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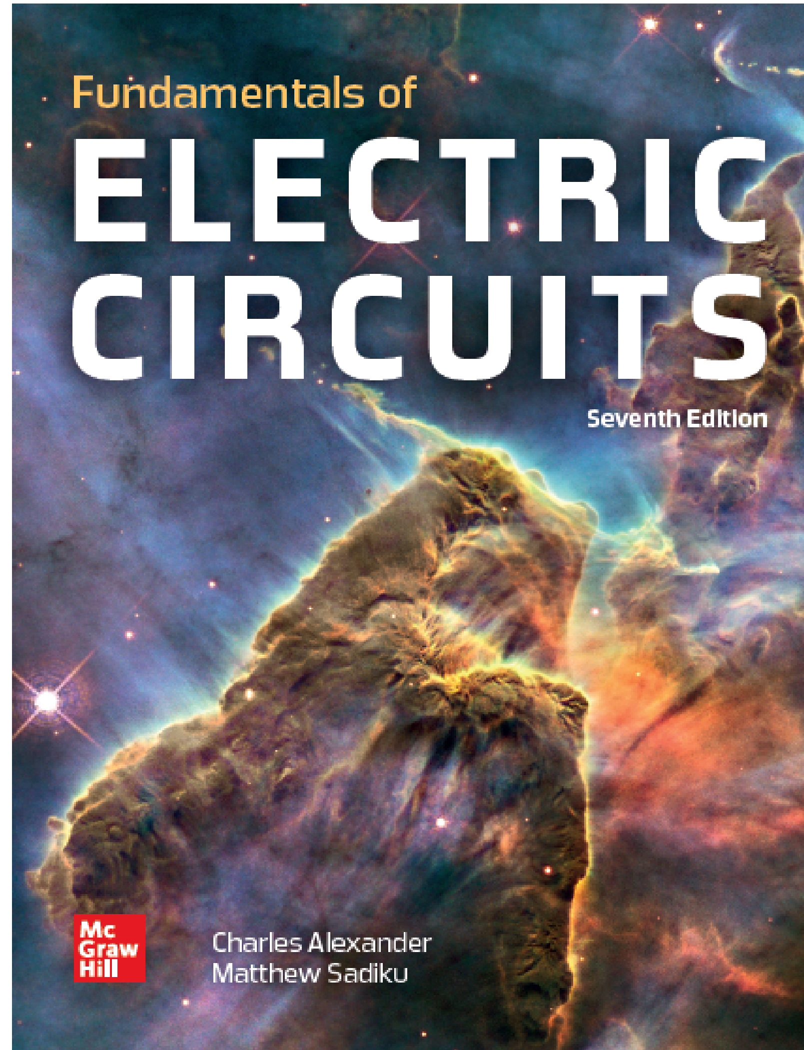
Fundamentals of Electric Circuits

Chapter 2



Overview

- This chapter will introduce Ohm's law: a central concept in electric circuits.
- Resistors will be discussed in more detail.
- Circuit topology and the voltage and current laws will be introduced.
- Finally, meters for measuring voltage, current, and resistivity will be presented.

Resistivity

- Materials tend to resist the flow of electricity through them.
- This property is called “resistance”.
- The resistance of an object is a function of its length, l, and cross sectional area, A, and the material’s resistivity:

$$R = \rho \frac{l}{A}$$

Ohm’s Law

- In a resistor, the voltage across a resistor is directly proportional to the current flowing through it.

$$V = IR$$

- The resistance of an element is measured in units of Ohms, Ω, (V/A).
- The higher the resistance, the less current will flow through for a given voltage.
- Ohm’s law requires conforming to the passive sign convention.

Resistivity of Common Materials

TABLE 2.1

Resistivities of common materials.

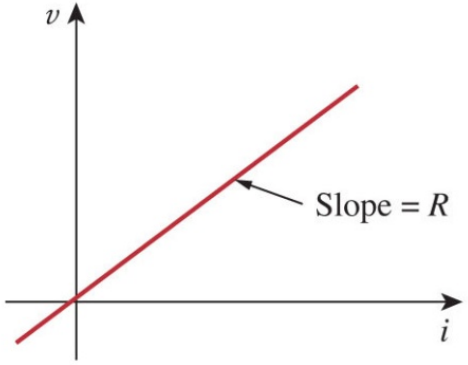
Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Short and Open Circuits

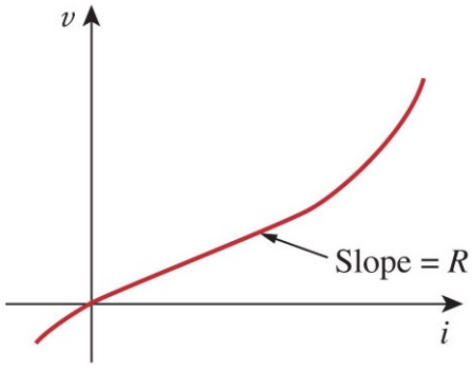
- A connection with almost zero resistance is called a short circuit.
- Ideally, any current may flow through the short.
- In practice this is a connecting wire.
- A connection with infinite resistance is called an open circuit.
- Here no matter the voltage, no current flows.

Linearity

- Not all materials obey Ohm's Law.
- Resistors that do are called linear resistors because their current voltage relationship is always linearly proportional.
- Diodes and light bulbs are examples of non-linear elements.



(a)



(b)

Power Dissipation

- Running current through a resistor dissipates power.

$$p = vi = i^2 R = \frac{v^2}{R}$$

- The power dissipated is a non-linear function of current or voltage.
- Power dissipated is always positive.
- A resistor can never generate power.

Nodes Branches and Loops

- Circuit elements can be interconnected in multiple ways.
- To understand this, we need to be familiar with some network topology concepts.
- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.

Network Topology

- A loop is independent if it contains at least one branch not shared by any other independent loops.
- Two or more elements are in series if they share a single node and thus carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and thus have the same voltage.

Kirchoff’s Laws

Ohm’s law is not sufficient for circuit analysis.

Kirchoff’s laws complete the needed tools.

There are two laws:

- Current law.
- Voltage law.

KCL

- Kirchoff’s current law is based on conservation of charge.
- It states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- It can be expressed as:

$$\sum_{n=1}^{N} i_n = 0$$

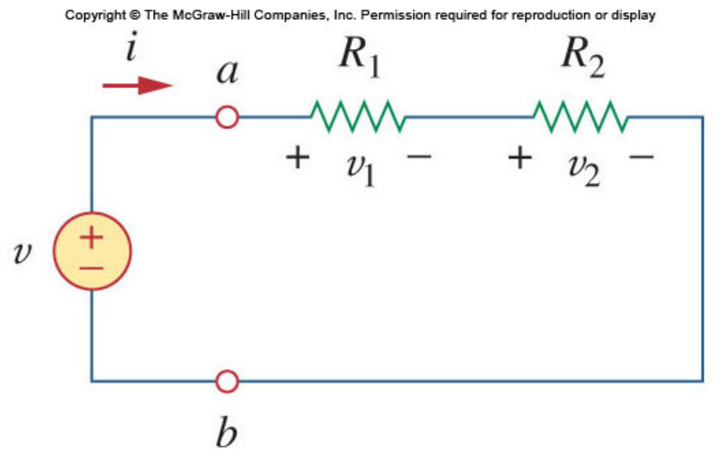
KVL

- Kirchoff's voltage law is based on conservation of energy.
- It states that the algebraic sum of currents around a closed path (or loop) is zero.
- It can be expressed as:

$$\sum_{m=1}^{M} v_m = 0$$

Series Resistors

- Two resistors are considered in series if the same current pass through them.
- Take the circuit shown:
- Applying Ohm's law to both resistors.



$$v_1 = iR_1 \quad v_2 = iR_2$$

- If we apply KVL to the loop we have:

$$-v + v_1 + v_2 = 0$$

Series Resistors 2

- Combining the two equations:

$$v = v_1 + v_2 = i\left(R_1 + R_2\right)$$

- From this we can see there is an equivalent resistance of the two resistors:

$$R_{eq} = R_1 + R_2$$

- For N resistors in series:

$$R_{eq} = \sum_{n=1}^{N} R_n$$

Voltage Division

- The voltage drop across any one resistor can be known.
- The current through all the resistors is the same, so using Ohm's law:

$$v_1 = \frac{R_1}{R_1 + R_2} v \quad v_2 = \frac{R_2}{R_1 + R_2} v$$

- This is the principle of voltage division.

Parallel Resistors

- When resistors are in parallel, the voltage drop across them is the same.

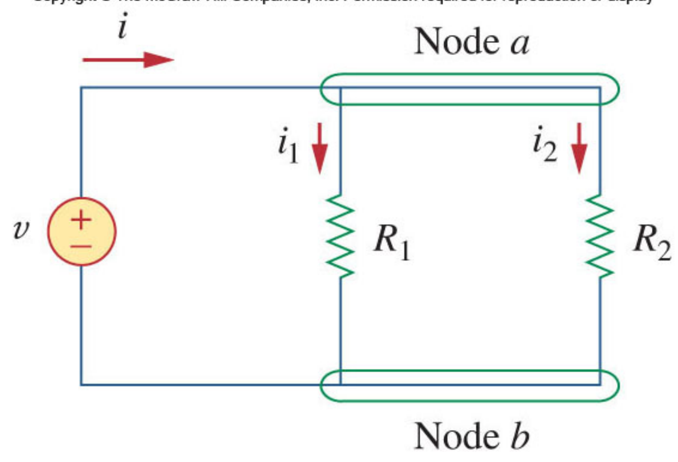
$$v = i_1 R_1 = i_2 R_2$$

- By KCL, the current at node *a* is

$$i = i_1 + i_2$$

- The equivalent resistance is:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



Current Division

- Given the current entering the node, the voltage drop across the equivalent resistance will be the same as that for the individual resistors.

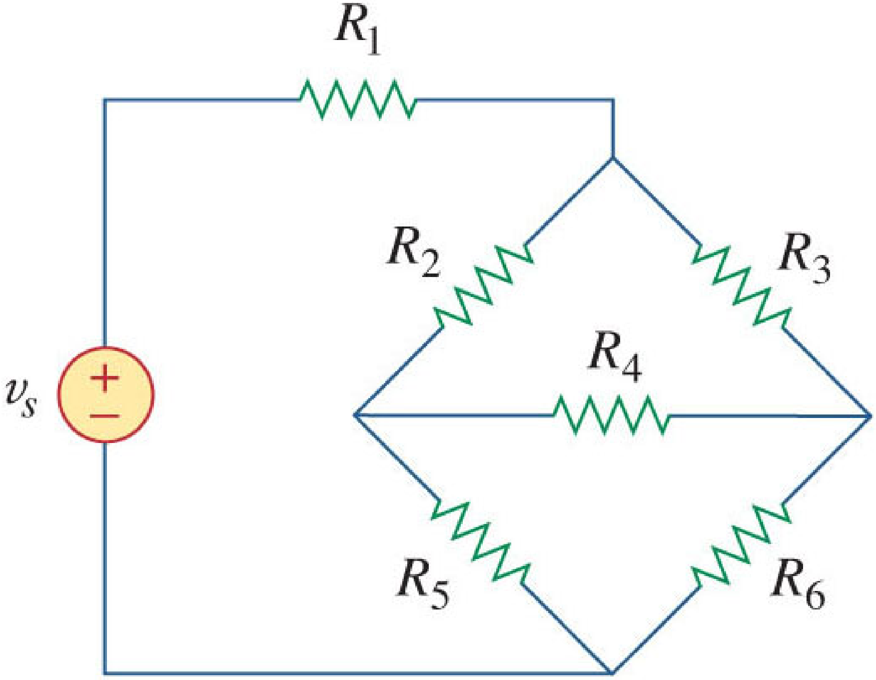
$$v = iR_{eq} = \frac{iR_1 R_2}{R_1 + R_2}$$

- This can be used in combination with Ohm's law to get the current through each resistor:

$$i_1 = \frac{iR_2}{R_1 + R_2} \qquad i_2 = \frac{iR_1}{R_1 + R_2}$$

Wye-Delta Transformations

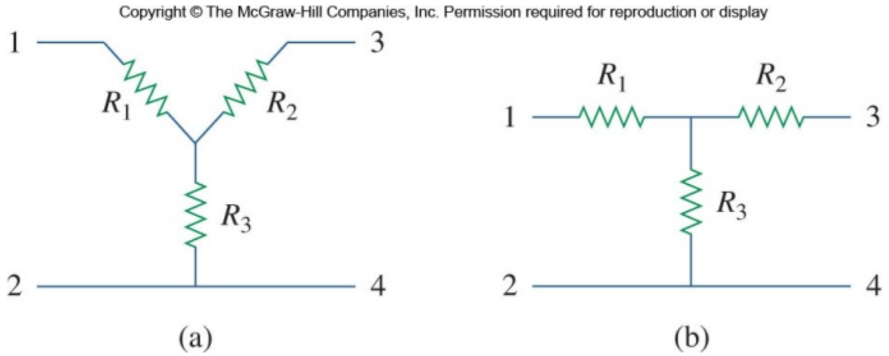
- There are cases where resistors are neither parallel nor series.
- Consider the bridge circuit shown here.
- This circuit can be simplified to a three-terminal equivalent.

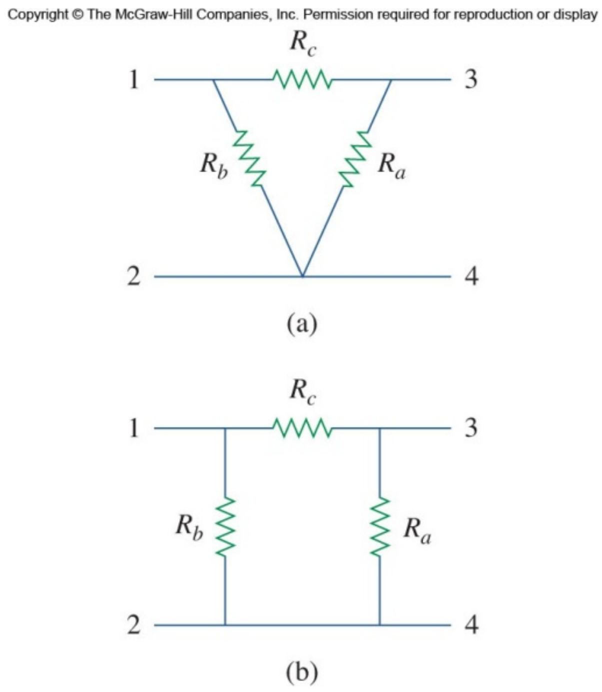


Wye-Delta Transformations II

Two topologies can be interchanged:

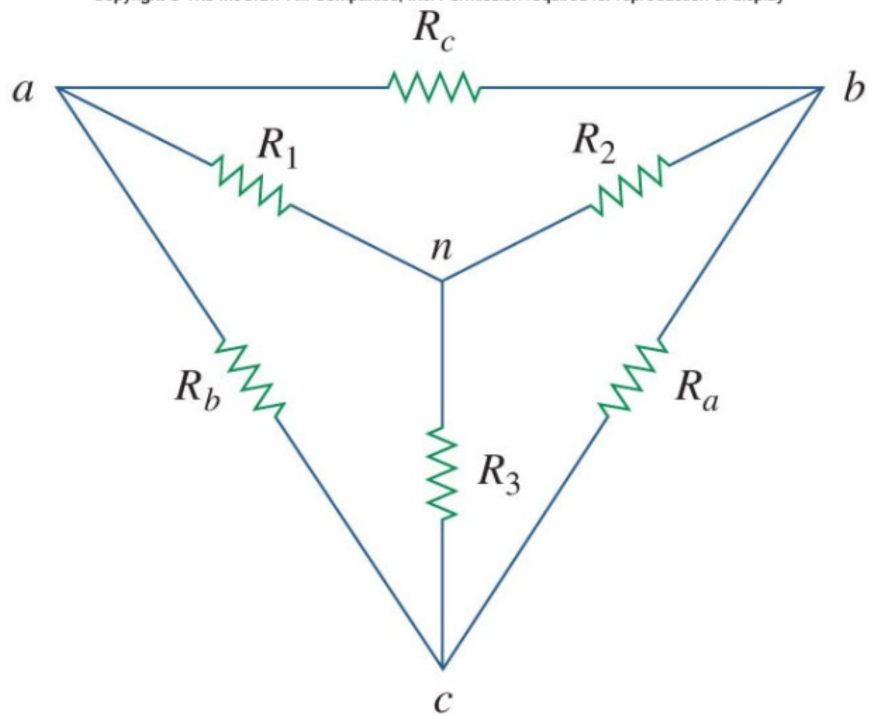
- Wye (Y) or tee (T) networks.
- Delta (Δ) or pi (Π) networks.
- Transforming between these two topologies often makes the solution of a circuit easier.





Wye-Delta Transformations III

- The superimposed wye and delta circuits shown here will used for reference.
- The delta consists of the outer resistors, labeled a,b, and c.
- The wye network are the inside resistors, labeled 1,2, and 3.



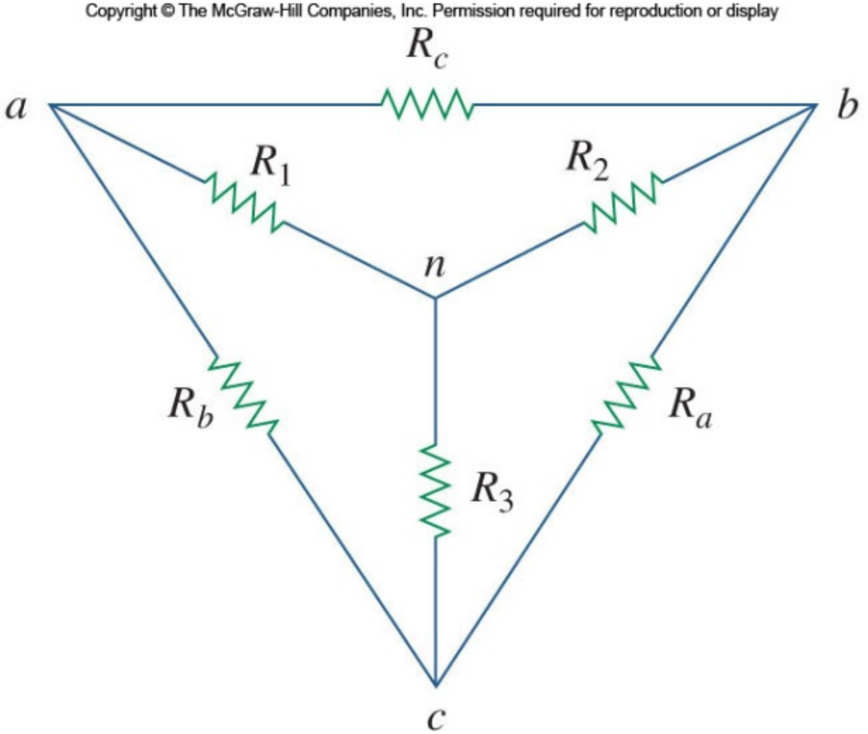
Delta to Wye

- The conversion formula for a delta to wye transformation are:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



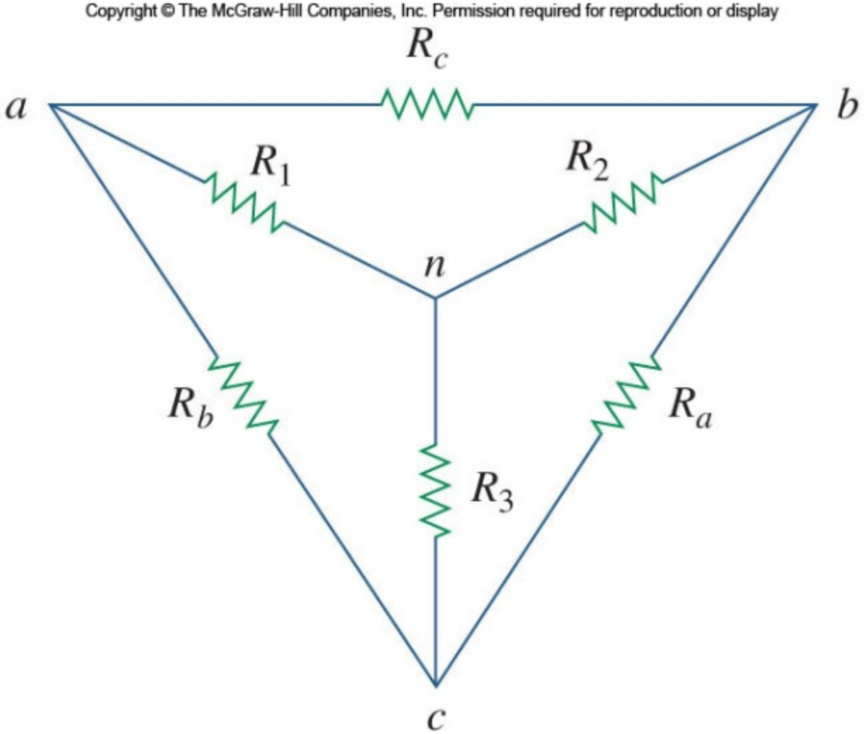
Wye to Delta

- The conversion formula for a wye to delta transformation are:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

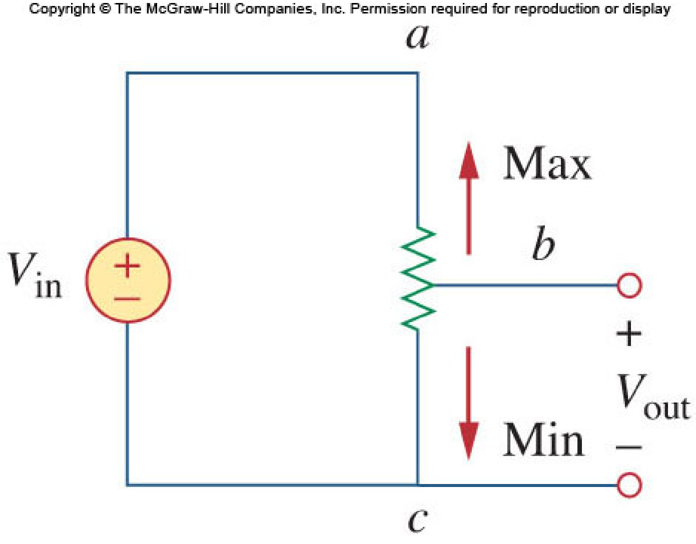


Design of DC Meters

- Resistors by their nature control current.
- This property may be used directly to control voltages, as in the potentiometer.
- The voltage output is:

$$V_{out} = V_{bc} = \frac{R_{bc}}{R_{ac}} V_{in}$$

- Resistors can also be used to make meters for measuring voltage and resistance.



D’Arsonval Meter Movement 1

Here we will look at DC analog meters.

The operation of a digital meter is beyond the scope of this chapter.

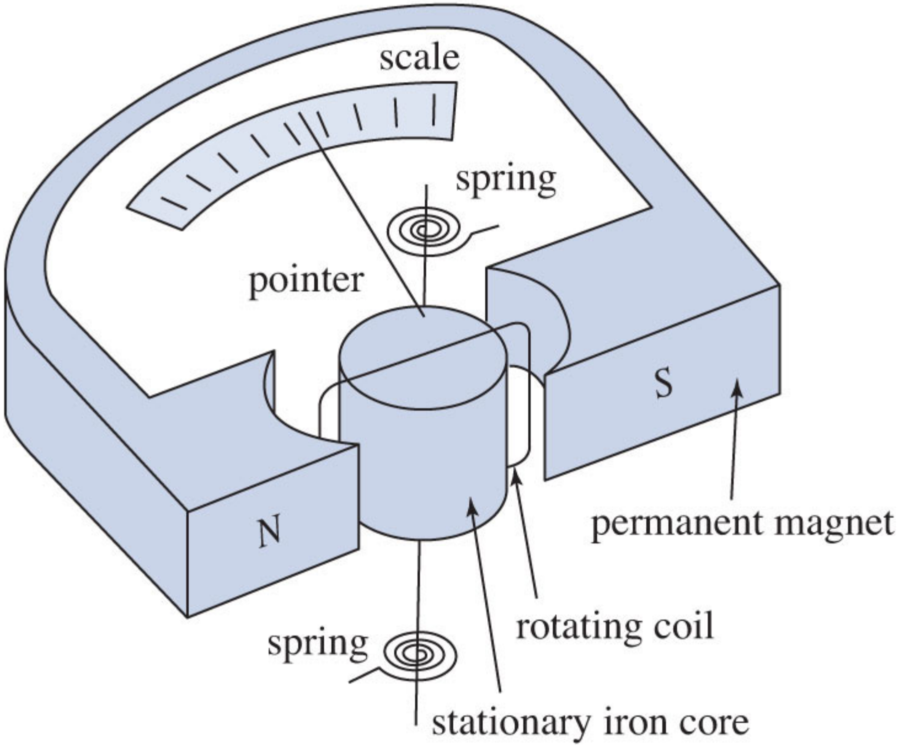
These are the meters where a needle deflection is used to read the measured value.

All of these meters rely on the D’Arsenol meter movement:

- This has a pivoting iron core coil.
- Current through this causes a deflection.

D’Arsonval Meter Movement 2

- Below is an example of a D’Arsonval Meter Movement.



Ammeter

- It should be clear that the basic meter movement directly measured current.
- The needle deflection is proportional to the current up to the rated maximum value.
- The coil also has an internal resistance.
- In order to measure a greater current, a resistor (shunt) may be added in parallel to the meter.
- The new max value for the meter is:

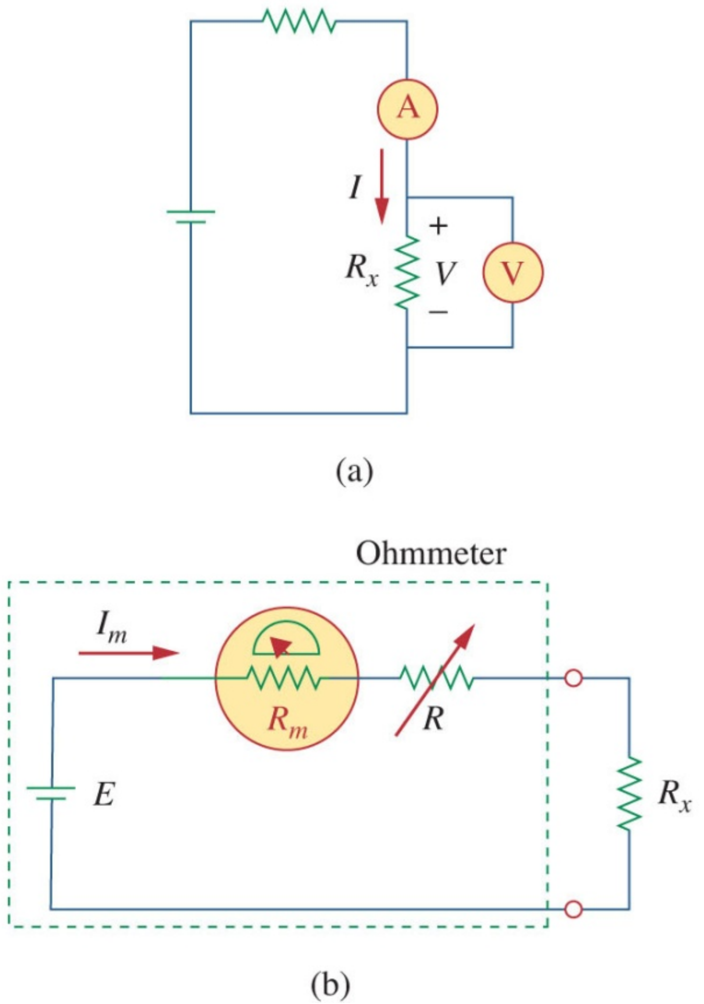
Voltmeter

- Ohm’s law can be used to convert the meter movement into a voltmeter.
- By adding a resistor in series with the movement, the sum of the meter’s internal resistance and the external resistor are combined.
- A voltage applied across this pair will result in a specific current, which can be measured.
- The full scale voltage measured is:

Ohmmeter [1]

- We know that resistance is related the voltage and current passing through a circuit element.
- The meter movement is already capable of measuring current.
- What is needed is to add a voltage source.
- By KVL:

$$R_x = \frac{E}{I_m} - (R + R_m)$$



Ohmmeter II

- The internal resistor is chosen such that when the external resistor is zero, the meter is at full deflection.
- This yields the following relationship between measured current and resistance.

$$R_x = \left(\frac{I_{fs}}{I_m} - 1 \right) (R + R_m)$$

- A consequence to measuring the current is that the readout of the meter will be the inverse of the resistance.

End of Main Content

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