

IX - I

Transconductance-C Filters

A Prototype Filter Implementation

Talk Outline

- Introduction
- Transconductance Elements
- Filter Topologies
- Simulation Results
- Layout of Prototype Filter
- Conclusions

1. Introduction

Why monolithic continuous-time filters?

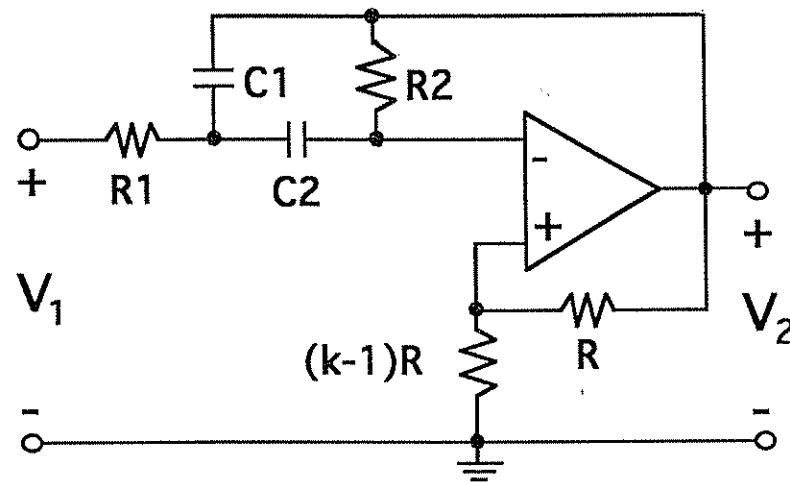
- Pre- and post filters for sampled-data systems
- Mixed analog digital systems
- Wide frequency range (<100MHz)
- Area and power efficient solution

Continuous-Time Filter Implementation Techniques

- RC-Active Circuits
- MOSFET-C Circuits
- Transconductance-C (Ota-C) Circuits

RC-active Filters

Example: 2nd Order Bandpass



Filter Characteristics

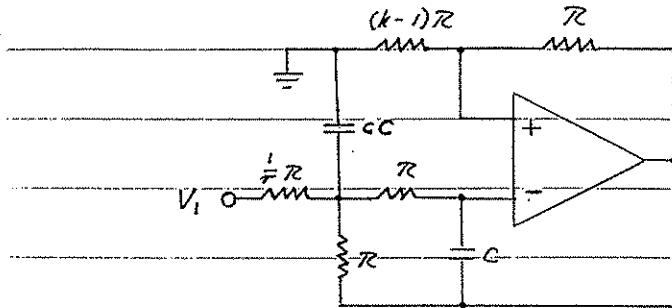
$$T_0 = -\frac{k R_2 C_2}{R_1(C_1 + C_2) - (k-1) R_2 C_2}$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\frac{1}{Q_0} = \sqrt{\frac{R_1}{R_2}} \left(\sqrt{\frac{C_1}{C_2}} + \sqrt{\frac{C_2}{C_1}} \right) - (k-1) \sqrt{\frac{R_2 C_2}{R_1 C_1}}$$

Summary: Single-opamp negative feedback topologies (2nd order)

Lowpass

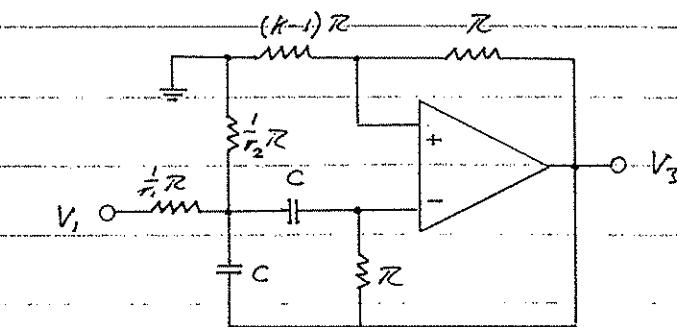


$$\omega_p = \frac{1}{RC} \sqrt{1 - r(k-1)}$$

$$Q_p = \frac{\sqrt{r}(1 - r(k-1))}{2 + r - c(k-1)}$$

$$T(s=0) = -\frac{kR}{1 - r(k-1)}$$

Bandpass

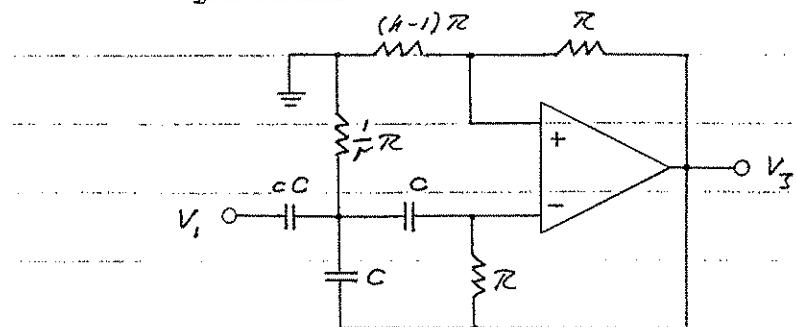


$$\omega_p = \frac{1}{RC} \sqrt{T_1 + T_2}$$

$$Q_p = \frac{\sqrt{T_1 + T_2}}{2 - (r_1 + r_2)(k-1)}$$

$$T(s=j\omega_p) = -\frac{k \cdot r_1}{2 - (r_1 + r_2)(k-1)}$$

Highpass



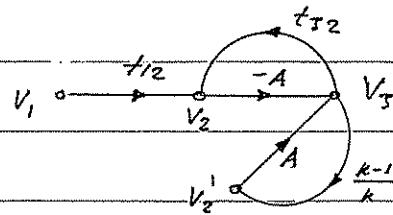
$$\omega_p = \frac{1}{RC} \sqrt{\frac{r}{1 - c(k-1)}}$$

$$Q_p = \frac{\sqrt{r(1 - c(k-1))}}{2 + c - r(k-1)}$$

$$T(s \rightarrow \infty) = -\frac{kc}{1 - c(k-1)}$$

Amplifier Finite Gain-Bandwidth Effects (neg. feedback topol.)

Voltage Transfer function:



$$T(s) = \frac{V_3}{V_1} = -\frac{A \cdot t_{12}}{1 + A(t_{32} - \frac{k-1}{k})} \frac{\frac{k}{A} \hat{d}}{\frac{k}{A} \hat{d}}$$

$$\Rightarrow \left| T(s) = -\frac{k n_{12}}{\hat{d}(1 + \frac{k}{A}) - k(\hat{d} - n_{32})} \right|$$

Example: 2nd order Lowpass filter

$$\hat{d} = s^2 + s \frac{1}{RC} (1 + \frac{2+r}{C}) + \frac{1}{(RC)^2} \frac{1+r}{C}$$

$$n_{32} = s^2 + s \frac{1}{RC} \frac{2+r}{C} + \frac{1}{(RC)^2} \frac{1}{C}$$

$$n_{12} = \frac{1}{(RC)^2} \frac{r}{C}$$

$$\text{Opamp: } A(s) \approx A_o \frac{\omega_o}{s} = \frac{G_B}{s}$$

GB: Gain-Bandwidth Pr.

$$\Rightarrow \left| T(s) = -\frac{k \frac{1}{(RC)^2} \frac{r}{C}}{(s^2 + s \frac{1}{RC} [1 + \frac{2+r}{C}] + \frac{1}{(RC)^2} \frac{1+r}{C})(1 + s \frac{k}{G_B}) - k(s \frac{1}{RC} + \frac{1}{(RC)^2} \frac{r}{C})} \right|$$

Approx.

$$\text{Solution: } \left| T(s) \approx -\frac{k \frac{1}{(RC)^2} \frac{r}{C}}{(1 + s \frac{k}{G_B})(s^2 [1 + \frac{k^2}{G_B RC}] + s \frac{1}{RC} [\frac{2+r-C(k-1)}{C} + \frac{k^2 r}{G_B R C}]) + \frac{1-r(k-1)}{C}} \right|$$

$$\text{Poles: neg. real: } \left| \omega_{p3} \approx \frac{G_B}{k} \right|$$

$$\text{Complex: } \left| \omega_p = \frac{1}{RC} \sqrt{\frac{1-r(k-1)}{C} \left(1 - \frac{k^2}{2 G_B R C} \right)} \right|$$

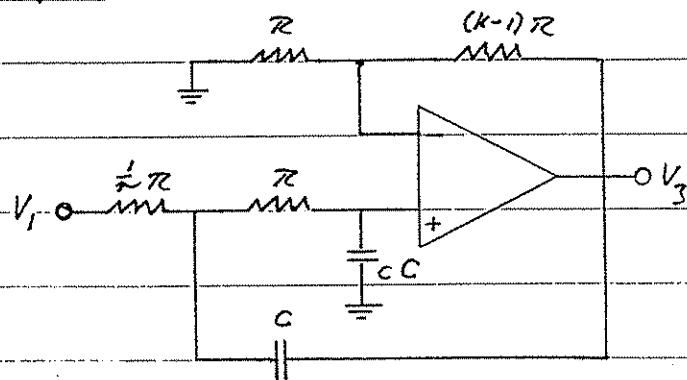
$$\left| Q_p \approx \frac{\sqrt{C(1-r(k-1))} \left(1 + \frac{k^2}{2 G_B R C} \right)}{2 + r - C(k-1) + \frac{k^2 r}{G_B R C}} \right|$$

$$\left| \omega_p \approx \omega_{po} \left(1 - \frac{k^2}{2 G_B R C} \right) \right|$$

$$\left| Q_p \approx Q_{po} \left(1 + \frac{k^2}{2 G_B R C} \frac{(2 + r - C(k-1))}{(2 + r - C(k-1))} \right) \right|$$

Summary: Single-opamp positive feedback topologies (2nd order)

Lowpass

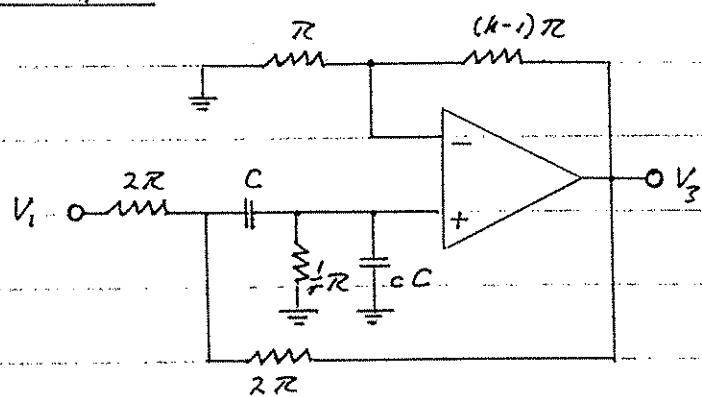


$$\omega_p = \sqrt{\frac{r}{c}} \cdot \frac{1}{RC}$$

$$Q_p = \frac{\sqrt{r \cdot c}}{c(1+r) - (k-1)}$$

$$T(s=0) = k$$

Bandpass

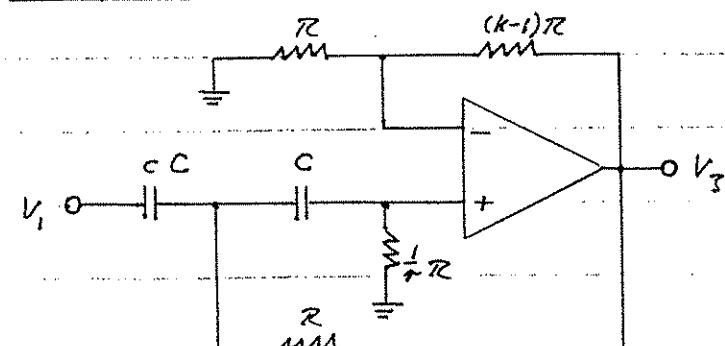


$$\omega_p = \sqrt{\frac{r}{c}} \cdot \frac{1}{RC}$$

$$Q_p = \frac{\sqrt{r \cdot c}}{r + c + \frac{1}{2} - \frac{1}{2}(k-1)}$$

$$T(s=j\omega_p) = \frac{k}{2(r+c)+1-(k-1)}$$

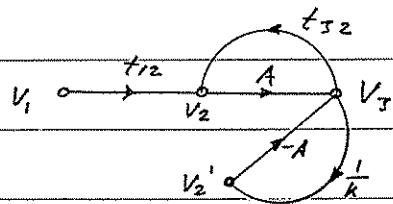
Highpass



$$\omega_p = \sqrt{\frac{r}{c}} \cdot \frac{1}{RC}$$

$$Q_p = \frac{\sqrt{r \cdot c}}{r(1+c) - (k-1)}$$

$$T(s \rightarrow \infty) = k$$

Amplifier Finite Gain-Bandwidth Effects (pos. feedback loop.)Voltage transfer Function:

$$T(s) = \frac{A \cdot t_{12}}{1 + A(\frac{1}{k} - t_{32})} \frac{\frac{k}{A} \hat{\omega}}{\frac{k}{A} \hat{\omega}}$$

$$\Rightarrow T(s) = \frac{k \cdot n_{12}}{\hat{\omega}(1 + \frac{k}{A}) - k \cdot n_{32}}$$

Example: 2nd order Lowpass Filter

$$\hat{\omega} = s^2 + s \frac{1}{RC} (1 + r + \frac{1}{c}) + \frac{1}{(RC)^2} \frac{r}{c}$$

$$n_{32} = s \frac{1}{RC} \frac{1}{c}$$

$$n_{12} = \frac{1}{(RC)^2} \frac{r}{c}$$

$$\text{Opamp: } A(s) \approx A_o \frac{\omega_o}{s} = \frac{G_B}{s}$$

G.B.: Gain-Bandwidth Pr.

$$\Rightarrow T(s) = \frac{\frac{1}{k(RC)^2} \frac{r}{c}}{(s^2 + s \frac{1}{RC} [1 + r + \frac{1}{c}] + \frac{1}{(RC)^2} \frac{r}{c})(1 + s \frac{k}{G_B}) - s \frac{1}{RC} \frac{k}{c}}$$

Approx.

solution:

$$T(s) \approx \frac{\frac{1}{k(RC)^2} \frac{r}{c}}{(1 + s \frac{k}{G_B})(s^2 [1 + \frac{k^2}{G_B RC c}] + s \frac{1}{RC} [1 + r - \frac{(k+1)}{c}] + \frac{1}{(RC)^2} \frac{r}{c})}$$

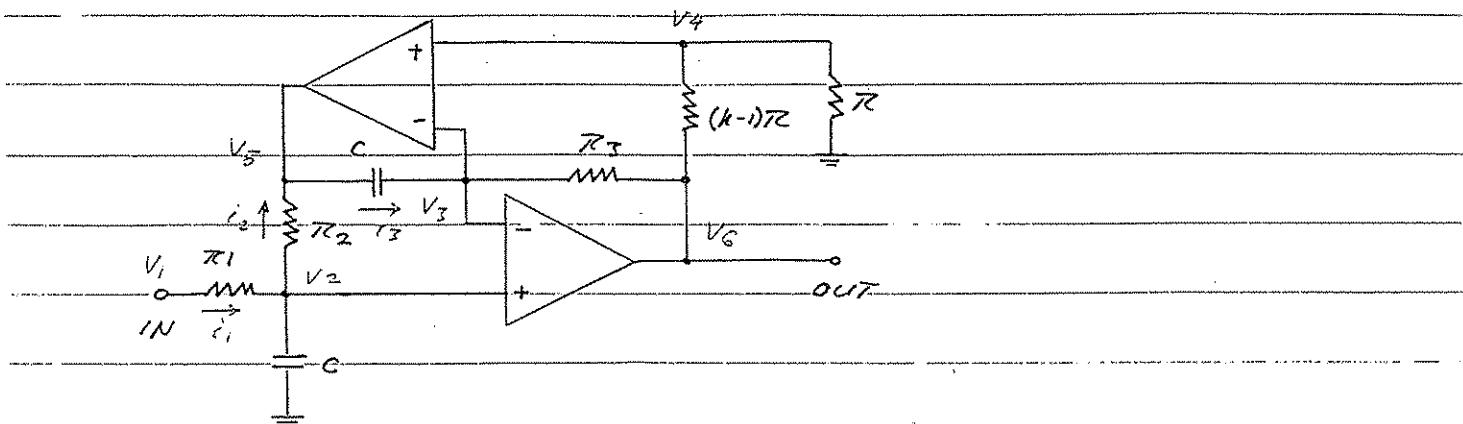
$$\text{Poles: neg. real: } \omega_{p_1} \approx \frac{G_B}{k}$$

$$\text{complex: } \omega_p \approx \frac{1}{RC} \sqrt{\frac{r}{c}} \left(1 - \frac{k^2}{2G_B R C c} \right)$$

$$Q_p \approx \frac{\sqrt{\frac{r}{c}}}{c(1+r) - (k-1)} \left(1 + \frac{k^2}{2G_B R C c} \right)$$

$$\omega_p \approx \omega_{p_0} \left(1 - \frac{k^2}{2G_B R C c} \right)$$

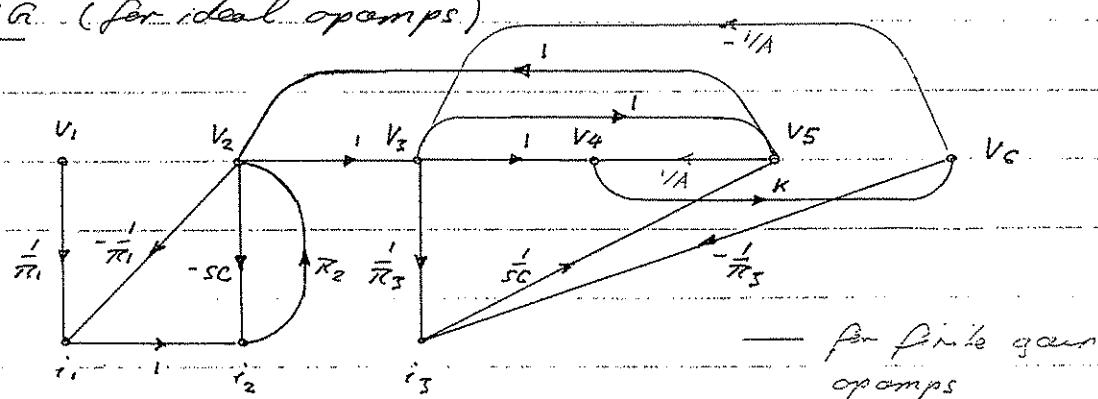
$$Q_p \approx Q_{p_0} \left(1 + \frac{k^2}{2G_B R C c} \right)$$

B) Sedra-Espinosa Bandpass Filter

assumption: opamps are ideal

$$\text{thus } V_2 = V_3 = V_4$$

SEA (for ideal opamps)



— for finite gain
opamps

$$\begin{aligned}
 T_{16}(s) &= \frac{V_6}{V_1} = \frac{\frac{R_2}{R_1}}{1 + \frac{R_2}{R_1} + sCR_2 - 1 + \frac{K}{sCR_3} - \frac{1}{sCR_3}} \times \frac{sCR_2}{sCR_2} \\
 &= \frac{K \cdot s \frac{1}{R_1 C}}{s^2 + s \frac{1}{R_1 C} + \frac{(K-1)}{R_2 R_3 C^2}}
 \end{aligned}$$

Note: K must be larger than 1 otherwise we obtain a 1st order lowpass ($K=1$) — or else the element $(K-1)R$ becomes negative!

Filter Parameters:

Sensitivities:

Special Cases		
$\omega_p = \sqrt{\frac{(k-1)^2}{R_2 R_3}} \cdot \frac{1}{C}$	$k=2$	$S_{R_2}^{\omega_p} = S_{R_3}^{\omega_p} = -\frac{1}{2}$
$\alpha_p = R_1 \sqrt{\frac{(k-1)^2}{R_2 R_3}}$	$\omega_p = \frac{1}{R C}$	$S_{R_1}^{\alpha_p} = 1$
$\tilde{\tau}(s=j\omega_p) = k$	$\alpha_p = \frac{R_1}{R}$	$S_{R_1}^{\alpha_p} = S_{R_3}^{\alpha_p} = -\frac{1}{2}$

Influence of amplifier gain-bandwidth product:

$$\tilde{T}_{16} = \frac{k \cdot s \frac{1}{R C} [1 + \frac{1}{s R_2 C} \frac{1}{A}]}{s^2 + s \frac{1}{R C} + \frac{(k-1)}{R_2 R_3 C^2} + \frac{k}{A} [\frac{s}{R C} + \frac{s}{R_2 C} + s^2]}$$

Replacing $A(s)$ by $\frac{1}{s} \omega_B = \frac{G B}{s}$ yields:

$$\begin{aligned} \tilde{T}_{16} &= \frac{k \cdot s \frac{1}{R C} [1 + \frac{1}{G B R_2 C}]}{s^2 \frac{k}{G B} + s^2 [1 + \frac{1}{G B} (\frac{1}{R C} + \frac{1}{R_2 C})] + \frac{1}{R_2 R_3 C^2}} \\ &\approx \frac{k \cdot s \frac{1}{R C} [1 + \frac{1}{G B R_3 C}]}{(s \frac{k}{G B} + 1) (s^2 [1 + \frac{1}{G B} \frac{1}{R_2 C}] + s [\frac{1}{R C} - \frac{k(k-1)}{G B R_2 R_3 C^2}] + \frac{(k-1)}{R_2 R_3 C^2})} \end{aligned}$$

Poles: $\omega_{p3} \approx \frac{k}{G B}$ reg. real pole

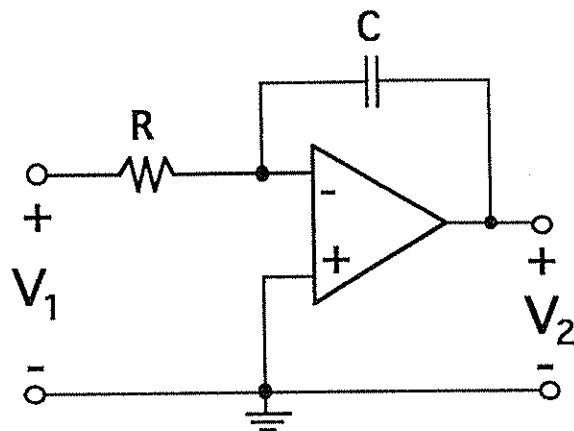
$\omega_p \approx \sqrt{\frac{(k-1)^2}{R_2 R_3}} \cdot \frac{1}{C} \left(1 - \frac{1}{2} \frac{k}{G B R_2 C}\right)$	complex pole-pair
$\alpha_p \approx R_1 \sqrt{\frac{(k-1)^2}{R_2 R_3}} \left(1 + \frac{1}{2} \frac{k}{G B R_2 C} \left[1 - 2(k-1) \frac{R_1}{R_3}\right]\right)$	

$k=2$	
$R_2=R_3=R$	$\Rightarrow \omega_p = \omega_{p0} \left(1 - \frac{\omega_{p0}}{G B}\right)$
$R_1=\alpha_p R$	$\alpha_p = \alpha_{p0} \left(1 + \frac{\omega_{p0}}{G B} [1 - 2\alpha_{p0}]\right)$

MOSFET-C Filters

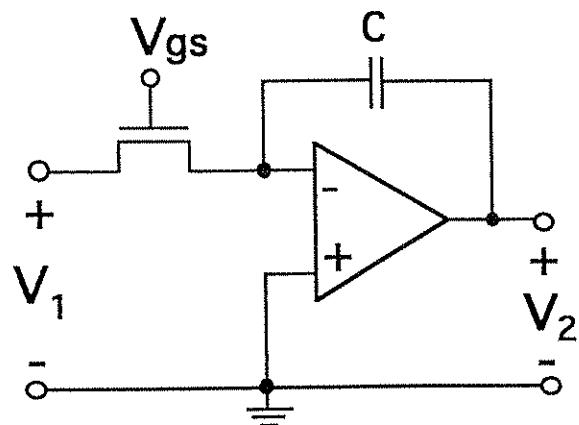
Example: Integrator

RC-active Implementation



$$T(s) = -\frac{1}{s RC}$$

MOSFET-C Implementation



$$T(s) = -\frac{g_{ds}}{s C}$$

where:

$$g_{ds} \approx \mu C_{ox} \frac{W}{L} [V_{gs} - V_T - V_{ds}]$$

RC-Active Filters

Properties

- Many filter topologies available
- Few active elements (op-amps)
- Excellent linearity and dynamic range (90dB)
- Frequency limited by op-amps (low MHz range)
- Passive elements require significant chip area (R_s : 10-100 Ω C 2 : 0.25-1fF/ μm^2)
- Poor control over RC product (20%-60% variation)
- Discrete tuning possible (e.g. by employing C-arrays)

MOSFET-C Filters

Properties

- Few op-amps required
- Area efficient implementation of resistors
- Filter topologies limited (due to tuning)
- Frequency limited by op-amps (low MHz range)
- Poor linearity (due to nonlinearity of MOSFET resistors)
- MOSFET resistors require tuning
- Additional circuitry for automatic tuning

Transconductance-C (Ota-C) Filters

Properties

- Well-suited for high-frequency applications (<100MHz)
- Filter topologies limited (due to tuning)
- Many transconductance elements required
- Limited dynamic range (due to Ota nonlinearities)
- Transconductance elements require tuning
- Additional circuitry for automatic tuning

Comparison

Filter Type	Accuracy	Dynamic Range	Impl. Cost	Frequency Range
RC-Active	20%-60%	90dB	low	< 1MHz
MOSFET-C	approx. 1%	60dB	medium	< 1MHz
Transcond.-C	approx. 1%	60dB	medium	< 100MHz

2. Transconductance Elements

Voltage Controlled Current Source

Special Requirements

- High input impedance ($>M\Omega$)
- High output impedance ($>M\Omega$)
- Large input voltage swing (e.g. $1V_{p-p}$)
- Good linearity over entire range
- Wide bandwidth ($<100MHz$)
- Transconductance electrically tunable
- Wide tuning range

Basic Implementation

MOS Transistor

Major Drawback

Poor Linearity

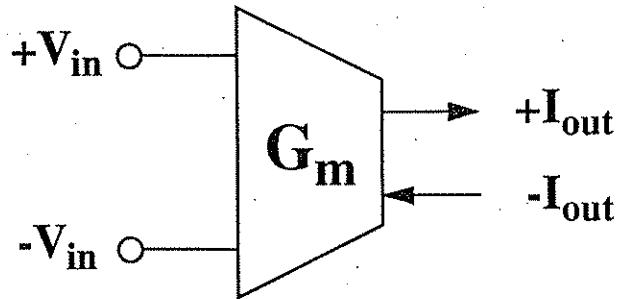
Remedies

Fully-differential implementation

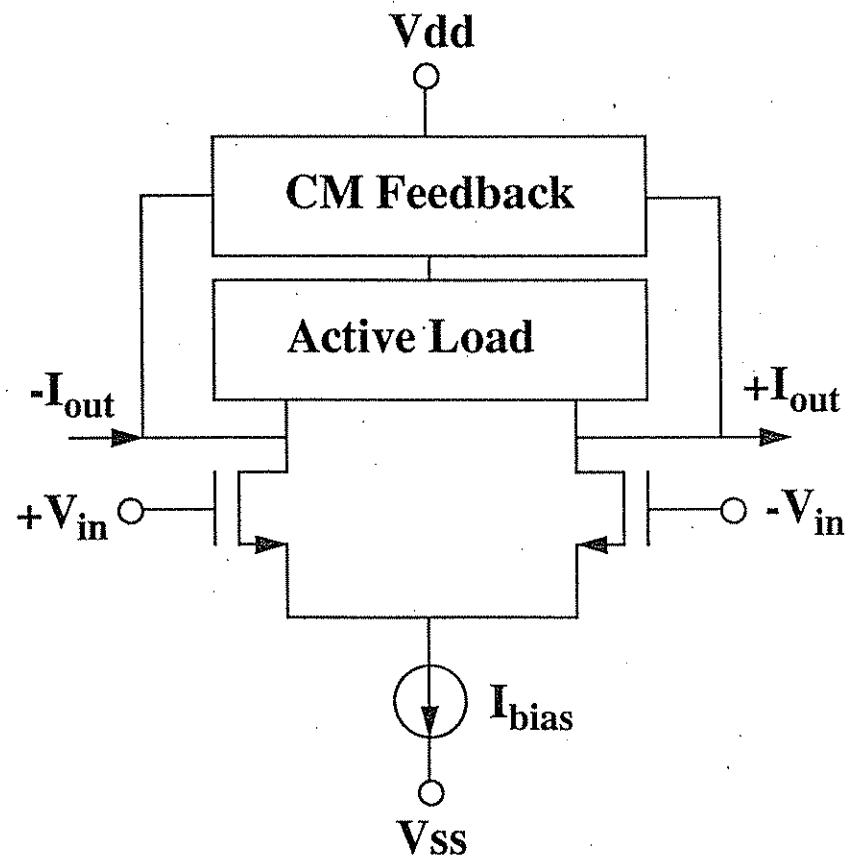
Additional compensating circuitry

Fully-differential transconductance element

Symbol

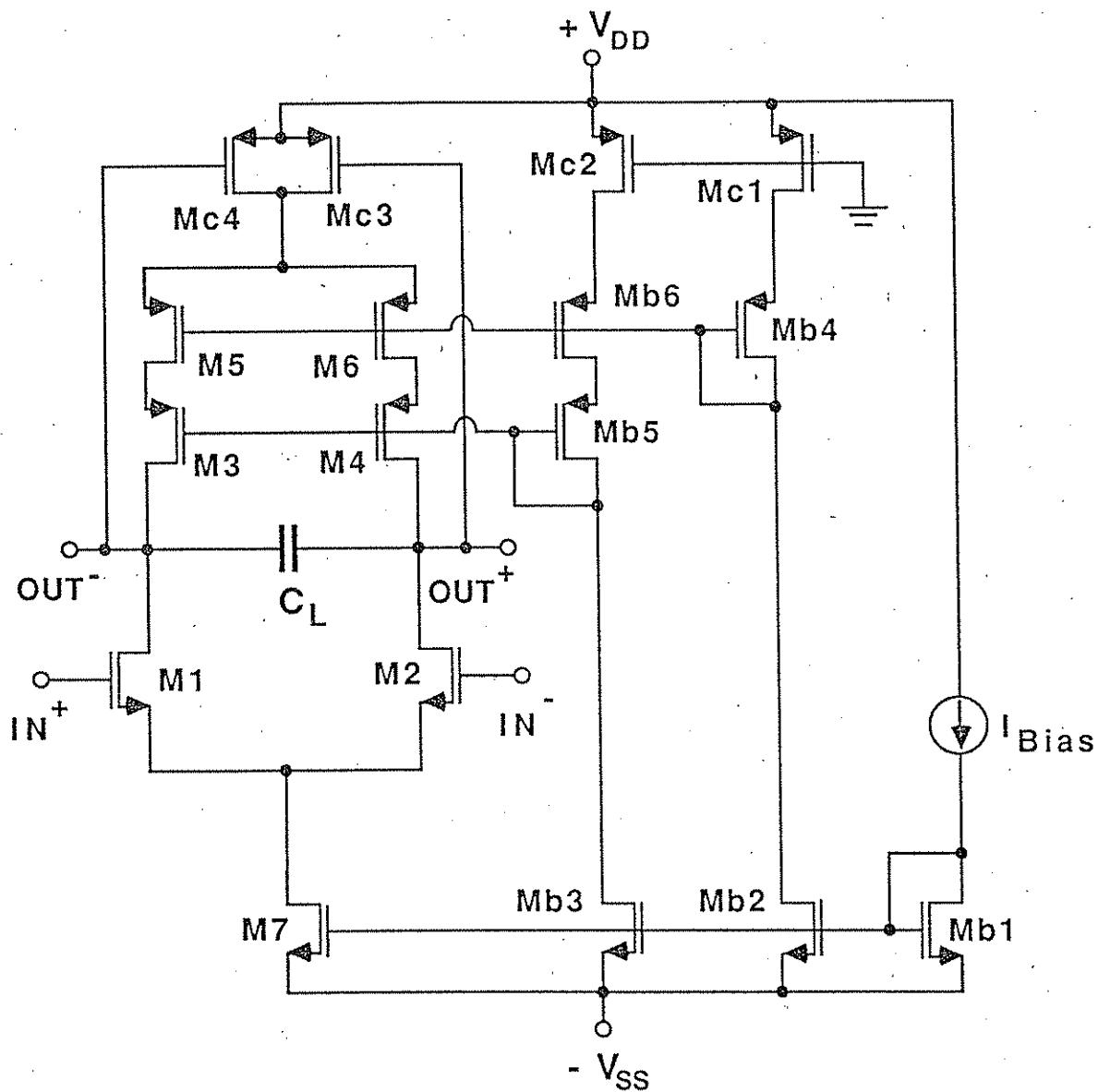


Implementation by CS Pair



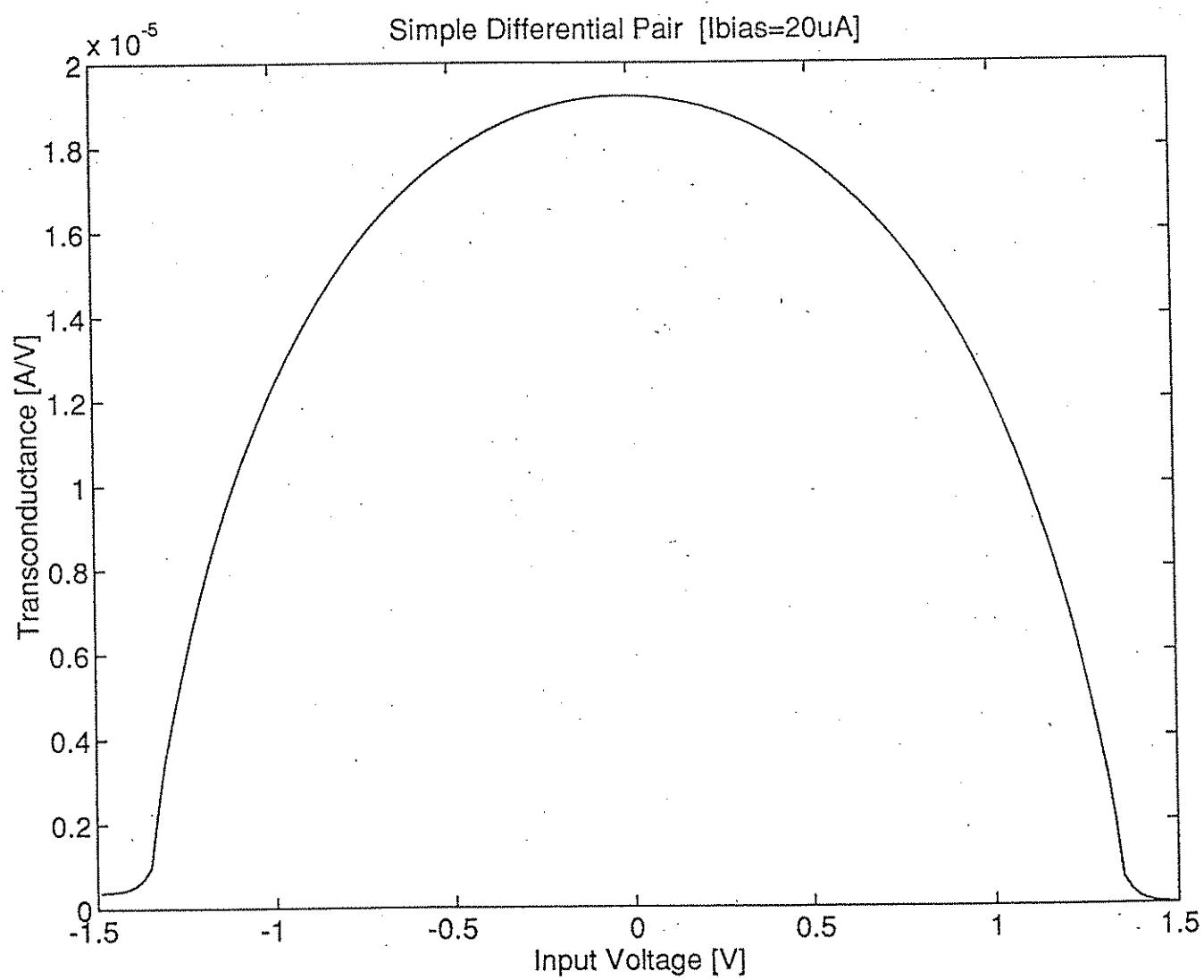
Fully-Differential Transconductance Amplifier

Version 0: Simple Source-Coupled Differential Pair



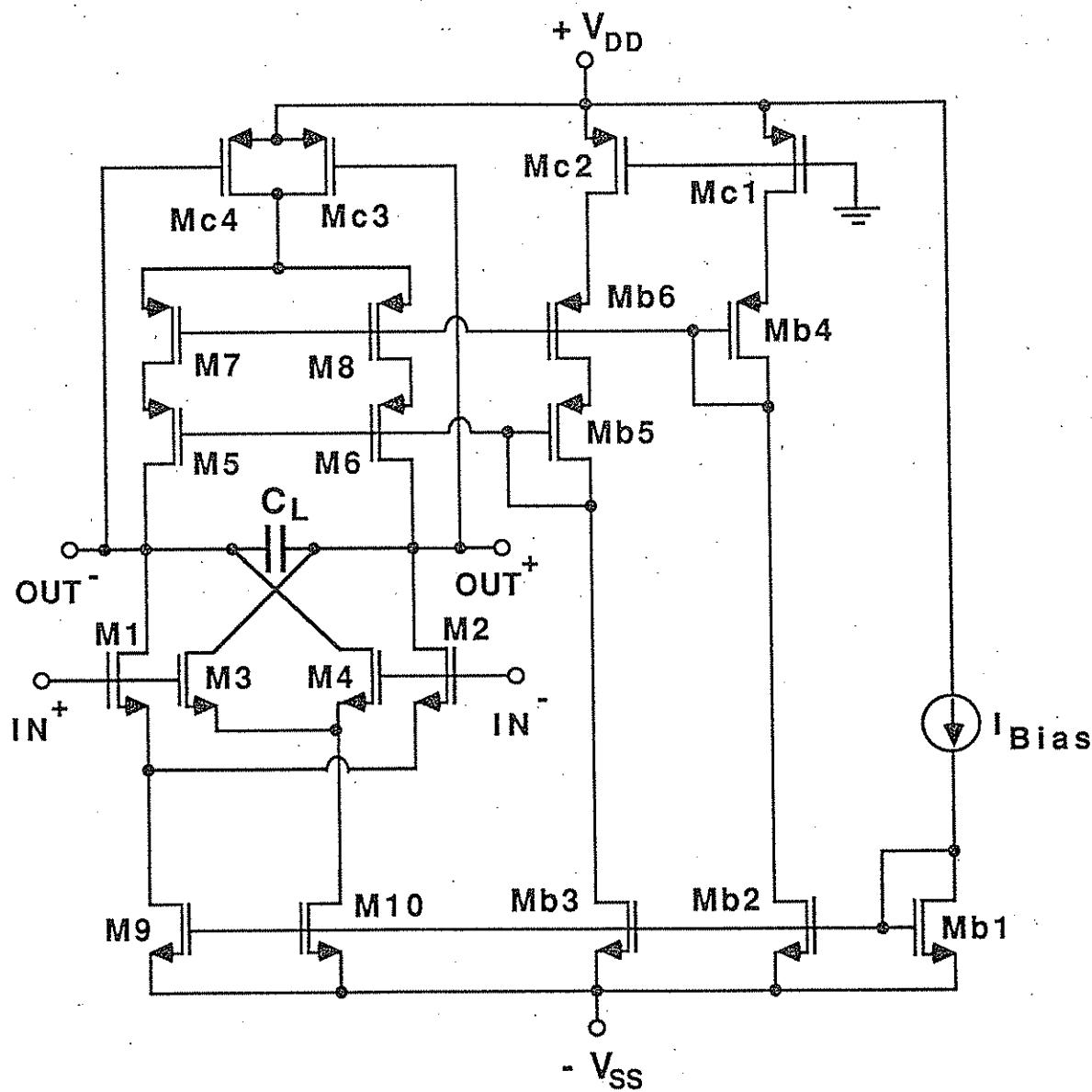
$$G_M = \frac{1}{2} g_{m1}$$

IX-16



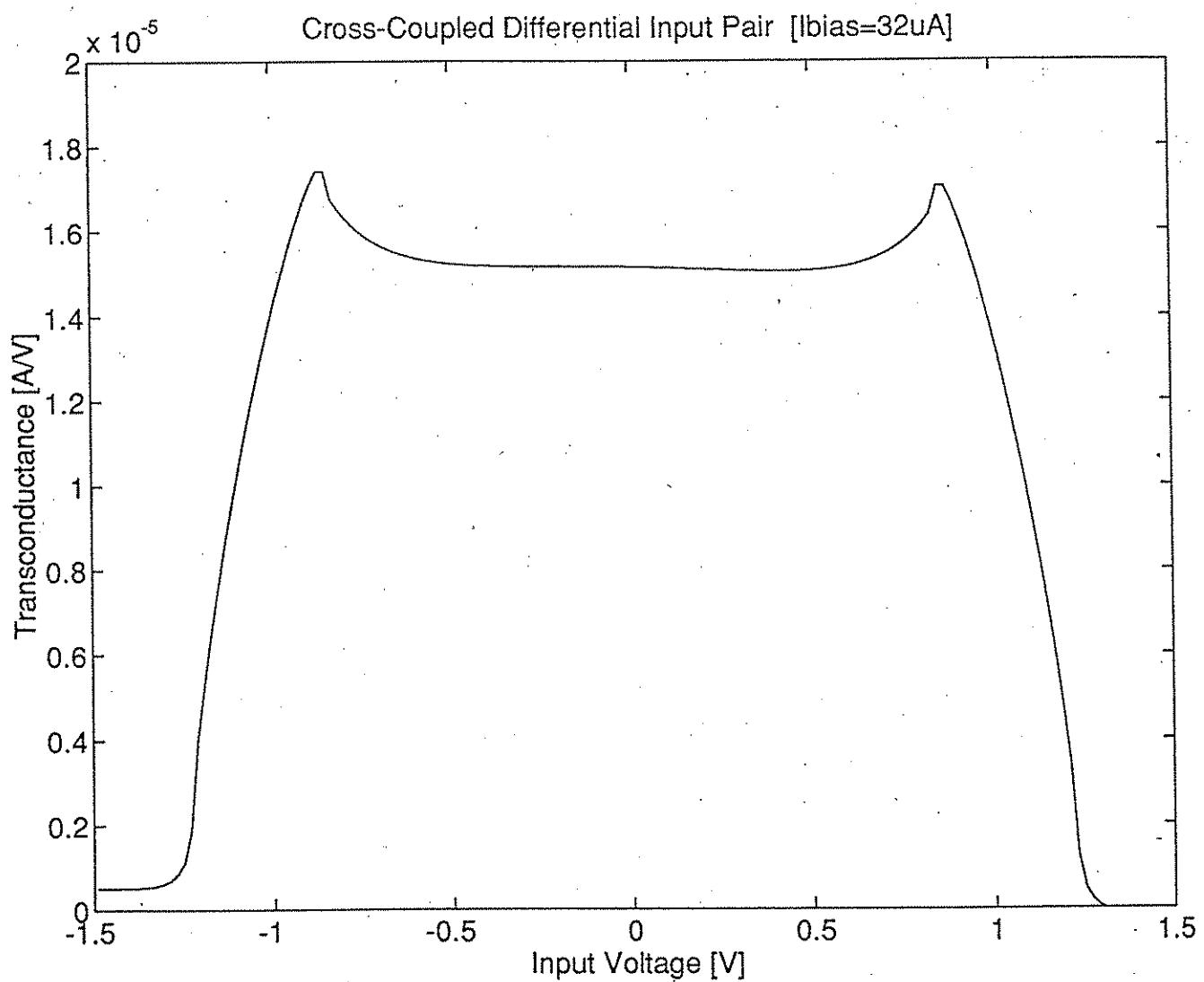
Fully-Differential Transconductance Amplifier

Version1: Differential Pair with Cross-Coupled Inputs



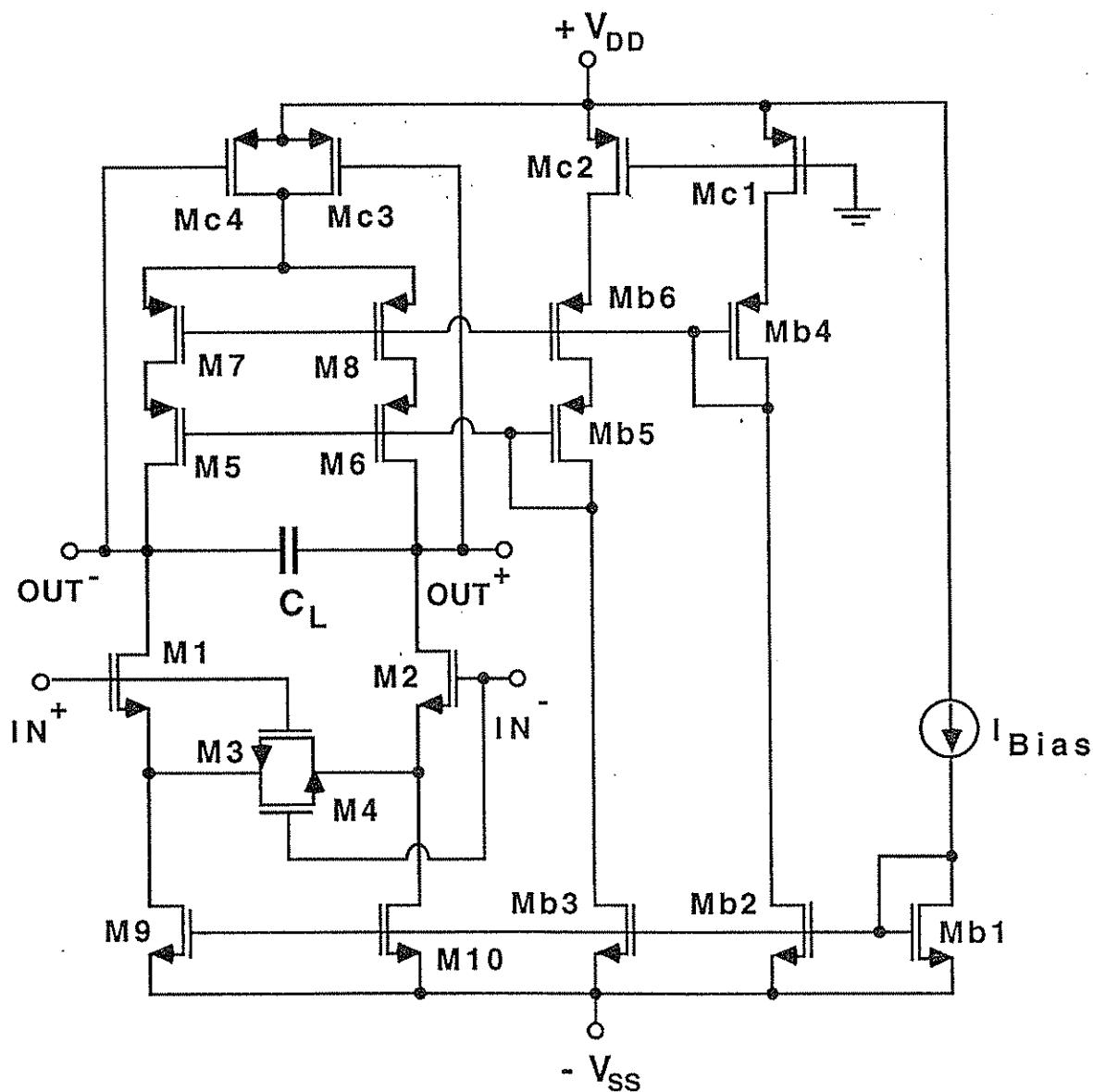
$$G_M = \frac{1}{2} (g_{m1} - g_{m3})$$

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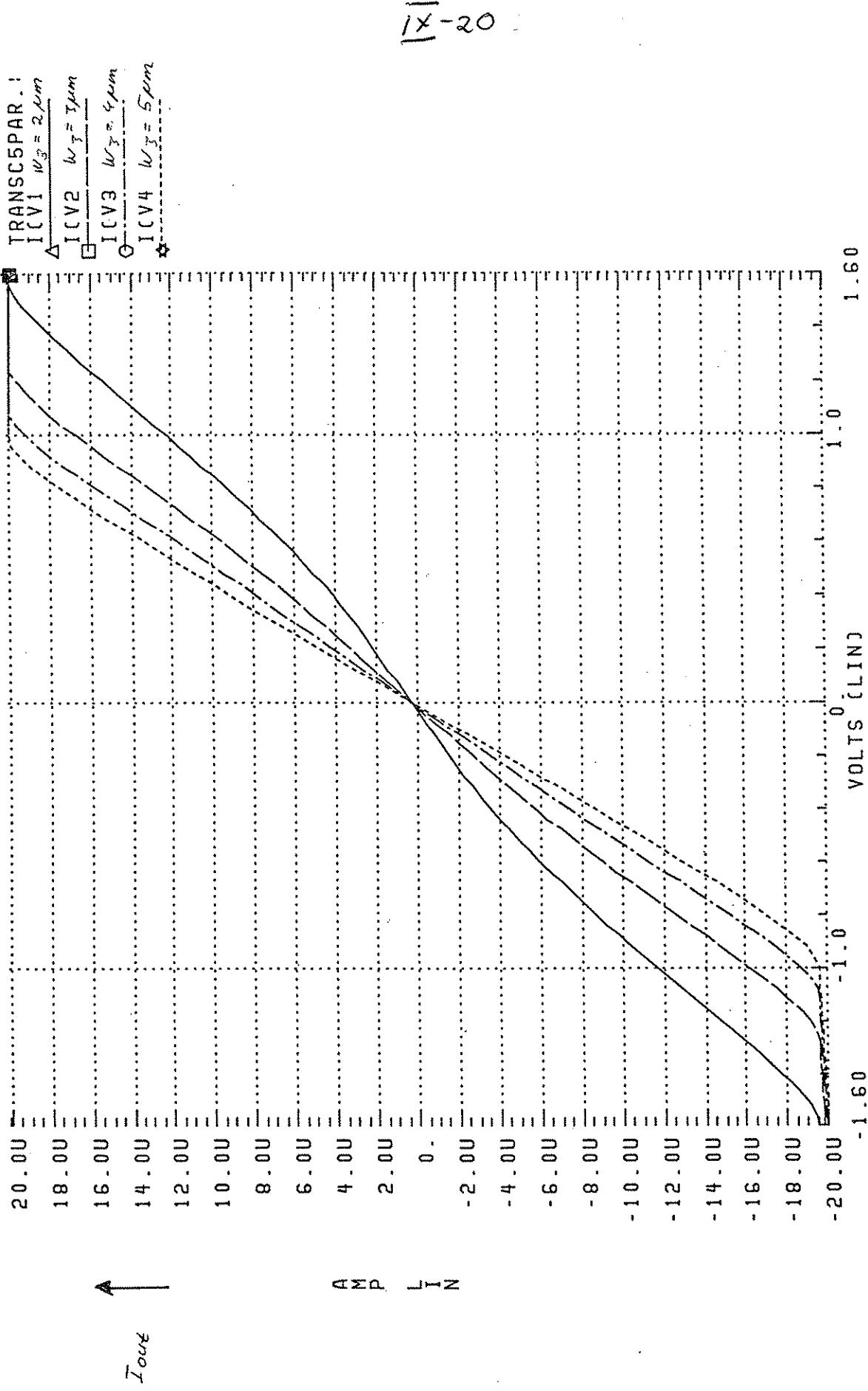
Fully-Differential Transconductance Amplifier

Version2: Differential Pair with Source Degeneration Devices

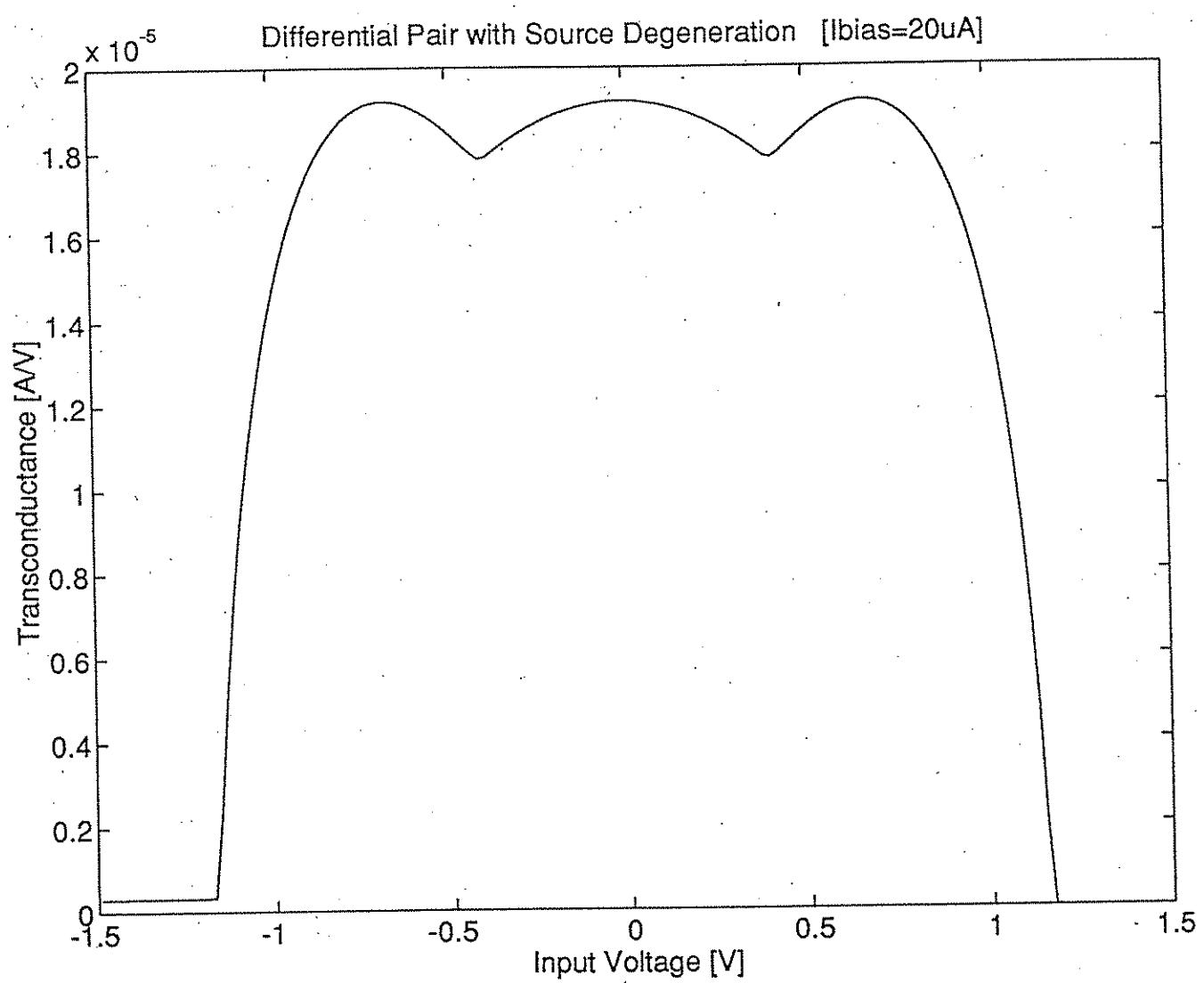


$$G_M = \frac{g_{m1}}{2 + g_{m1}/(g_{d3} + g_{d4})}$$

FULLY-DIFFERENTIAL TRANSCONDUTTANCE AMPLIFIER
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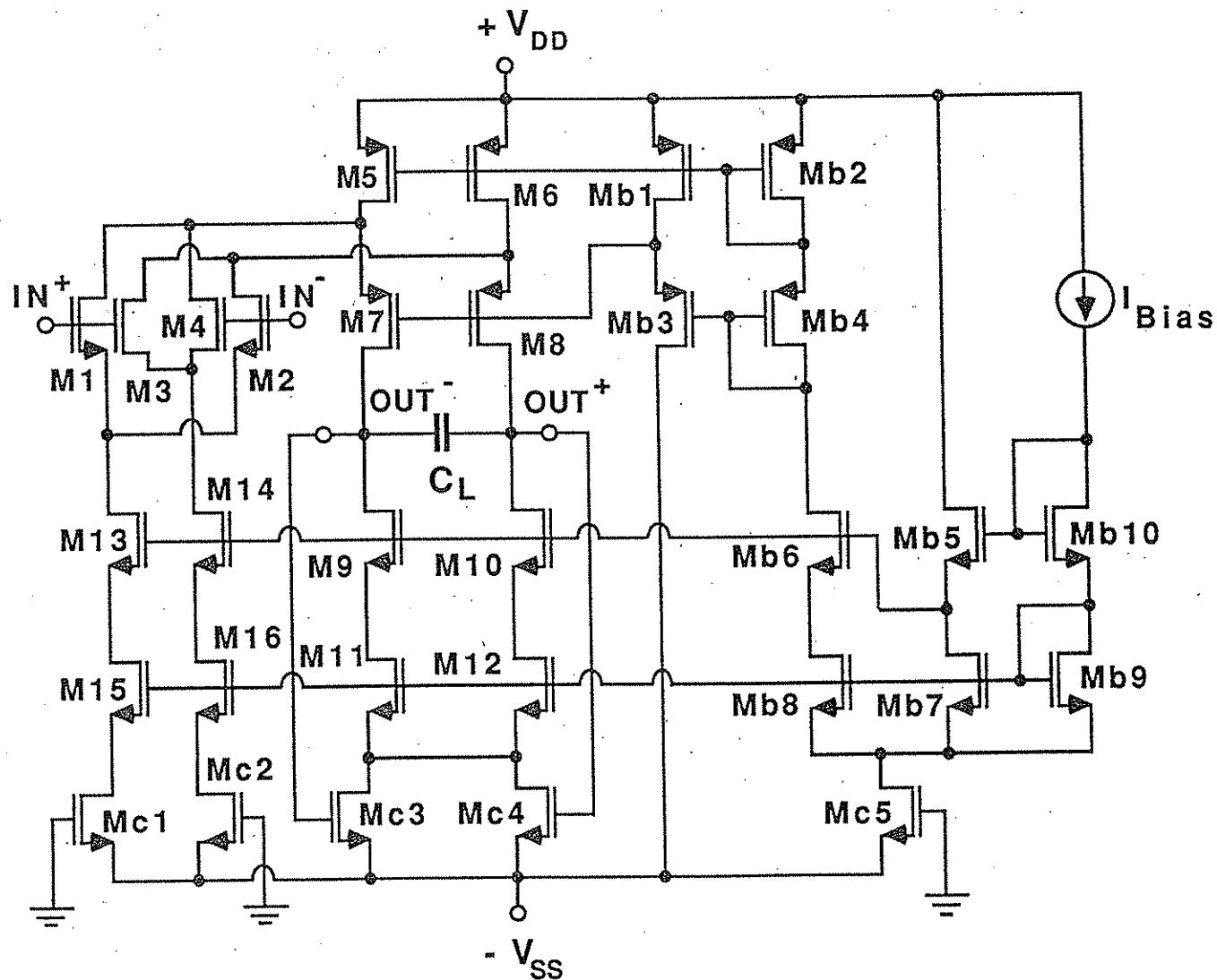


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Fully-Differential Transconductance Amplifier

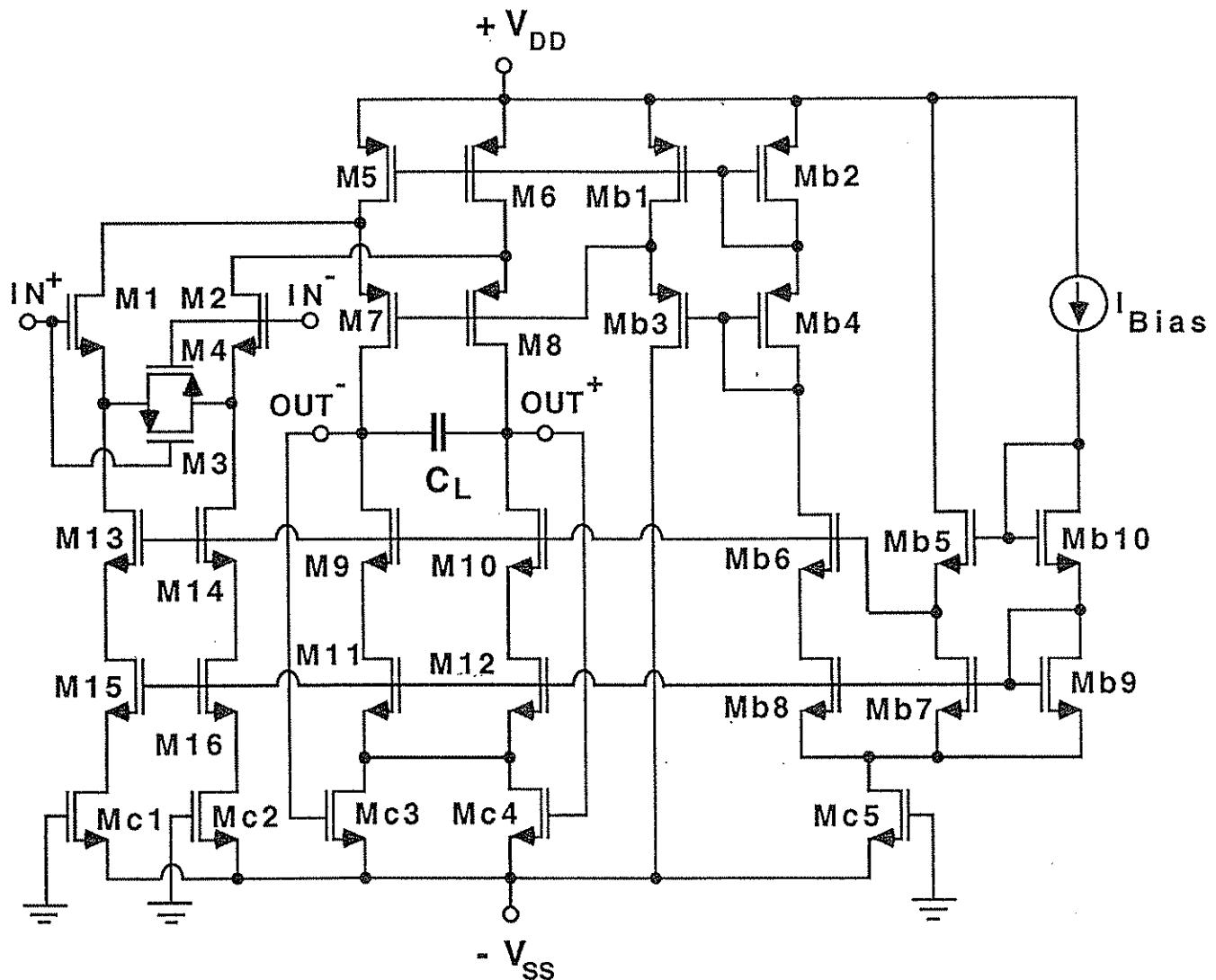
Version3: Folded Cascode with Cross-Coupled Inputs



$$G_M = \frac{1}{2} (g_{m1} - g_{m3})$$

Fully-Differential Transconductance Amplifier

Version4: Folded Cascode with Source Degeneration Devices

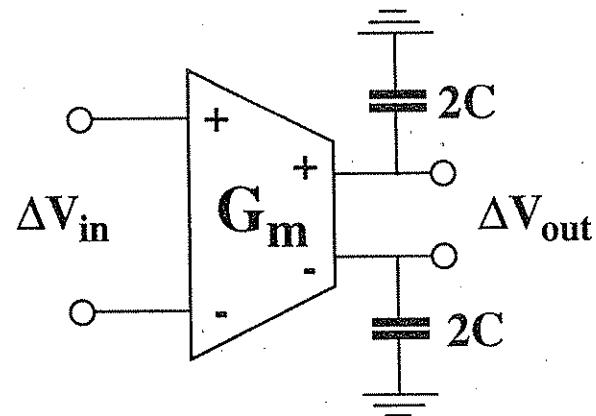


$$G_M = \frac{g_{m1}}{2 + g_{m1}/(g_{d3} + g_{d4})}$$

3. Filter Topologies

Basic building block

Ota-C Integrator



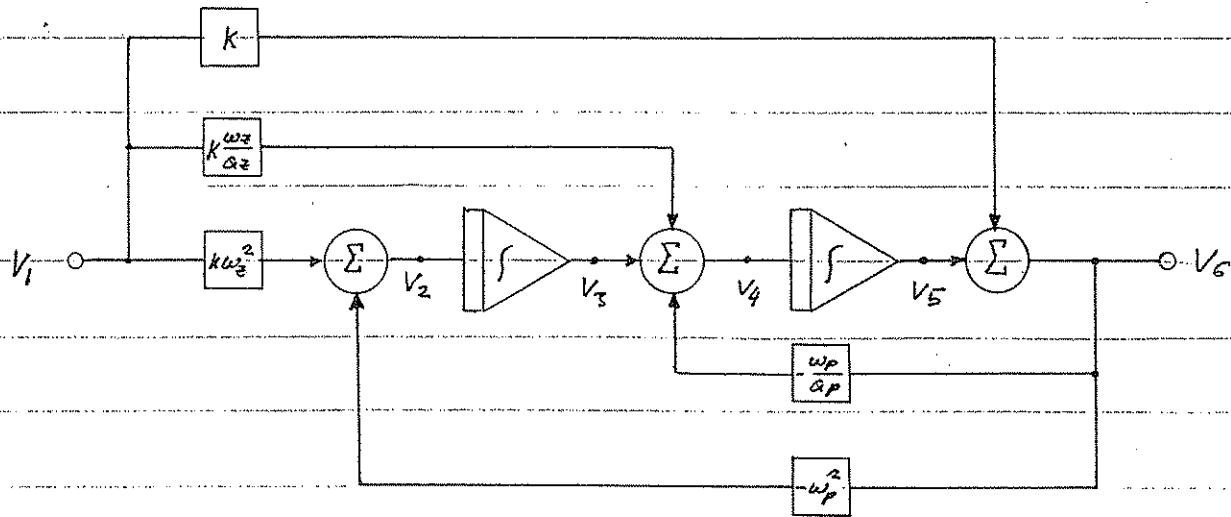
$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{G_m}{sC}$$

State-variable filter topologies

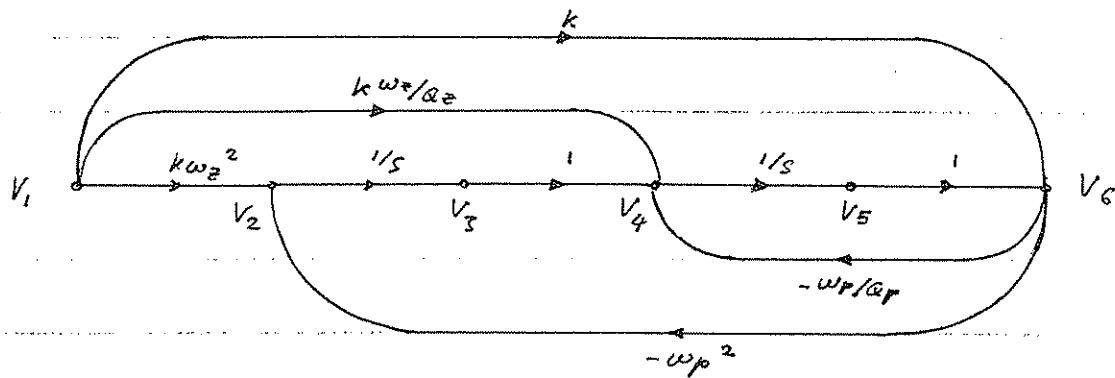
State-variable Biquadrealic Filters

General 2nd order configuration

a) Block Diagram representation:



b) SFG representation: (in s-domain)



c) Voltage Transfer Function:

$$T_{16}(s) = \frac{V_6(s)}{V_1(s)} = k \frac{s^2 + s \frac{\omega_z}{Q_z} + \omega_z^2}{s^2 + s \frac{w_p}{Q_p} + w_p^2}$$

State-Variable Systems

The state-space method is based on the description of system eq. in terms of n 1st order differential (or difference) equations, which may be combined into a 1st order vector matrix equation.

For linear, time-invariant systems, the state equation and the output equation are given by

$$\dot{\bar{x}}(t) = [A] \bar{x}(t) + [\bar{B}] u(t)$$

$$\bar{y}(t) = [C] \bar{x}(t) + [D] u(t)$$

where:

$\bar{x}(t)$: state variable vector

$\bar{y}(t)$: output variable vector

$u(t)$: input vector

Example:

$$\dot{x}_1 = a_1 y + b_1 u$$

$$\dot{x}_2 = a_2 x_1 + a_3 y + b_2 u$$

$$y = x_2 + b_3 u$$

where:

$$\bar{x} = (x_1, x_2)$$

$$[A] = \begin{bmatrix} 0 & a_1 \\ a_2 & a_3 \end{bmatrix}$$

$$[\bar{B}] = \begin{bmatrix} a_1 b_3 + b_1 & 0 \\ a_3 b_3 + b_2 & 0 \end{bmatrix}$$

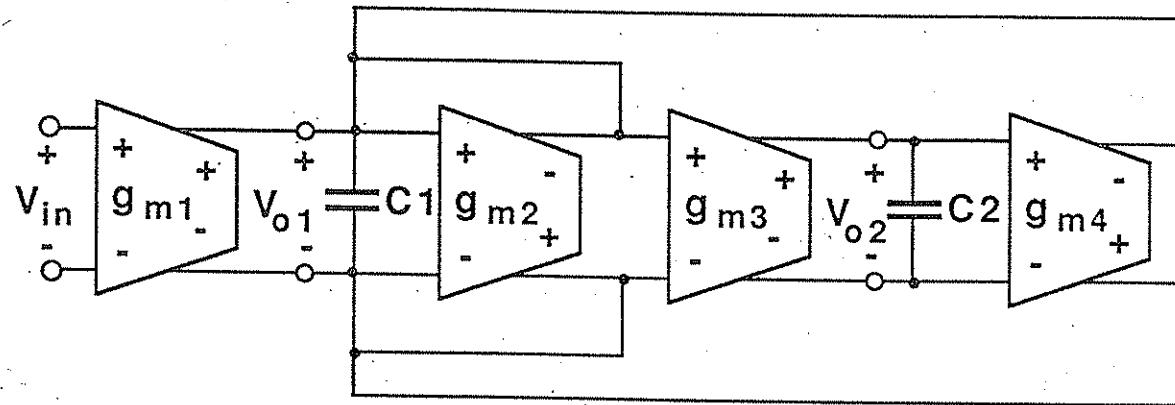
$$\bar{y} = (y, 0)$$

$$[C] = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$[D] = \begin{bmatrix} b_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{u} = (u, 0)$$

2nd Order Ota-C Lowpass/Bandpass Filter



V_{o1} : Bandpass Output

V_{o2} : Lowpass Output

Pole Locations:

$$\omega_p = \sqrt{\frac{g_{m3}g_{m4}}{C_1 C_2}}$$

$$Q_p = \sqrt{\frac{g_{m3}g_{m4}C_1}{C_2}} \cdot \frac{1}{g_{m2}}$$

2nd Order OTA-C Lowpass / Bandpass Filter

a) Ideal system (Root of o/p's are infinite)

equations:

$$(1) \quad V_{o1} = V_{in} \frac{g_{m1}}{sC_1} - V_{o1} \frac{g_{m2}}{sC_1} - V_{o2} \frac{g_{m4}}{sC_1}$$

$$(2) \quad V_{o2} = V_{o1} \frac{g_{m3}}{sC_2}$$

$$(2) \text{ into } (1) \quad \therefore V_{o1} = V_{in} \frac{\frac{g_{m1}}{sC_1}}{(1 + \frac{g_{m2}}{sC_1} + \frac{g_{m3}g_{m4}}{s^2C_1C_2})}$$

transfer functions:

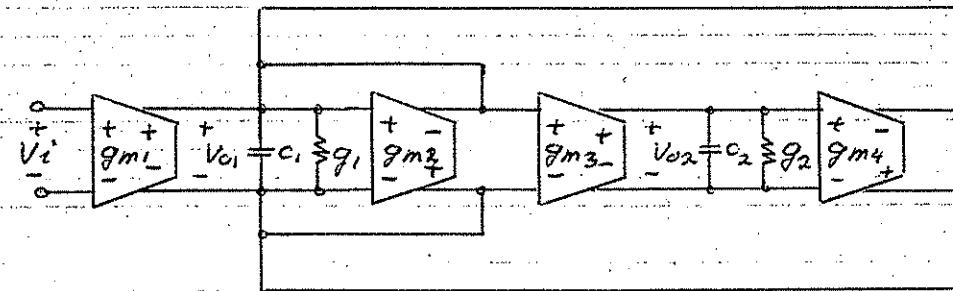
$$\left| \begin{array}{l} \frac{V_{o1}}{V_{in}} = \frac{s \frac{g_{m1}}{C_1}}{s^2 + s \frac{g_{m2}}{C_1} + \frac{g_{m3}g_{m4}}{C_1C_2}} \\ \frac{V_{o2}}{V_{in}} = \frac{\frac{g_{m1}g_{m3}}{C_1C_2}}{s^2 + s \frac{g_{m2}}{C_1} + \frac{g_{m3}g_{m4}}{C_1C_2}} \end{array} \right| \quad \begin{array}{l} 2^{\text{nd}} \text{ order BP} \\ 2^{\text{nd}} \text{ order LP} \end{array}$$

Poles:

$$\left| \begin{array}{l} \omega_p = \sqrt{\frac{g_{m3}g_{m4}}{C_1C_2}} \\ Q_p = \sqrt{\frac{C_1}{C_2}} \cdot \frac{\sqrt{g_{m3}g_{m4}}}{g_{m2}} \end{array} \right|$$

Trimming: use g_{m3} and g_{m4} for ω_p

use g_{m2} for Q_p

Fully-Differential Ota-C BioccelTopology for Lowpass and BandpassTransfer Functions

$$\frac{V_{o1}}{V_i} = \frac{\frac{g_{m1}}{C_1} (s + \frac{g_2}{g_1})}{s^2 + s \frac{g_{m2}}{C_1} \left[1 + \frac{g_1}{g_{m2}} + \frac{g_2}{g_{m2}} \frac{C_1}{C_2} \right] + \frac{g_{m3}g_{m4}}{C_1 C_2} \left[1 + \frac{g_{m2}g_2 + g_1g_2}{g_{m3}g_{m4}} \right]}$$

$$\frac{V_{o2}}{V_i} = \frac{\frac{g_{m1}g_{m3}}{C_1 C_2}}{s^2 + s \frac{g_{m2}}{C_1} \left[1 + \frac{g_1}{g_{m2}} + \frac{g_2}{g_{m2}} \frac{C_1}{C_2} \right] + \frac{g_{m3}g_{m4}}{C_1 C_2} \left[1 + \frac{g_{m2}g_2 + g_1g_2}{g_{m3}g_{m4}} \right]}$$

$$\omega_p = \sqrt{\frac{g_{m3}g_{m4}}{C_1 C_2}} \sqrt{1 + \frac{g_{m2}g_2 + g_1g_2}{g_{m3}g_{m4}}}$$

$$Q_p = \sqrt{\frac{g_{m3}g_{m4}}{g_{m2}^2 C_2}} \frac{\sqrt{1 + \frac{g_{m2}g_2 + g_1g_2}{g_{m3}g_{m4}}}}{1 + \frac{g_1}{g_{m2}} + \frac{g_2}{g_{m2}} \frac{C_1}{C_2}}$$

If $g_1, g_2 \ll g_m$ If $\frac{g_i}{g_m} = \epsilon \ll 1 \quad i=1,2$

$$\omega_p \approx \sqrt{\frac{g_{m3}g_{m4}}{C_1 C_2}}$$

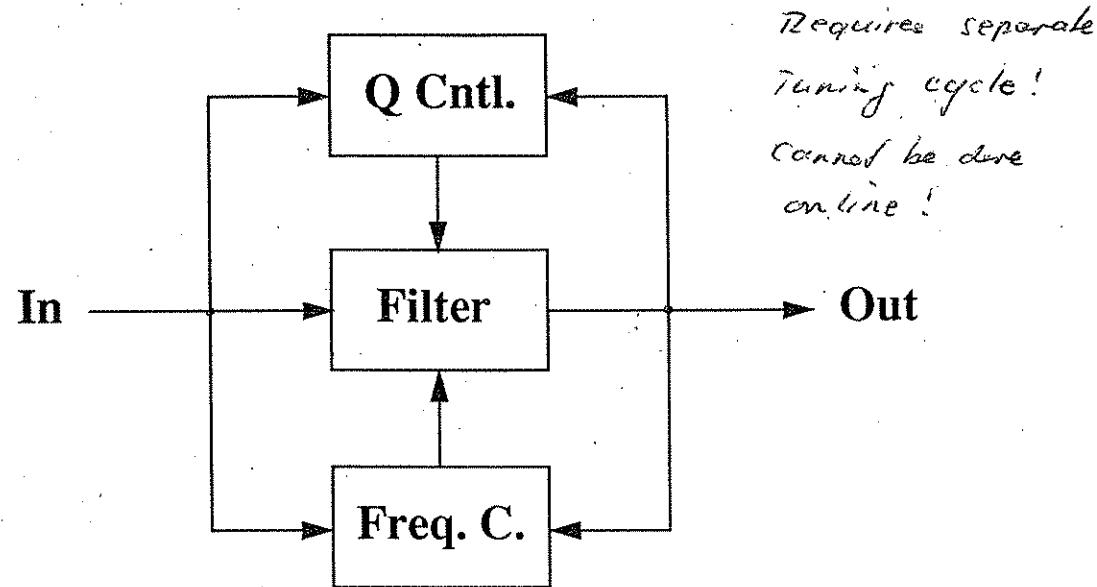
$$\omega_p = \omega_{pid} \left(1 + \frac{1}{2} \epsilon \right)$$

$$Q_p \approx \sqrt{\frac{g_{m3}g_{m4}}{g_{m2}^2 C_2}}$$

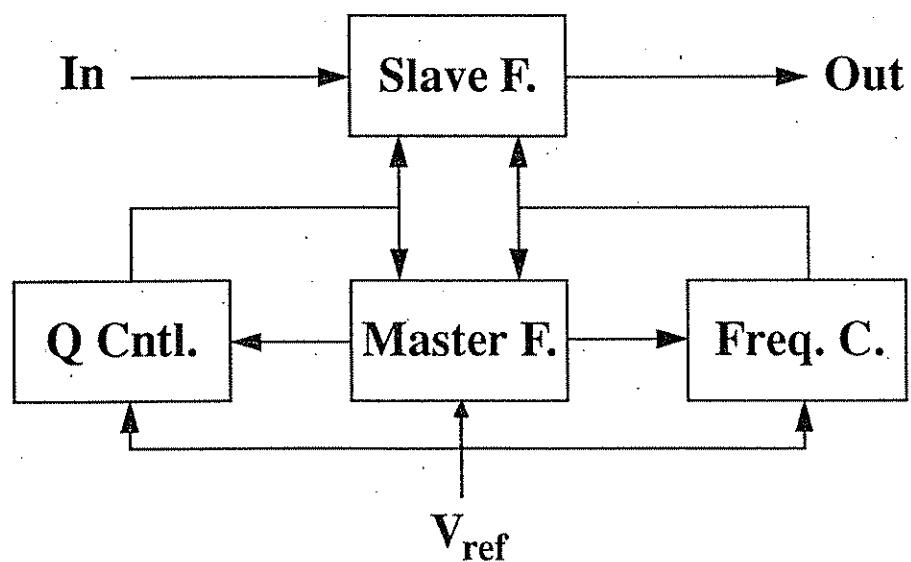
$$Q_p = Q_{pid} \left(1 - \frac{3}{2} \epsilon \right)$$

Filter Tuning

Direct Tuning



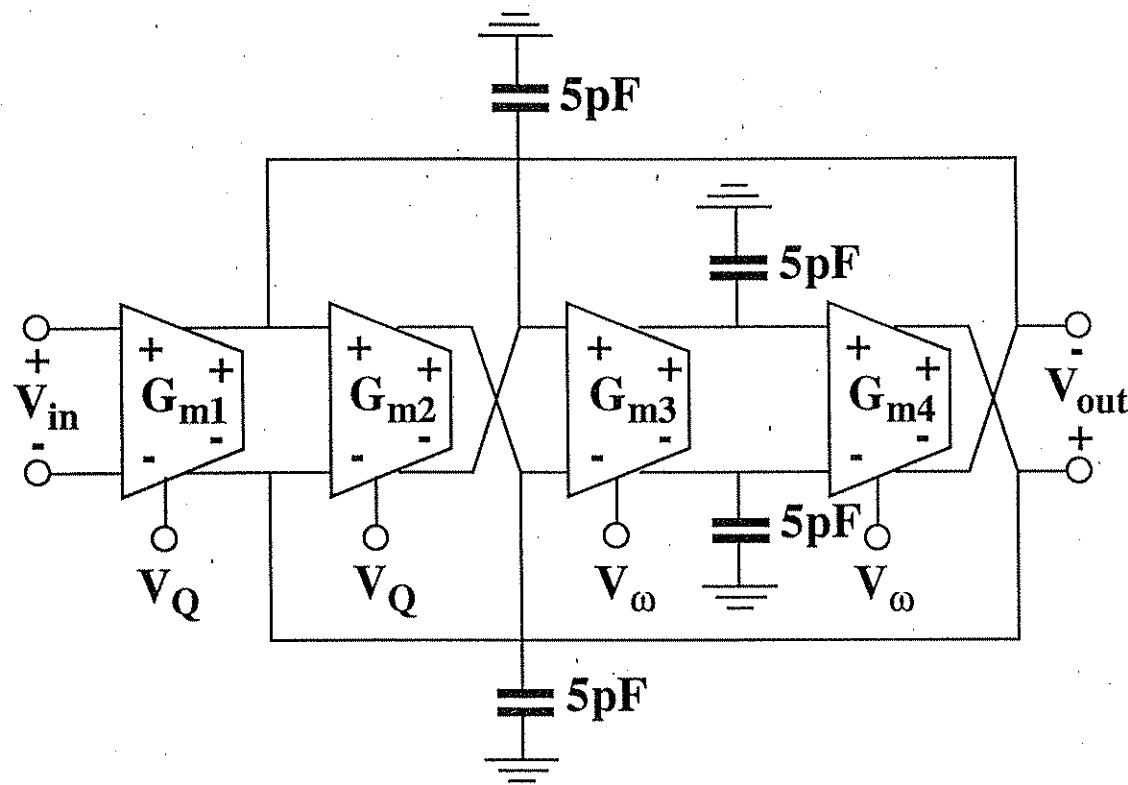
Master-Slave Tuning



4. Simulation Results

Test Circuit

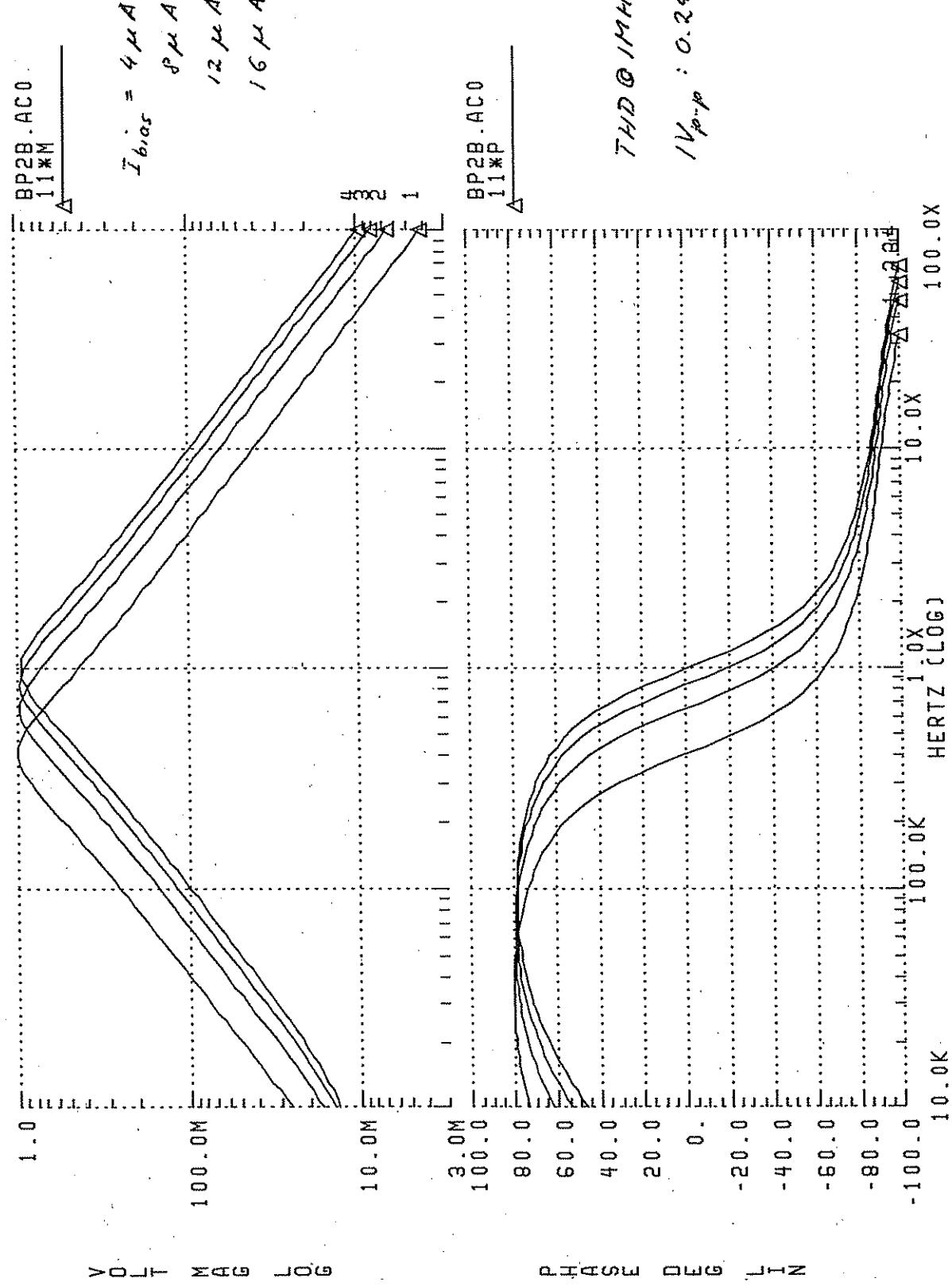
2nd Order Bandpass



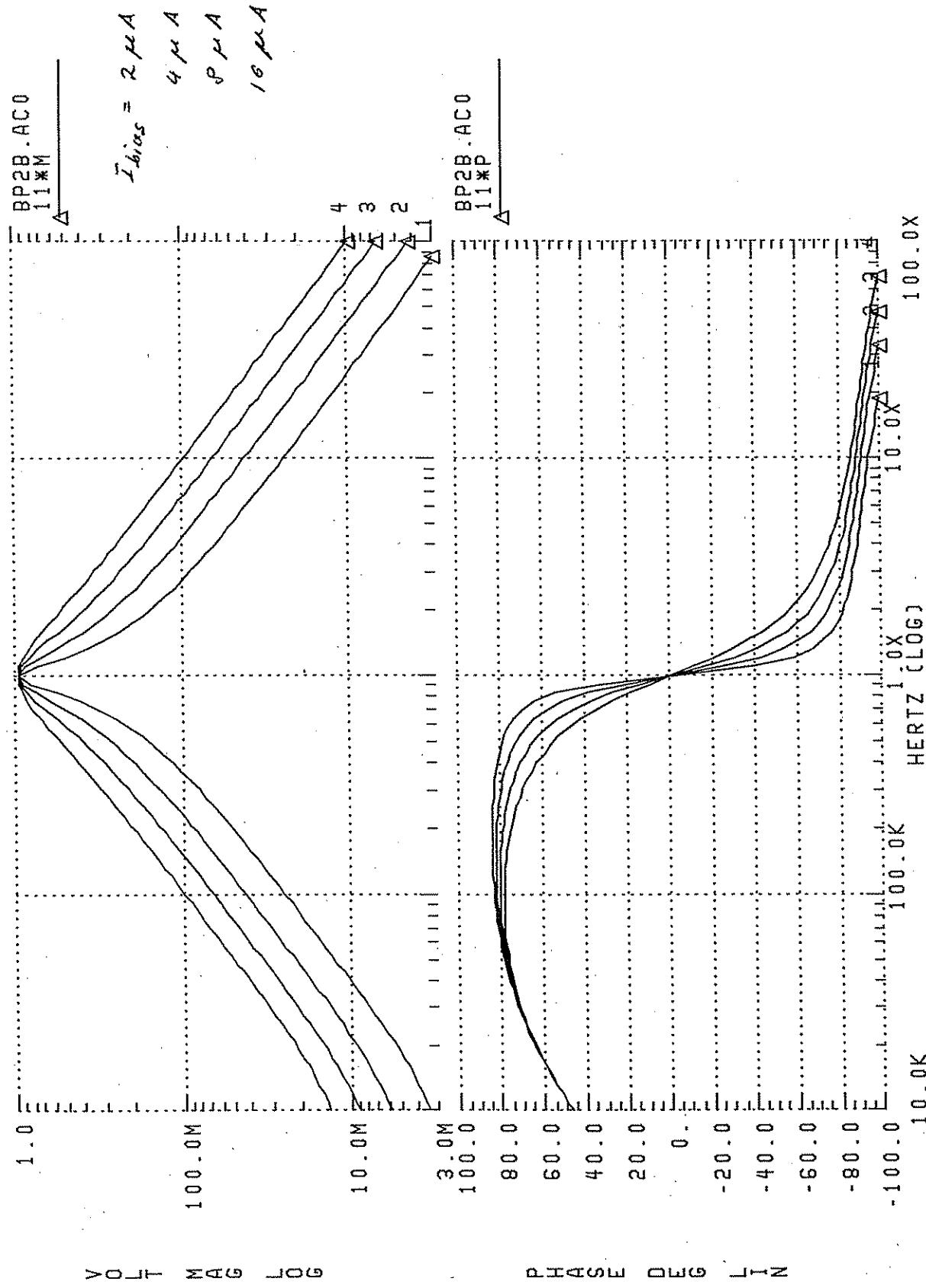
nominal bias: $G_{mi} = 16\mu\text{A/V}$ $i=1,2,3,4$

$$\boxed{\begin{aligned}\omega_p &= 2\pi \cdot 1\text{MHz} \\ Q_p &= 1\end{aligned}}$$

2ND ORDER OTA-C BANDPASS CIRCUIT
25-NOV-93 12:21:47

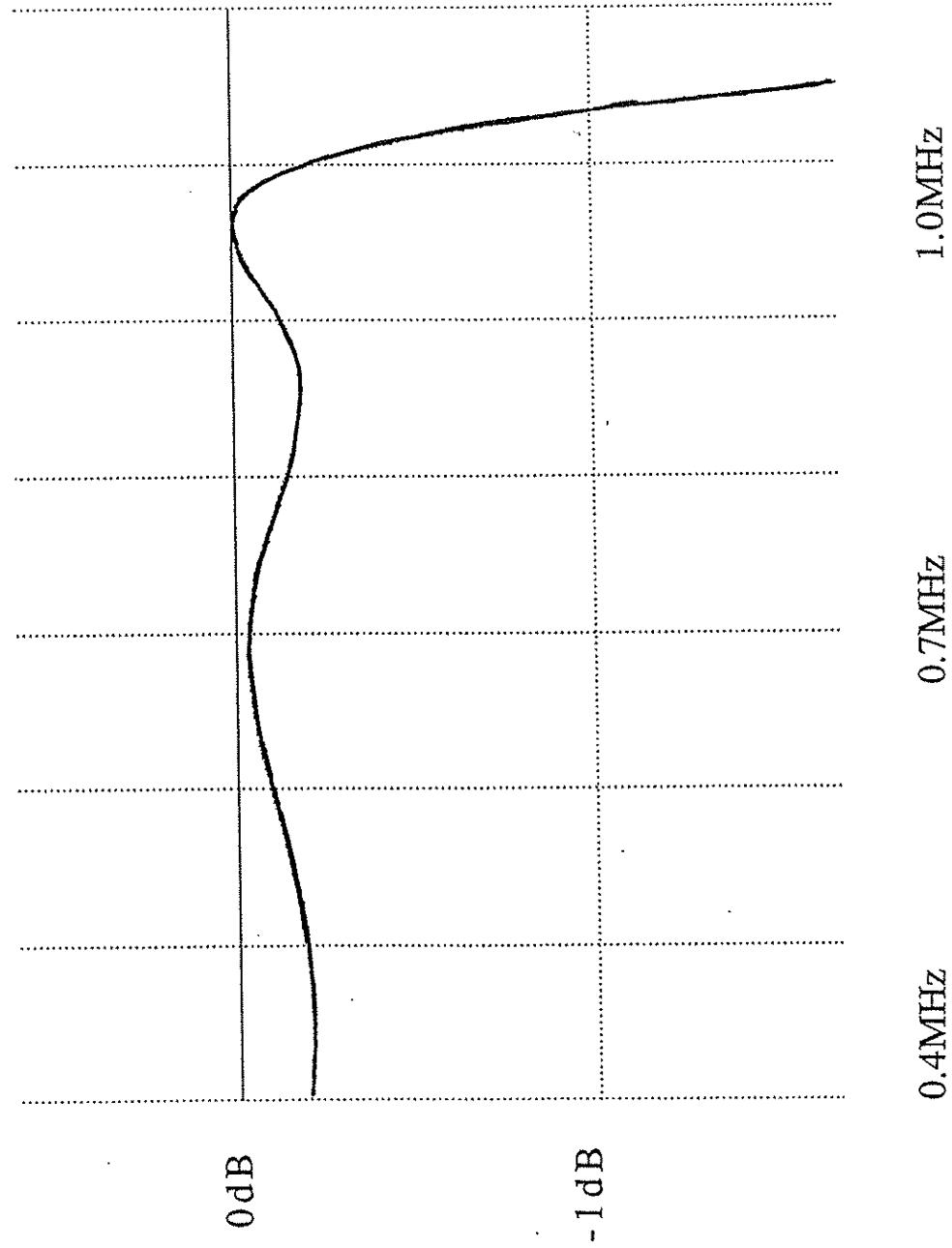


2ND ORDER OTA-C BANDPASS CIRCUIT
25-NOV-93 12:21:47



6th Order Ota-C Lowpass Filter

Passband Response

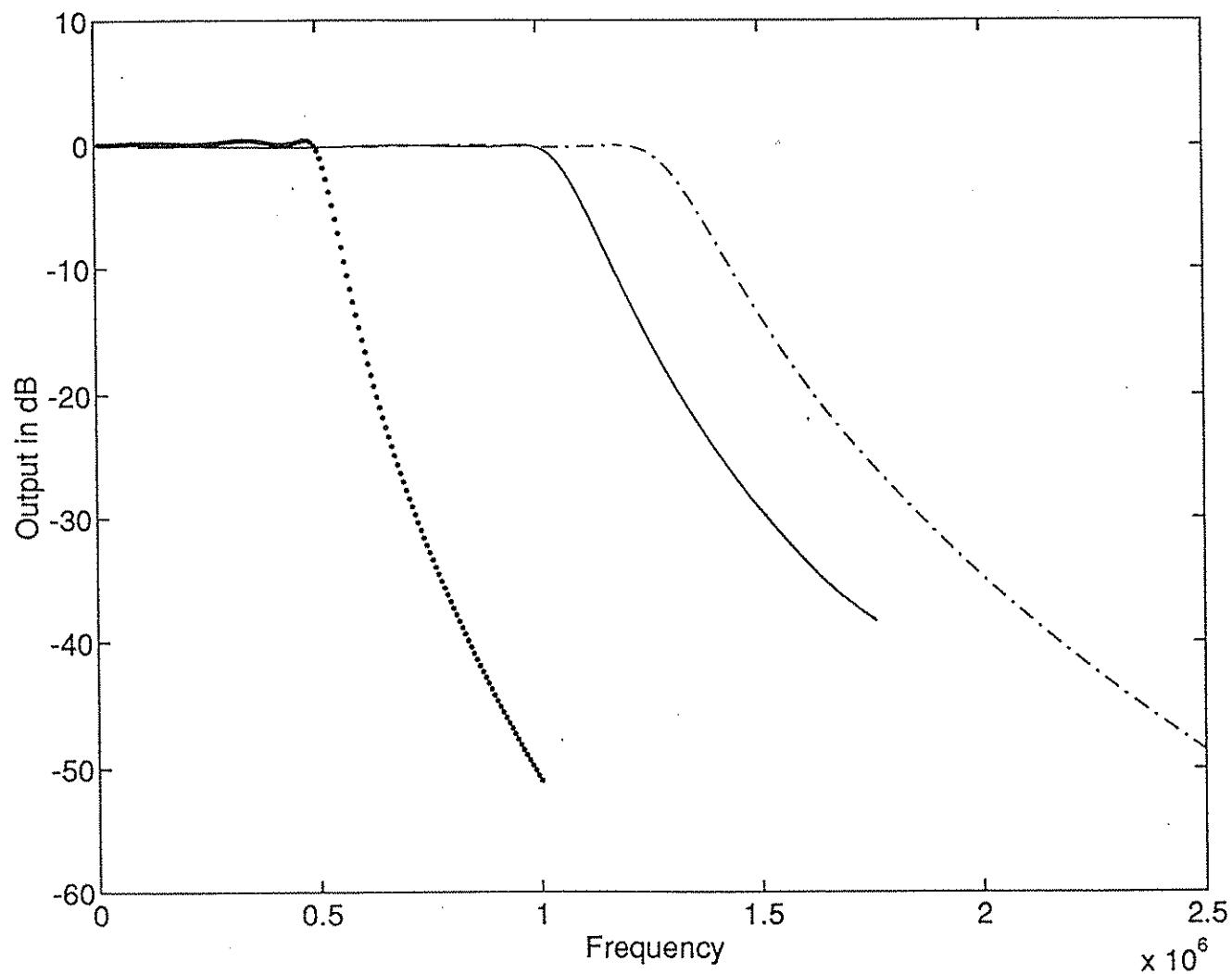


IX-34

$I_x \sim 35$

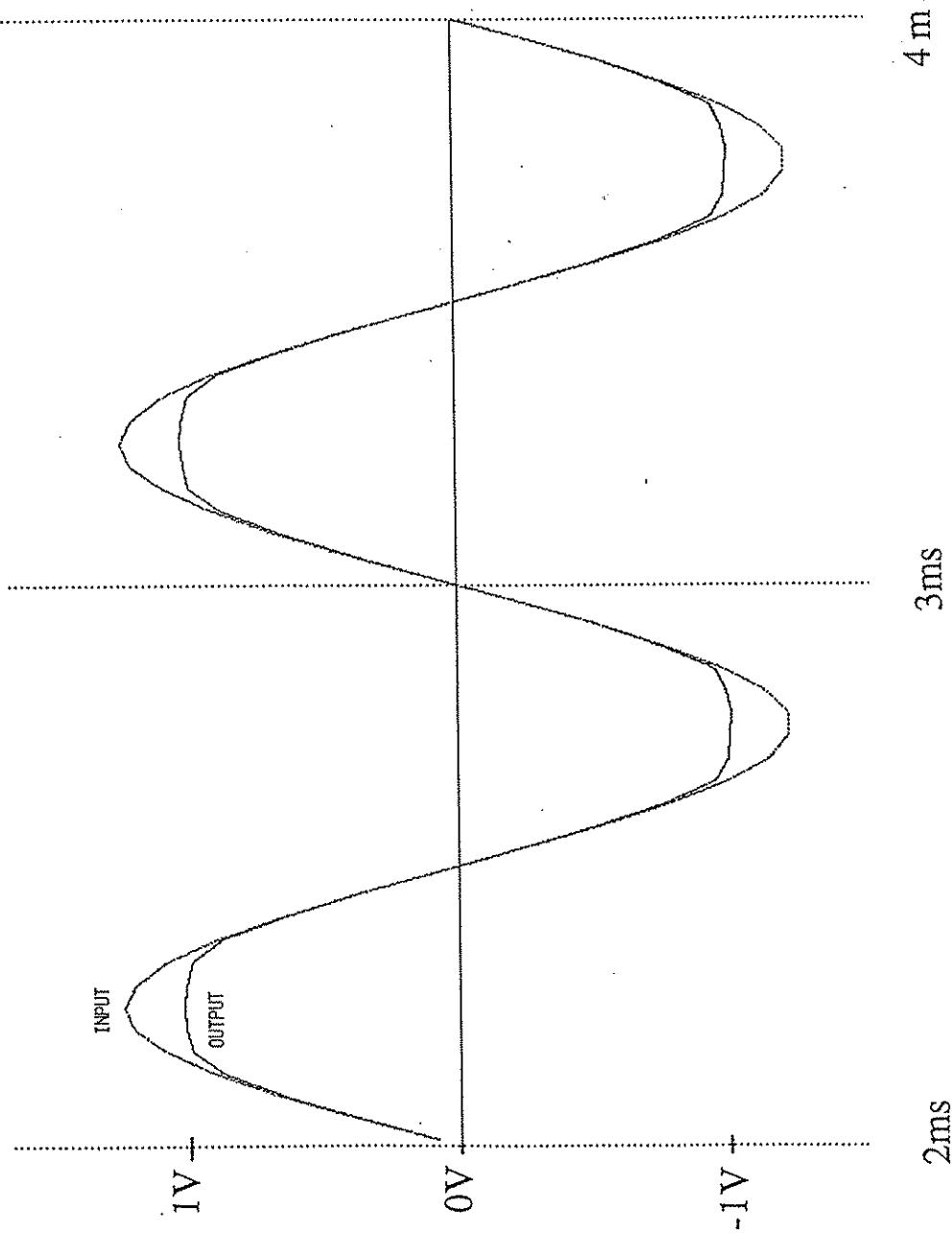
Filter Tuning Range

$I_b1=4\mu m, I_b2=16\mu m, I_b3=24\mu m,$



IX-36

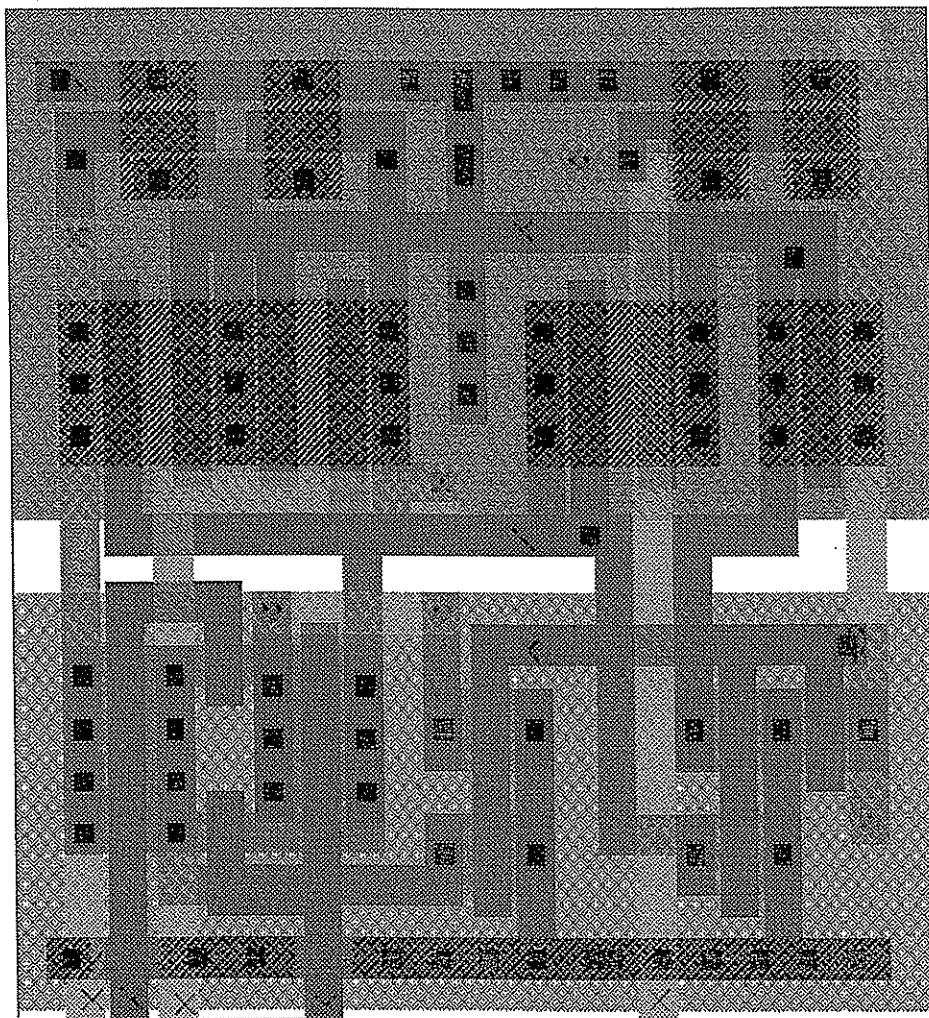
Swing Limitation



5. Layout of Prototype Filter

Transconductance Element ($1.2 \mu\text{m CMOS}$)

(size: $85\mu\text{m} \times 88\mu\text{m}$)



6. Conclusions

- Simple design procedure (for state variable topologies)
- Power and area efficient filter implementation
- Well suited for high frequency applications ($\omega_p = G_m/C$)
- Dynamic range limited by transconductors (Large input swing \rightarrow THD < 60dB)
- Transconductors require tuning
- Accuracy depending on device matching (for on-line master-slave tuning)
- Additional circuitry for automatic tuning required