BME464: Beamforming

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Basics

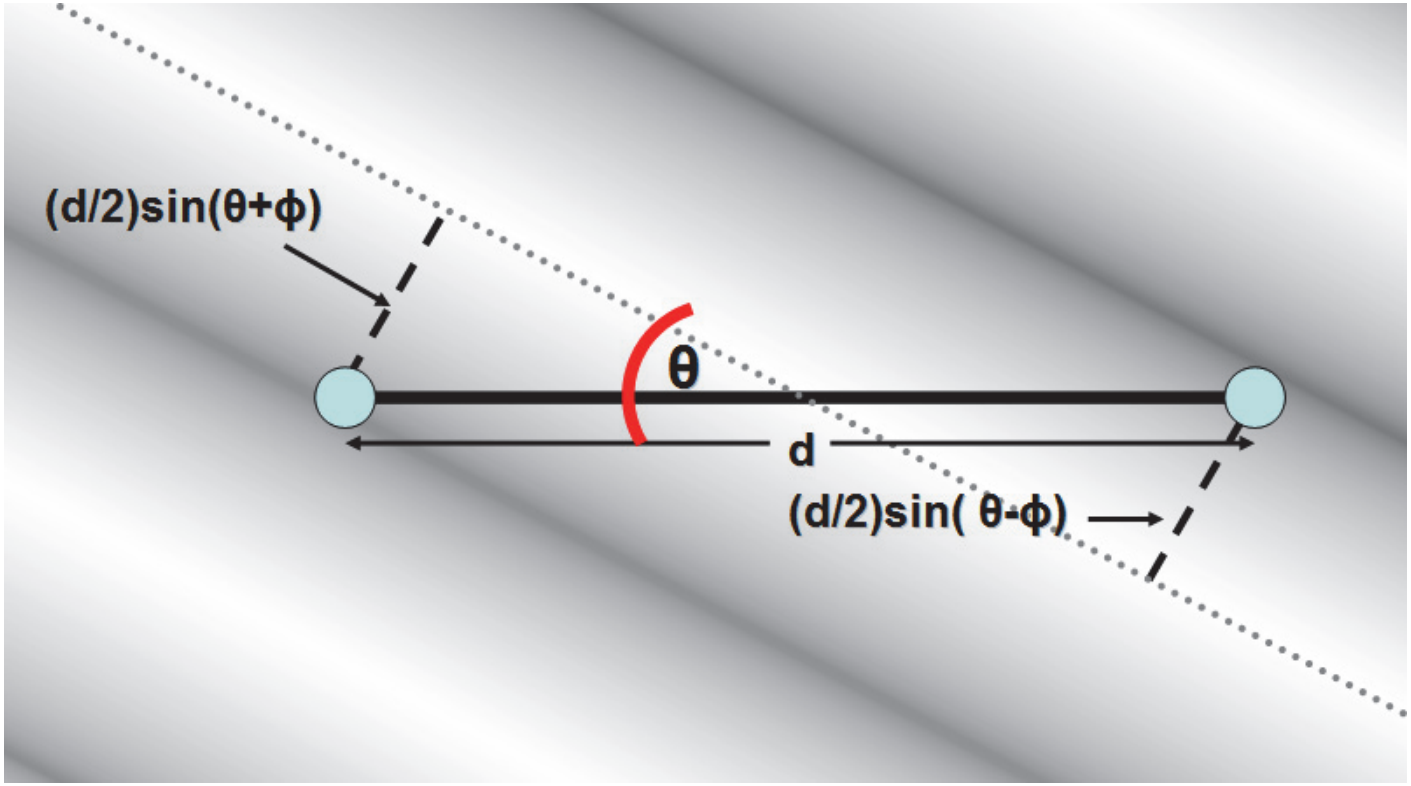

Figure: For a signal arriving from right to left at angle θ, the signal arrives at the right element first. The distance between the two elements is d, so halfway between the elements is d/2. Since the elements don't receive the signal at the same time, a phase shift ($\pm\pi$ in this case) is added such that it appears the peak of the signal is at both elements at the same time. This amplifies the arriving signal.

Math

$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi) \quad (1)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \quad (2)$$

The second identity is really just the first if $\theta = \phi$.

$$E(\theta, \phi) = \sin(\theta + \phi) + \sin(\theta - \phi) \quad (3)$$

$$E(\theta, \phi) = (\sin(\theta)\cos(\phi) + \cancel{\cos(\theta)\sin(\phi)}) + (\sin(\theta)\cos(\phi) - \cancel{\cos(\theta)\sin(\phi)}) \quad (4)$$

$$E(\theta, \phi) = 2\sin(\theta)\cos(\phi) \quad (5)$$

$$E(\theta, \phi) = \sin(\theta)\cancel{2\cos(\phi)} \cdot \frac{\sin(2\phi)}{\sin(\phi)} \quad (6)$$

$$\textbf{since} \quad \sin(2\phi) = 2\sin(\phi)\cos(\phi) \quad (7)$$

$$E(\theta, \phi) = \sin\theta \frac{\sin(2\phi)}{\sin(\phi)} \quad (8)$$

Cont.

Equation 8 is the standard closed-form solution to the beam pattern equation and it is generalized to an arbitrary number of elements N and normalized (so the maximum value is one) as

$$E(\theta, \phi) = \sin\theta \frac{\sin(N\phi)}{N\sin(\phi)} \tag{9}$$

It is not uncommon to see ϕ represented by ψ or u or any number of other variables. This is simply to make the equation look more compact since the phase associated with the delay is

$$\phi = \frac{2\pi}{\lambda} d \sin\theta \tag{10}$$

Beampattern