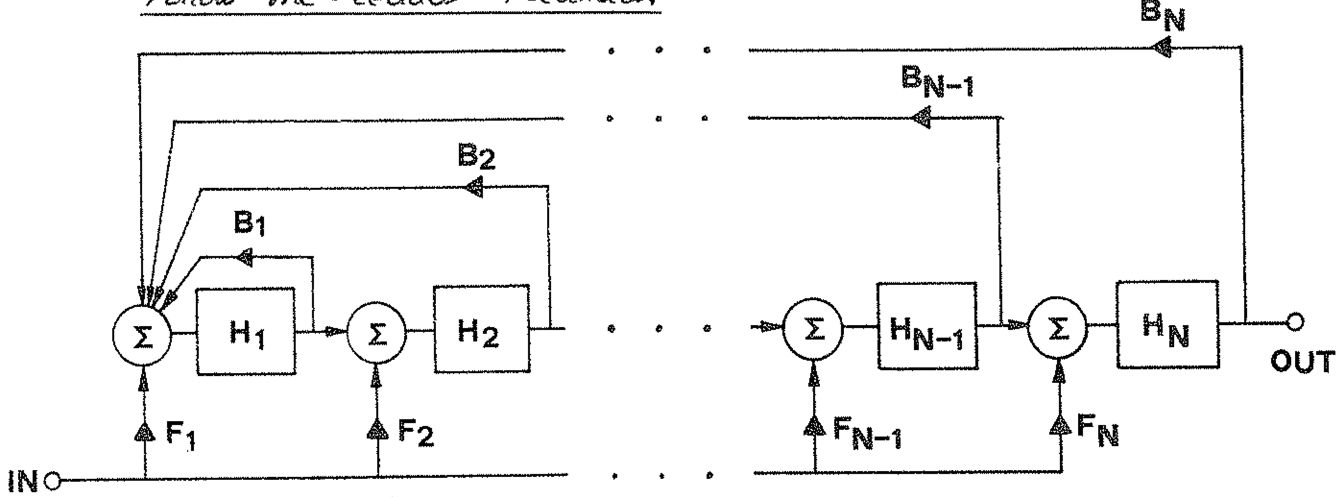
C-Spread $= \beta^{-1}$

e.g. $\alpha = 0.01$ $\therefore \beta = 0.1105$
$\therefore$ Spread $= 9.05$

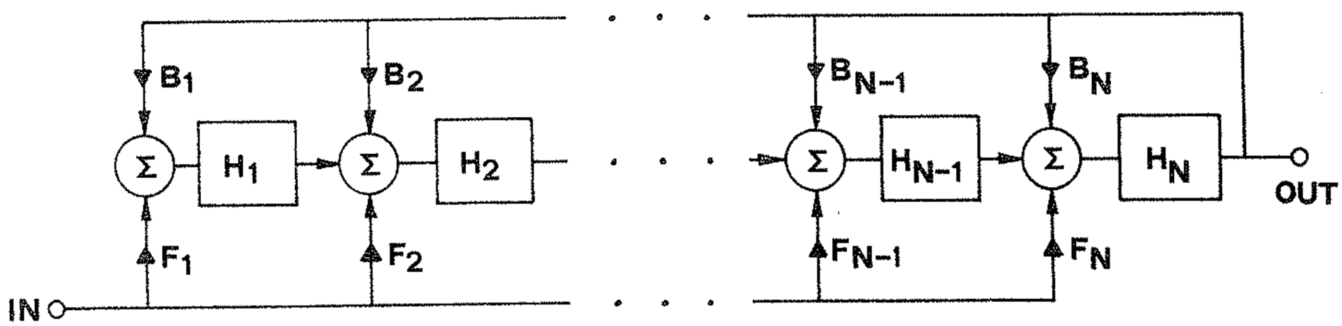
Leapfrog Structure



Follow-the-Leader Feedback

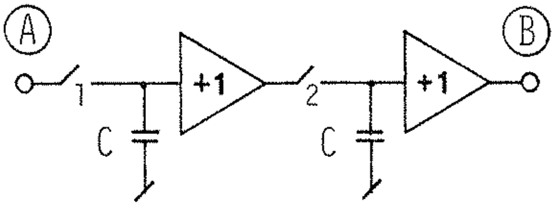


Inverse Follow-the-Leader Feedback

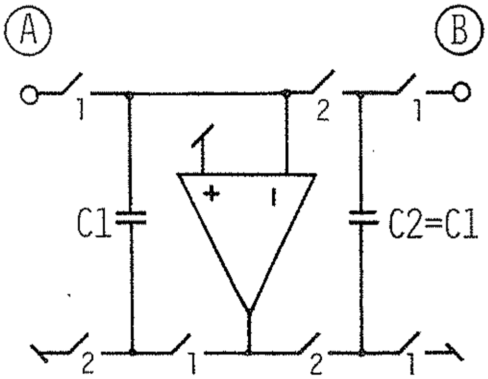


4. Synthesis via Digital Filter Architecture

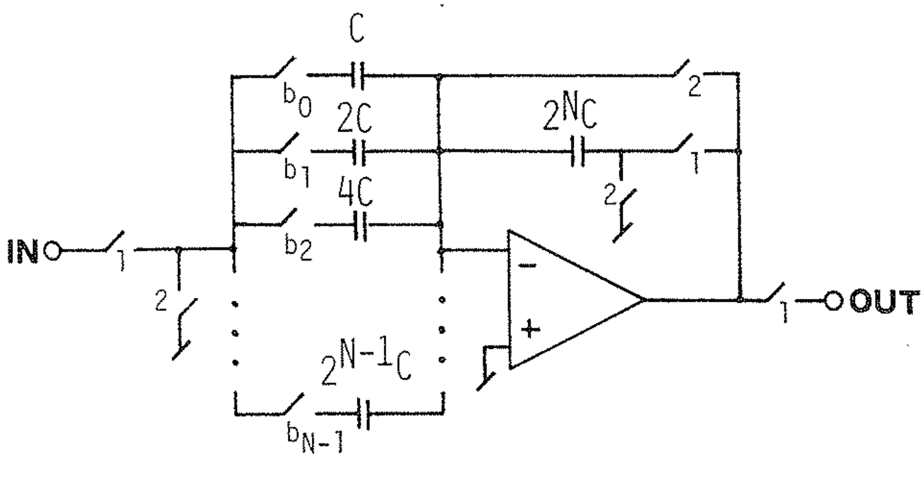
a) 2-opamp unit delay cell



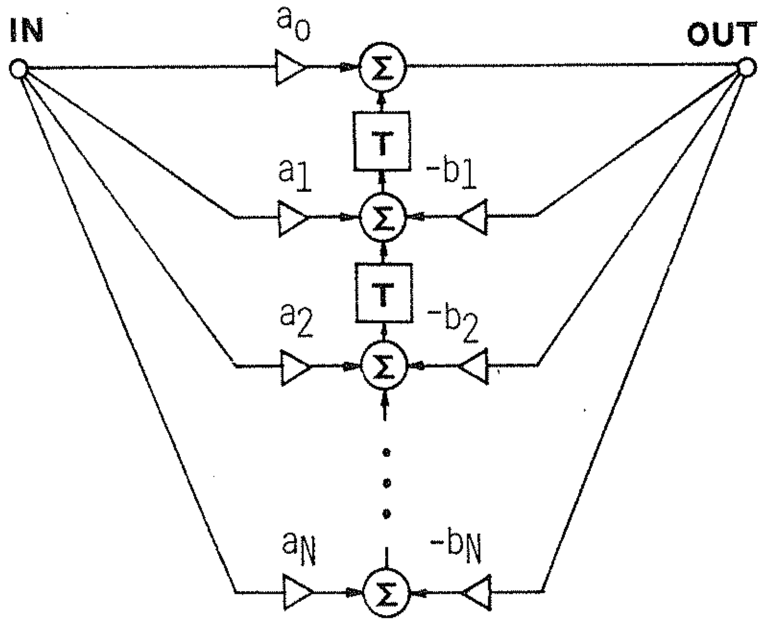
b) single opamp unit delay



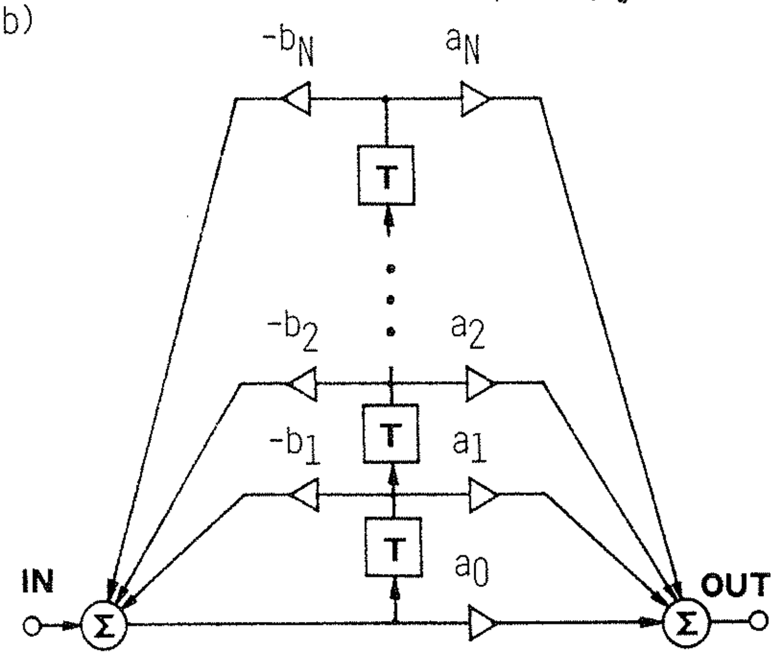
SC Multiplier (digitally controlled) ∴ mult. DAC



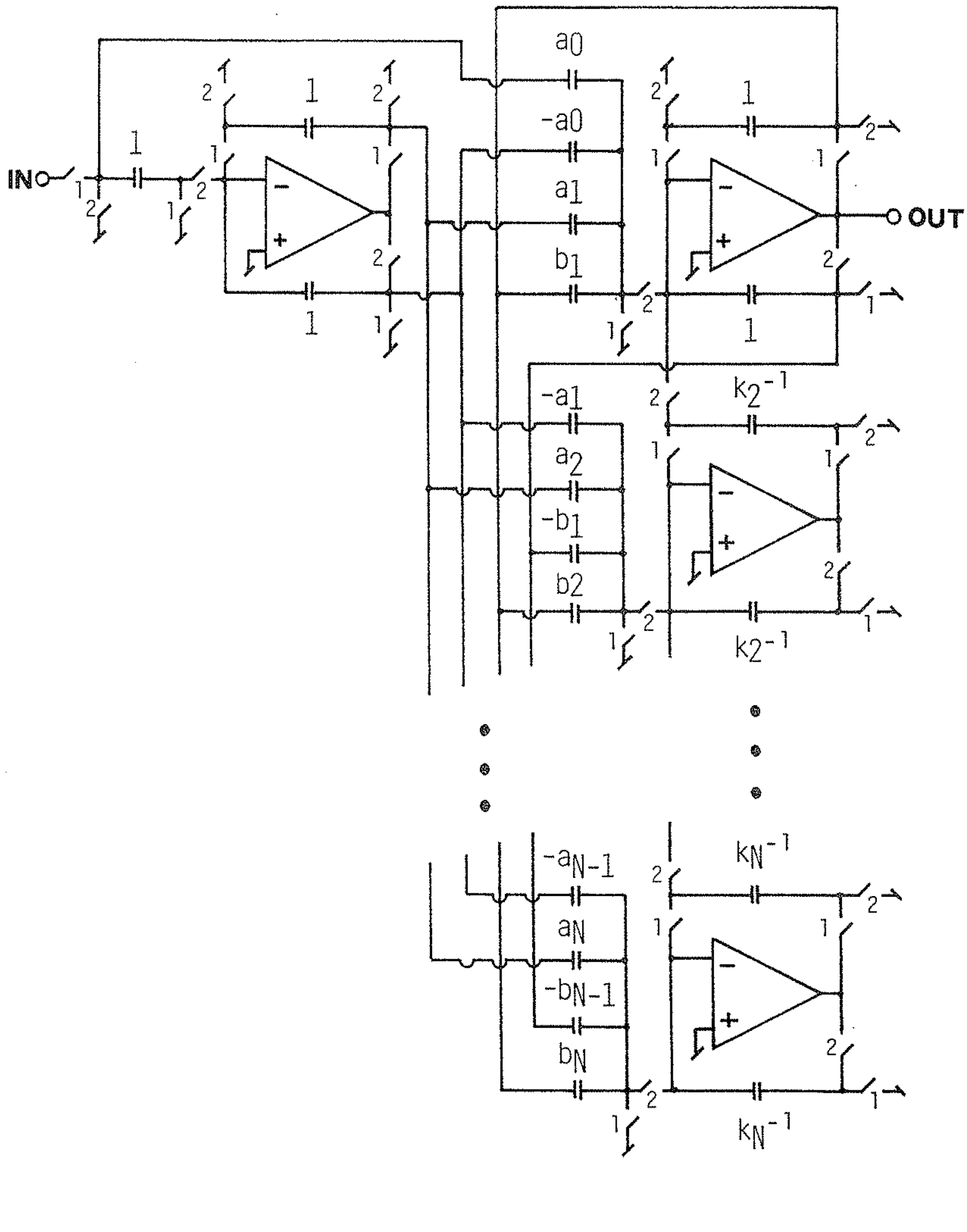
a) Direct Form I Topology



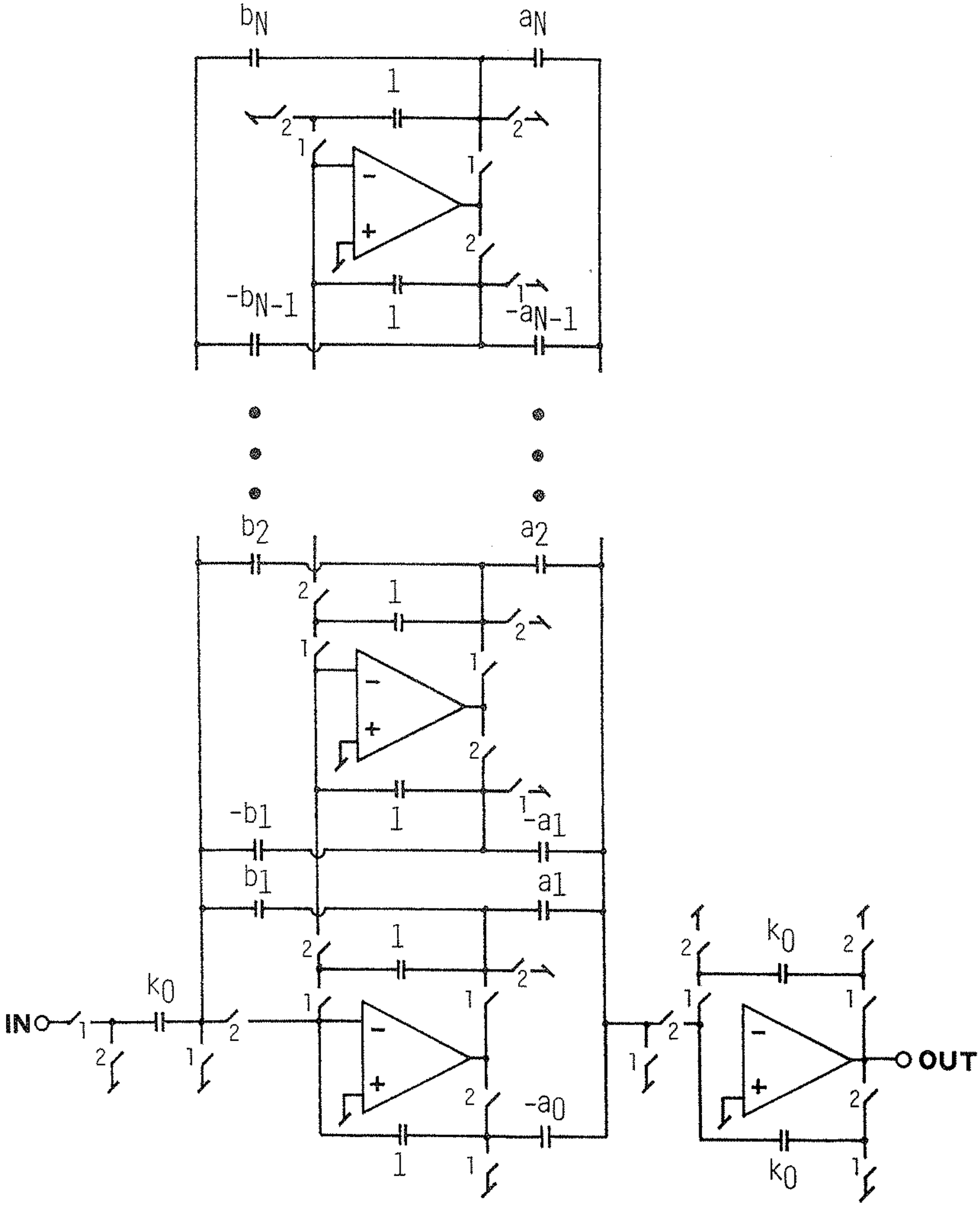
b) Direct Form II Topology



N-th order Circuit in Direct Form I Realization

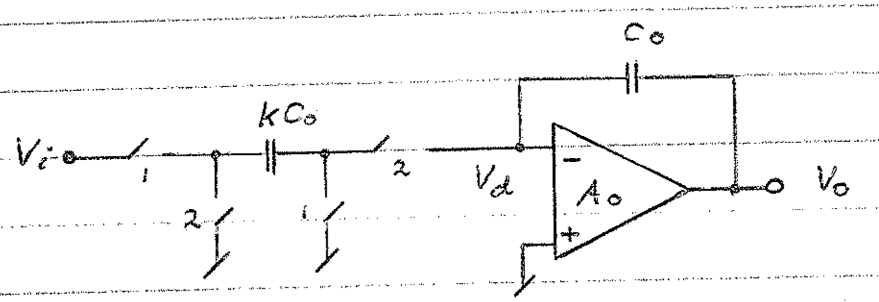


N-th order Circuit in Direct Form II Realization



Influence of the amplifier finite gain

discrete Integrator



$$\underline{V_o = -A_0 V_d}$$

Analysis

charge at end of φ_1 — charge at end of φ_2

Charge conservation: $Q_1 = Q_2$ $\qquad 1 \rightarrow 2$

$$\left| \begin{array}{l} Q_1 = Q_1(n+\tfrac{1}{2}) = kC_0 V_{i1} + C_0 (V_{o1} - V_{d1}) \\ Q_2 = Q_2(n+1) = -kC_0 V_{d2} + C_0 (V_{o2} - V_{d2}) \end{array} \right|$$

replace V_d by $-\frac{1}{A_0} V_o$

$$\left| \begin{array}{l} Q_1 = kC_0 V_{i1} + C_0 V_{o1} (1 + \frac{1}{A_0}) \\ Q_2 = \frac{k}{A_0} C V_{o2} + C_0 V_{o2} (1 + \frac{1}{A_0}) \end{array} \right|$$

V_o does not change during φ_1; i.e. $\underline{V_o(n+\tfrac{1}{2}) = V_o(n)}$

$$\left| \begin{array}{l} Q_1(n+\tfrac{1}{2}) = kC_0 V_i(n+\tfrac{1}{2}) + C_0 V_o(n) [1 + \frac{1}{A_0}] \\ Q_2(n+1) = \frac{k}{A_0} C_0 V_o(n+1) + C_0 V_o(n+1) [1 + \frac{1}{A_0}] \end{array} \right|$$

$\underline{Q_1 = Q_2}$

$$\Rightarrow k\cancel{C_0}V_i(n+\tfrac{1}{2}) + \cancel{C_0}V_0(n)\left[1+\tfrac{1}{A_0}\right] = \cancel{C_0}V_0(n+1)\left[1+\tfrac{1}{A_0}+\tfrac{k}{A_0}\right]$$

$$\Big\downarrow\ z\text{-Transform}$$

$$kV_i(z)\,z^{+1/2} + V_0(z)\left[1+\tfrac{1}{A_0}\right] = V_0(z)\,z^{+1}\left[1+\tfrac{1}{A_0}+\tfrac{k}{A_0}\right]$$

$$\Rightarrow \frac{V_0(z)}{V_i(z)} = \frac{k\cdot z^{1/2}}{z\left[1+\frac{1}{A_0}+\frac{k}{A_0}\right]-\left[1+\frac{1}{A_0}\right]}$$

$$= \underbrace{\frac{k\,z^{-1/2}}{(1-z^{-1})}}_{\text{Ideal}=H_{id}(z)}\ \underbrace{\frac{(1-z^{-1})}{\left(1+\frac{1}{A_0}+\frac{k}{A_0}-z^{-1}\left[1+\frac{1}{A_0}\right]\right)}}_{\text{Parasitic}=E(z)}$$

$$E(z) \cong \left(1-\tfrac{1}{A_0}[1+k]\right)\frac{(1-z^{-1})}{1-z^{-1}\left[1-\frac{k}{A_0}\right]}$$

$$\downarrow$$

$$E(\omega) = \left(1-\tfrac{1}{A_0}[1+k]\right)\frac{2j\,\cancel{e^{-j\omega T/2}}\sin\omega T/2}{2j\,\cancel{e^{-j\omega T/2}}\sin\omega T/2+\frac{k}{A_0}e^{-j\omega T/2}}$$

$$\cong \frac{1-\frac{1}{A_0}[1+k]}{1-j\frac{k}{A_0 2}\left(\frac{1}{tg\,\omega T/2}-j\right)} = \frac{1-\frac{1}{A_0}[1+k]}{1-\frac{k}{A_0 2}-j\frac{k}{A_0 2}[tg\,\omega T/2]^{-1}}$$

$$\cong \left(1-\tfrac{1}{A_0}[1+k]\right)\left(1+j\frac{k}{A_0 2}\left[\frac{1}{tg\,\omega T/2}-j\right]\right)$$

$$\left\| E(\omega) \cong 1-\frac{1}{A_0}\left[1+\frac{k}{2}\right]+j\frac{k}{A_0 2}\frac{1}{tg\,\omega T/2}\right\|$$

$$\boxed{\begin{aligned} m_{A_0}(\omega) &\cong -\frac{1}{A_0}\left[1+\frac{k}{2}\right] \\ \theta_{A_0}(\omega) &\cong tg^{-1}\left[\frac{k}{2A_0\,tg\,\omega T/2}\right]\end{aligned}}$$

Influence of the finite amplifier open-loop Gain A_0

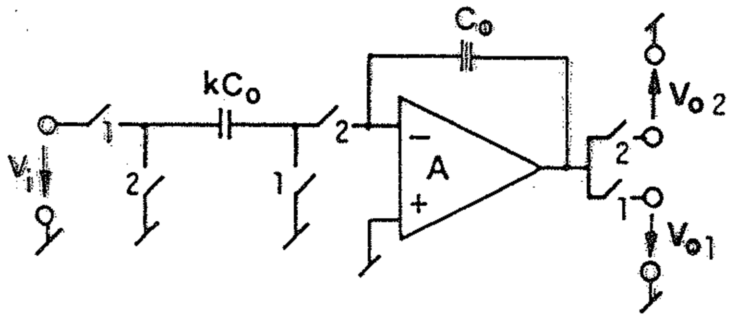
$$\boxed{\begin{aligned} m_{A_0}(\omega) &\cong -\frac{1}{A_0}\left(1+\frac{k}{2}\right) \\ \theta_{A_0}(\omega) &\cong \text{Arc tan}\left(\frac{1}{A_0}\,\frac{k}{2\tan\omega T/2}\right) \end{aligned}}$$

Influence of the Stray-Capacitances

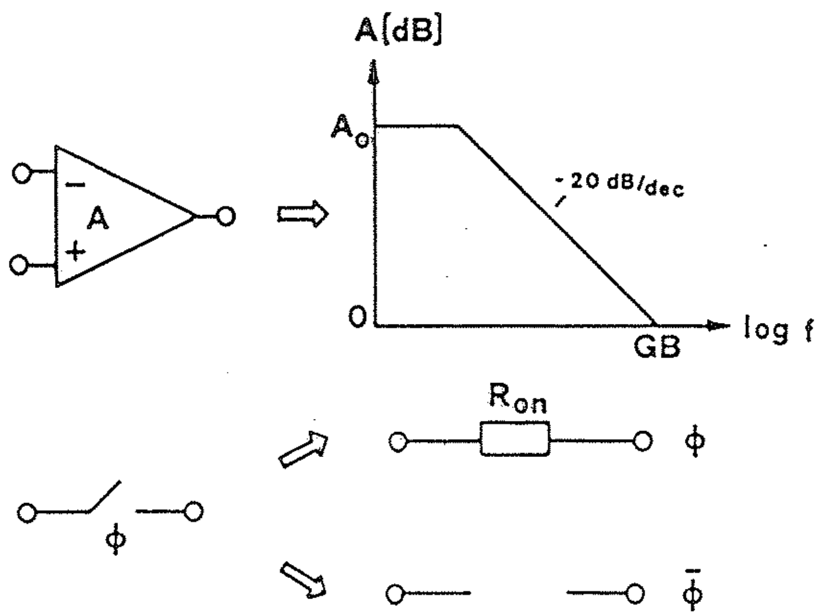
$$\boxed{\begin{aligned} m_s(\omega) &\cong -\frac{1}{A_0}\left(\frac{C_{sd}}{C_0}+\frac{C_{st}}{2\,C_0}\right) \\ \theta_s(\omega) &\cong \text{Arc tan}\left(\frac{1}{A_0}\,\frac{C_{st}/C_0}{2\tan\omega T/2}\right) \end{aligned}}$$

Influence of the amplifier unity-gain bandwidth and the switch ON resistances

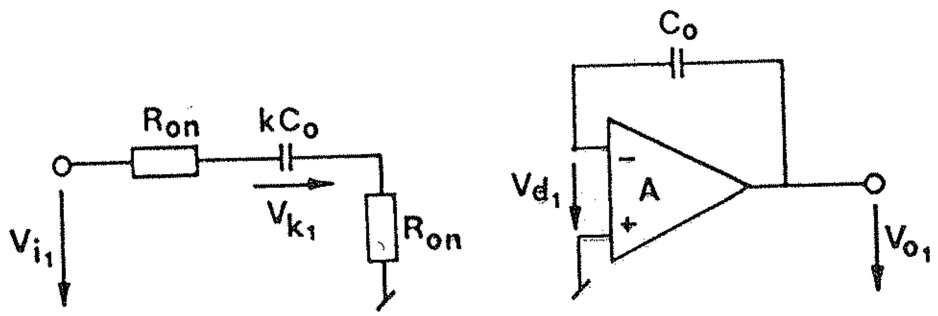
SC Integrator



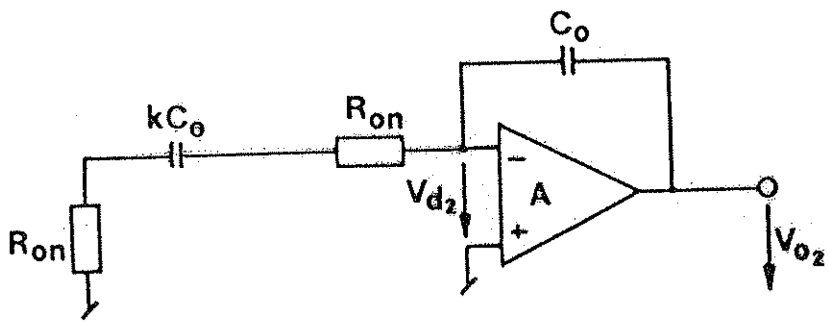
Applied Models for amplifier and switches



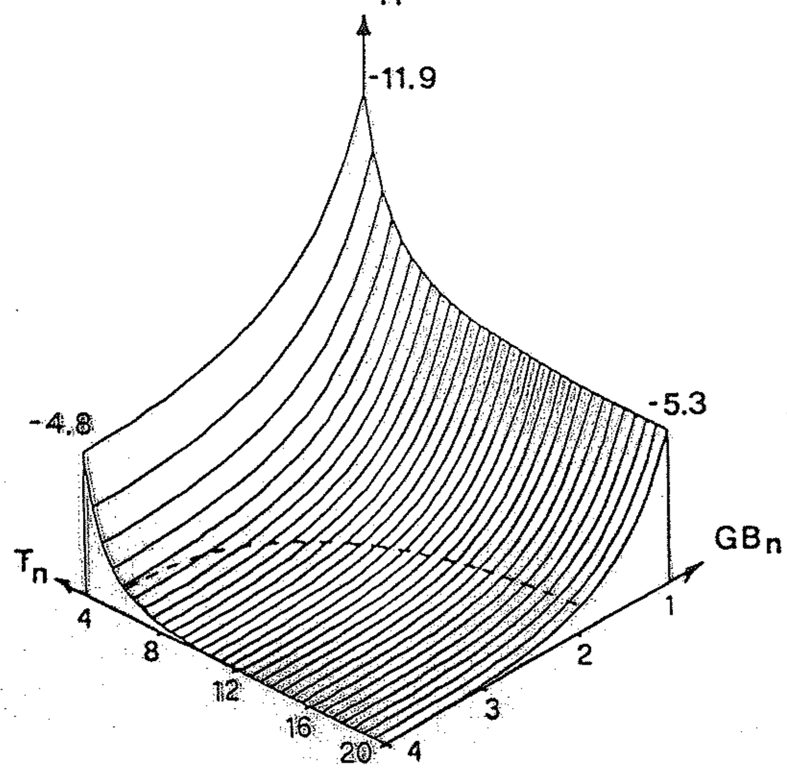
SC Integrator during Phase 1

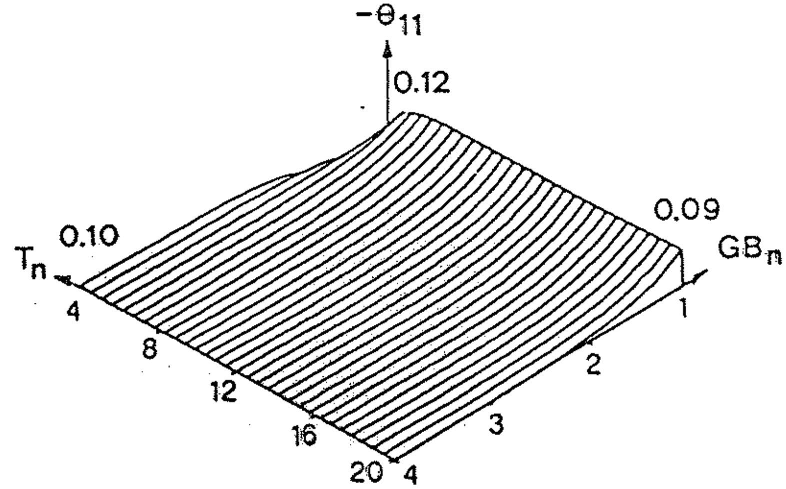


SC Integrator during Phase 2

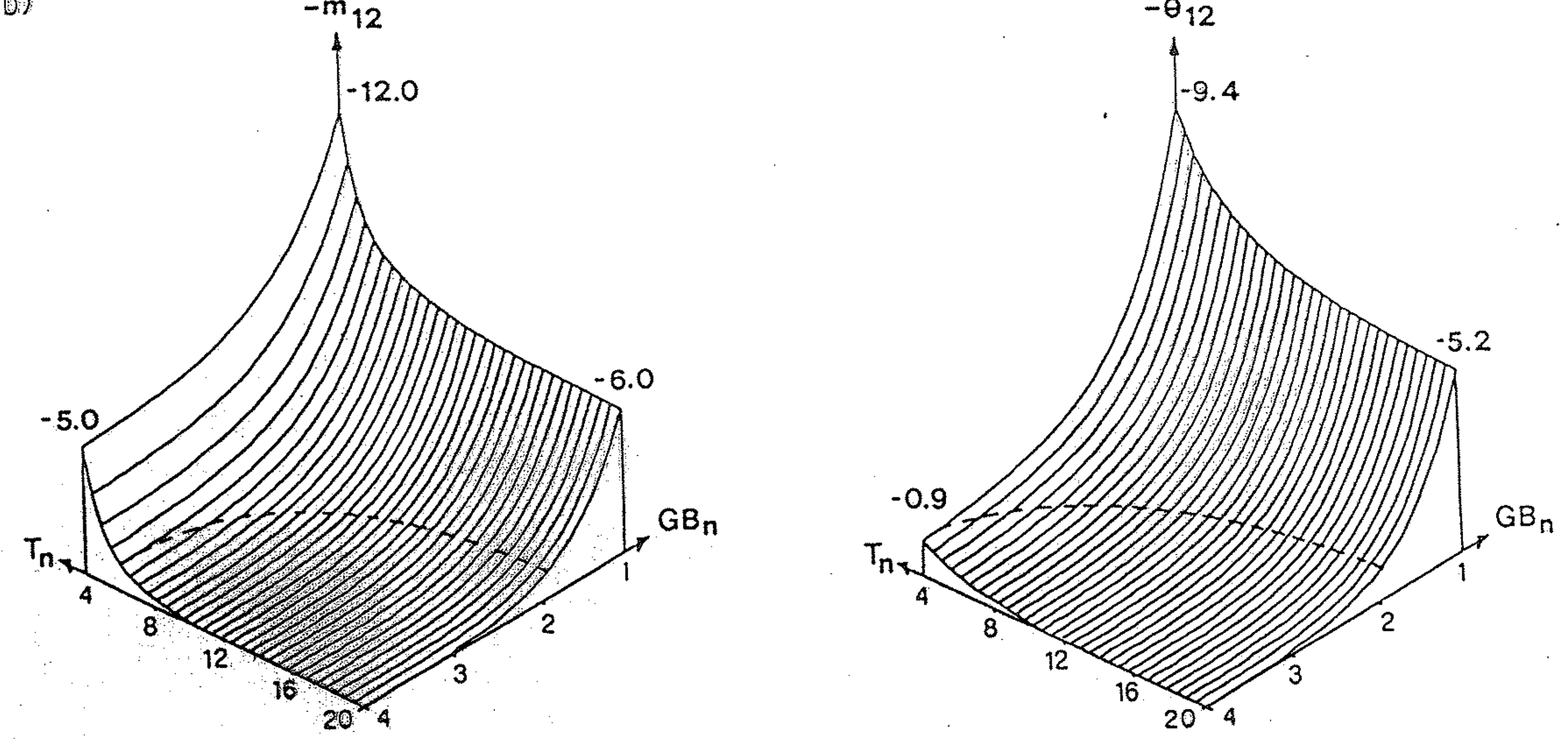


a)


-m11
-11.9
-4.8
-5.3
GBn
Tn
4
8
12
16
20
4
3
2
1

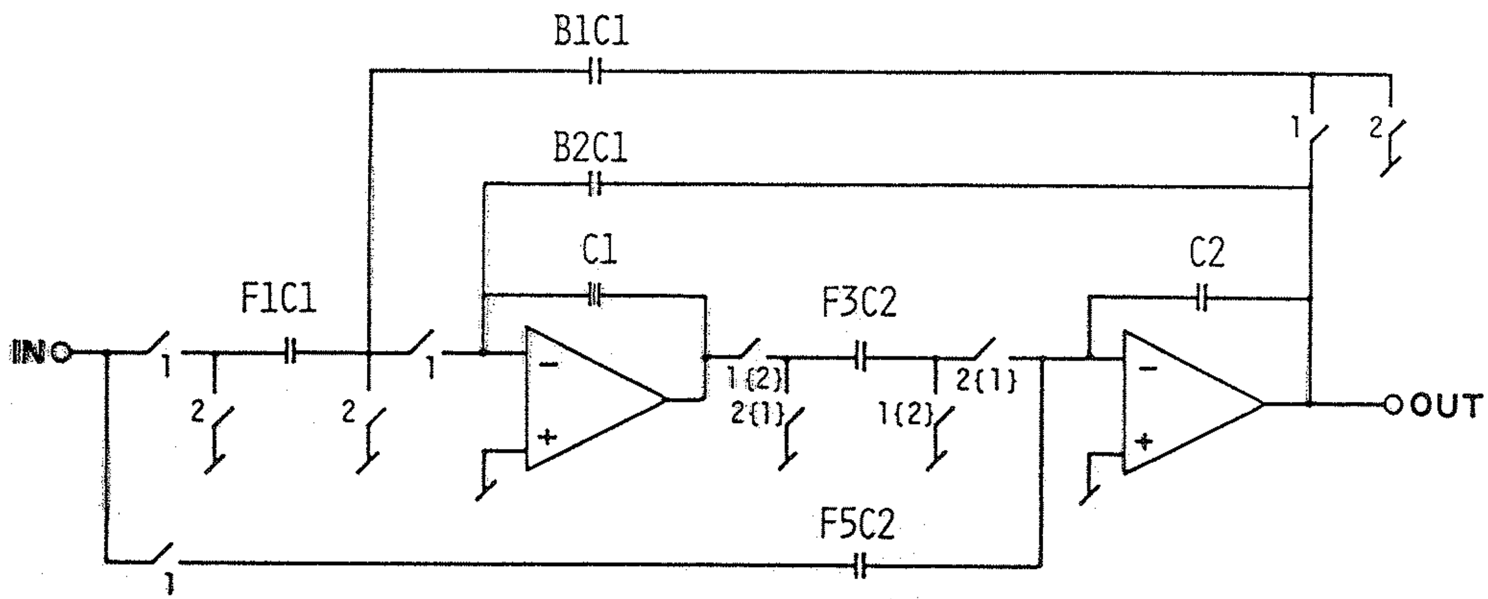

-θ11
0.12
0.10
0.09
GBn
Tn
4
8
12
16
20
4
3
2
1

b)



$-m_{12}$
-12.0
-5.0
-6.0
T_n
GB_n
4
8
12
16
20
1
2
3
4
$-\theta_{12}$
-9.4
-0.9
-5.2
T_n
GB_n
4
8
12
16
20
1
2
3
4

Second-order SC Lowpass Circuit with finite Passband Zero



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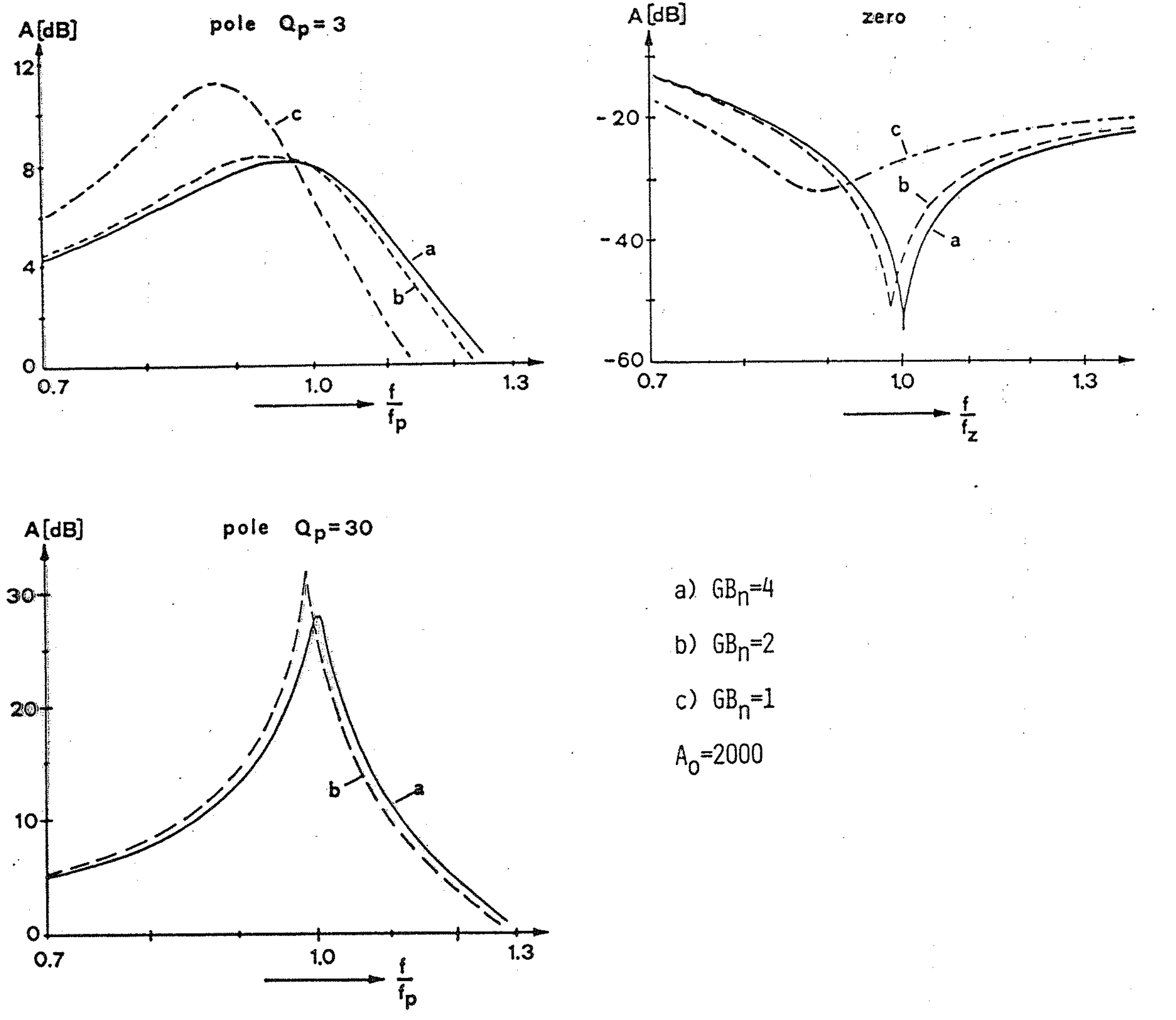
A[dB]
pole $Q_p = 3$
12
8
4
0
0.7
1.0
1.3
$\frac{f}{f_p}$
a
b
c
A[dB]
zero
-20
-40
-60
0.7
1.0
1.3
$\frac{f}{f_z}$
a
b
c
A[dB]
pole $Q_p = 30$
30
20
10
0
0.7
1.0
1.3
$\frac{f}{f_p}$
a
b
a) $GB_n = 4$
b) $GB_n = 2$
c) $GB_n = 1$
$A_0 = 2000$