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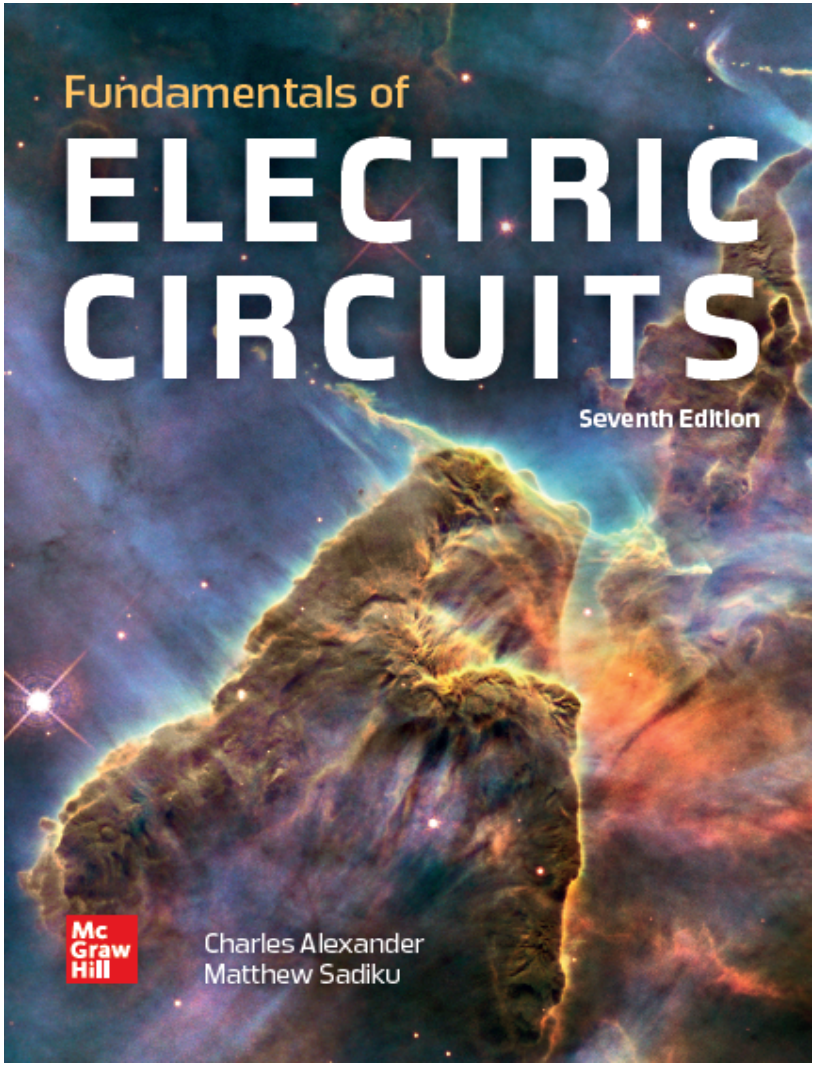
Methods of Analysis

Chapter 3



Overview

With Ohm's and Kirchhoff's law established, they may now be applied to circuit analysis.

Two techniques will be presented in this chapter:

- Nodal analysis, which is based on Kichhoff current law (KCL).
- Mesh analysis, which is based on Kichhoff voltage law (KVL).

Any linear circuit can be analyzed using these two techniques.

The analysis will result in a set of simultaneous equations which may be solved by Cramer's rule or computationally (using MATLAB for example).

Computational circuit analysis using PSpice will also be introduced here.

Nodal Analysis

- If instead of focusing on the voltages of the circuit elements, one looks at the voltages at the nodes of the circuit, the number of simultaneous equations to solve for can be reduced.
- Given a circuit with n nodes, without voltage sources, the nodal analysis is accomplished via three steps:

  1. Select a node as the reference node. Assign voltages $v_1, v_2, \ldots v_n$ to the remaining *n-1* nodes, voltages are relative to the reference node.
  2. Apply KCL to each of the $n - 1$ non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
  3. Solve the resulting $n - 1$ simultaneous equations to obtain the unknown node voltages.

- The reference, or datum, node is commonly referred to as the ground since its voltage is by default zero.

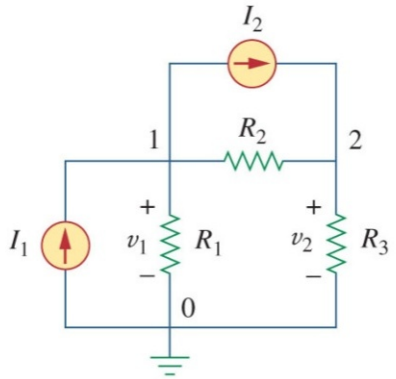
Applying Nodal Analysis

- Let's apply nodal analysis to this circuit to see how it works.
- This circuit has a node that is designed as ground. We will use that as the reference node (node 0).
- The remaining two nodes are designed 1 and 2 and assigned voltages v_1 and v_2.
- Now apply KCL to each node:
- At node 1.

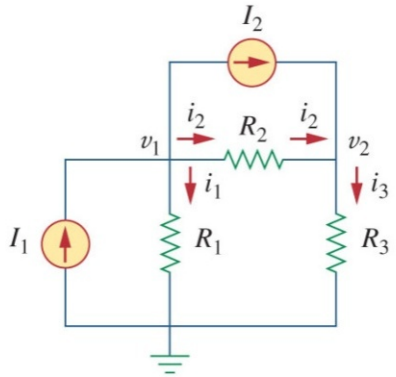
$$I_1 = I_2 + i_1 + i_2$$

- At node 2.

$$I_2 + i_2 = i_3$$



(a)



(b)

Apply Nodal Analysis II

- We can now use OHM's law to express the unknown currents i_1, i_2, *and* i_3 in terms of node voltages.
- In doing so, keep in mind that current flows from high potential to low.
- From this we get:

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2 (v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2$$

Substituting back into the node equations →

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

or

$$I_1 = I_2 + G_1 v_1 + G_2 (v_1 - v_2)$$

$$I_2 + G_2 (v_1 - v_2) = G_3 v_2$$

- The last step is to solve the system of equations.

Including voltage sources

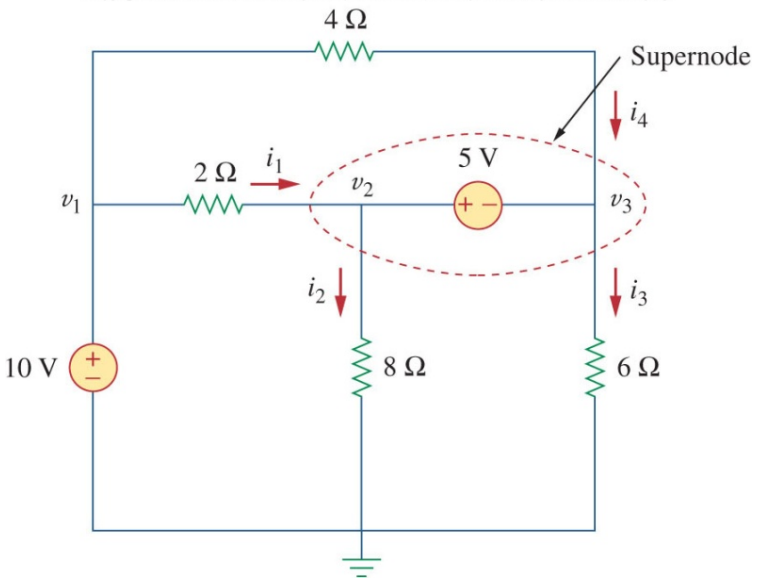
Depending on what nodes the source is connected to, the approach varies.

Between the reference node and a non-reference mode:

- Set the voltage at the non-reference node to the voltage of the source.
- In the example circuit $v_1 = 10V.$

Between two non-reference nodes.

- The two nodes form a supernode.



Supernode

A supernode is formed by enclosing a voltage source (dependant or independent) connected between two non-reference nodes and any elements connected in parallel with it.

Why?

- Nodal analysis requires applying KCL.
- The current through the voltage source cannot be known in advance (Ohm's law does not apply).
- By lumping the nodes together, the current balance can still be described.

In the example circuit node 2 and 3 form a supernode.

The current balance would be: $i_1 + i_4 = i_2 + i_3$

Or this can be expressed as:

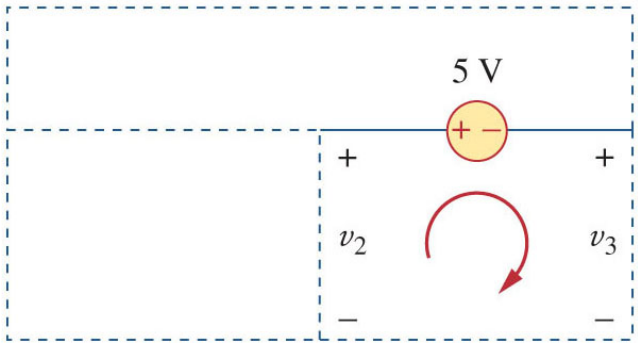
$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

Analysis with a supernode

- In order to apply KVL to the supernode in the example, the circuit is redrawn as shown.
- Going around this loop in the clockwise direction gives:

$$-v_2 + 5 + v_3 = 0 \quad \Rightarrow \quad v_2 - v_3 = 5$$

- Note the following properties of a supernode:
  1. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
  2. A supernode has no voltage of its own.
  3. A supernode requires the application of both KCL and KVL.



Mesh Analysis

Another general procedure for analyzing circuits is to use the mesh currents as the circuit variables.

Recall:

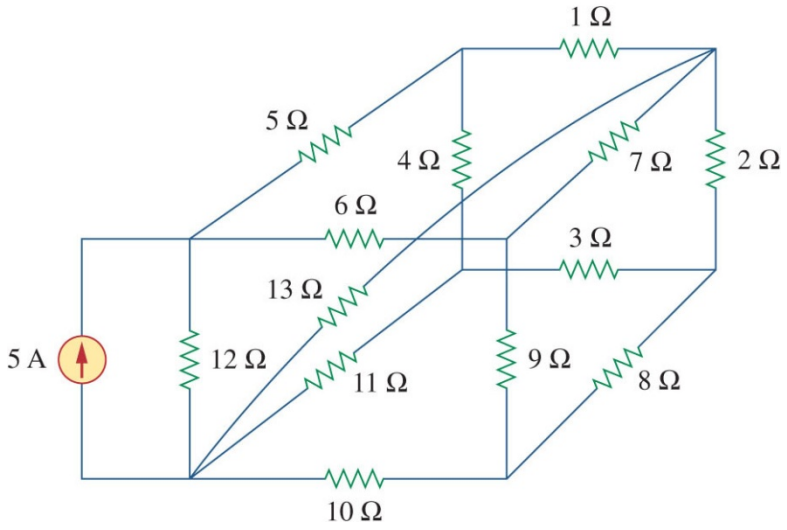
- A loop is a closed path with no node passed more than once.
- A mesh is a loop that does not contain any other loop within it.

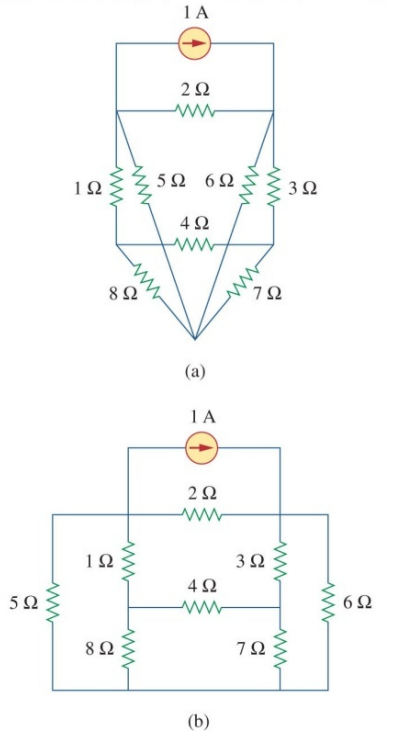
Mesh analysis uses KVL to find unknown currents.

Mesh analysis is limited in one aspect: It can only apply to circuits that can be rendered planar.

A planar circuit can be drawn such that there are no crossing branches.

Planar versus Nonpalanar





The figure on the left is a nonplanar circuit: The branch with the 13Ω resistor prevents the circuit from being drawn without crossing branches

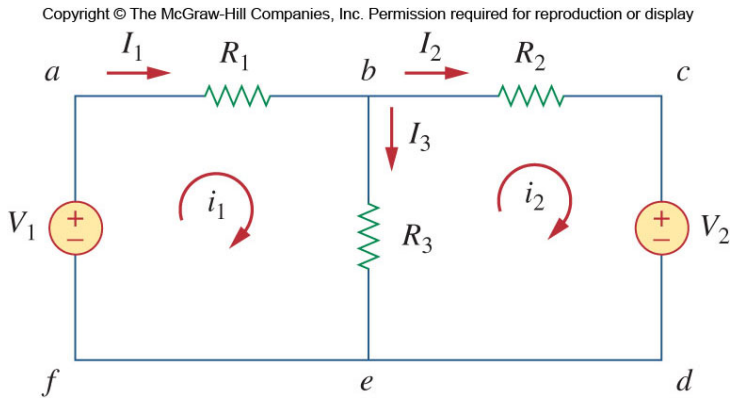
The figure on the right is a planar circuit: It can be redrawn to avoid crossing branches

Mesh Analysis Steps

Mesh analysis follows these steps:

1. Assign mesh currents $i_1, i_2, \ldots i_n$ to the n meshes.
2. Apply KVL to each of the n mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Mesh Analysis Example



- The above circuit has two paths that are meshes (abefa and bcdeb).
- The outer loop (abcdefa) is a loop, but not a mesh.
- First, mesh currents i_1 and i_2 are assigned to the two meshes.
- Applying KVL to the meshes:

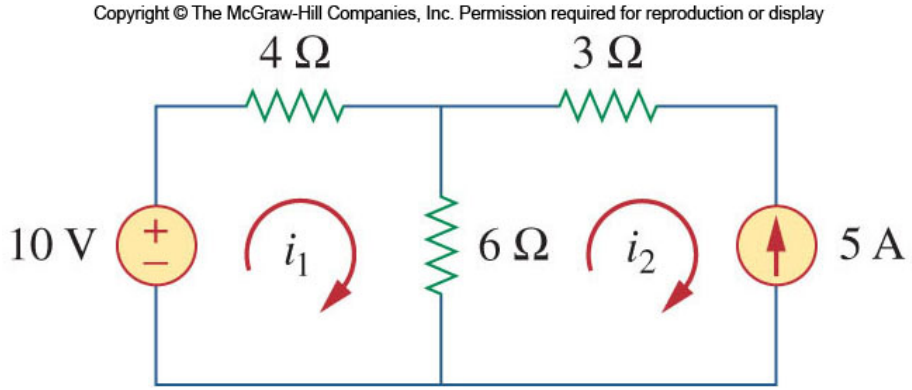
$$-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0 \qquad R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$(R_1 + R_3) i_1 - R_3 i_2 = V_1 \qquad -R_3 i_1 + (R_2 + R_3) i_2 = -V_2$$

Mesh Analysis with Current Sources

- The presence of a current source makes the mesh analysis simpler in that it reduces the number of equations.
- If the current source is located on only one mesh, the current for that mesh is defined by the source.
- For example:

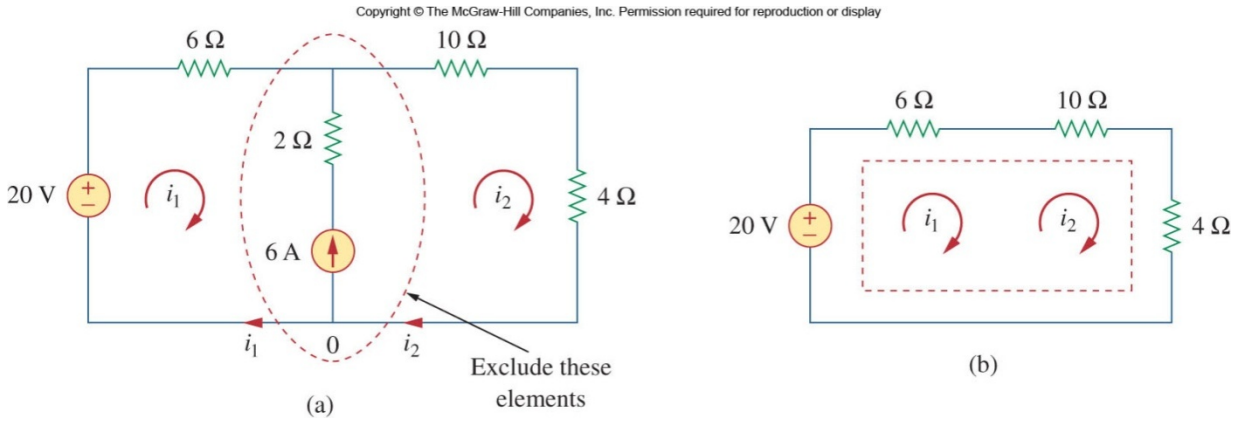


- Here, the current i_2 is equal to $-2A$.

Supermesh

- Similar to the case of nodal analysis where a voltage source shared two non-reference nodes, current sources (dependent or independent) that are shared by more than one mesh need special treatment.
- The two meshes must be joined together, resulting in a supermesh.
- The supermesh is constructed by merging the two meshes and excluding the shared source and any elements in series with it.
- A supermesh is required because mesh analysis uses KVL.
- But the voltage across a current source cannot be known in advance.
- Intersecting supermeshes in a circuit must be combined to for a larger supermesh.

Creating a Supermesh



- In this example, a 6A current course is shared between mesh *1* and *2*.
- The supermesh is formed by merging the two meshes.
- The current source and the 2Ω resistor in series with it are removed.

Supermesh Example

- Using the circuit from the last slide:
- Apply KVL to the supermesh.

$$-20+6i_1+10i_2+4i_2=0 \quad \text{or} \quad 6i_1+14i_2=20$$

- We next apply KCL to the node in the branch where the two meshes intersect.

$$i_2=i_1+6$$

- Solving these two equations we get:

$$i_1=-3.2\text{A} \quad i_2=2.8\text{A}$$

- Note that the supermesh required using both KVL and KCL.

Nodal Analysis by Inspection

- There is a faster way to construct a matrix for solving a circuit by nodal analysis.
- It requires that all current sources within the circuit be independent.
- In general, for a circuit with *N* nonreference nodes, the node-voltage equations may be written as:

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

Nodal Analysis by Inspection II [1]

- Each diagonal term on the conductance matrix is the sum of conductances connected to the node indicated by the matrix index.
- The off diagonal terms, G_{jk} are the negative of the sum of all conductances connected between nodes j and k with $j \neq k$.

Nodal Analysis by Inspection II 2

- The unknown voltages are denoted as v_k
- The sum of all independent current sources directly connected to node *k* are denoted as i_k. Current entering the node are treated as positive and vice versa.
- This matrix equation can be solved for the unknown values of the nodal voltages.

Mesh Analysis by Inspection

- There is a similarly fast way to construct a matrix for solving a circuit by mesh analysis.
- It requires that all voltage sources within the circuit be independent.
- In general, for a circuit with *N* meshes, the mesh-current equations may be written as:

$$\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

- Each diagonal term on the resistance matrix is the sum of resistances in the mesh indicated by the matrix index.

Mesh Analysis by Inspection II

- The off diagonal terms, R_{jk} are the negative of the sum of all resistances in common with meshes j and k with $j \neq k$.
- The unknown mesh currents in the clockwise direction are denoted as i_k.
- The sum taken clockwise of all voltage sources in mesh k are denoted as v_k. Voltage rises are treated as positive.
- This matrix equation can be solved for the values of the unknown mesh currents.

Selecting an Appropriate Approach

In principle both the nodal analysis and mesh analysis are useful for any given circuit.

What then determines if one is going to be more efficient for solving a circuit problem?

There are two factors that dictate the best choice:

- The nature of the particular network is the first factor.
- The second factor is the information required.

Mesh analysis when…

If the network contains:

- Many series connected elements.
- Voltage sources.
- Supermeshes.
- A circuit with fewer meshes than nodes.

If branch or mesh currents are what is being solved for.

Mesh analysis is the only suitable analysis for transistor circuits.

It is not appropriate for operational amplifiers because there is no direct way to obtain the voltage across an op-amp.

Nodal analysis if…

If the network contains:

- Many parallel connected elements.
- Current sources.
- Supernodes.
- Circuits with fewer nodes than meshes.

If node voltages are what are being solved for.

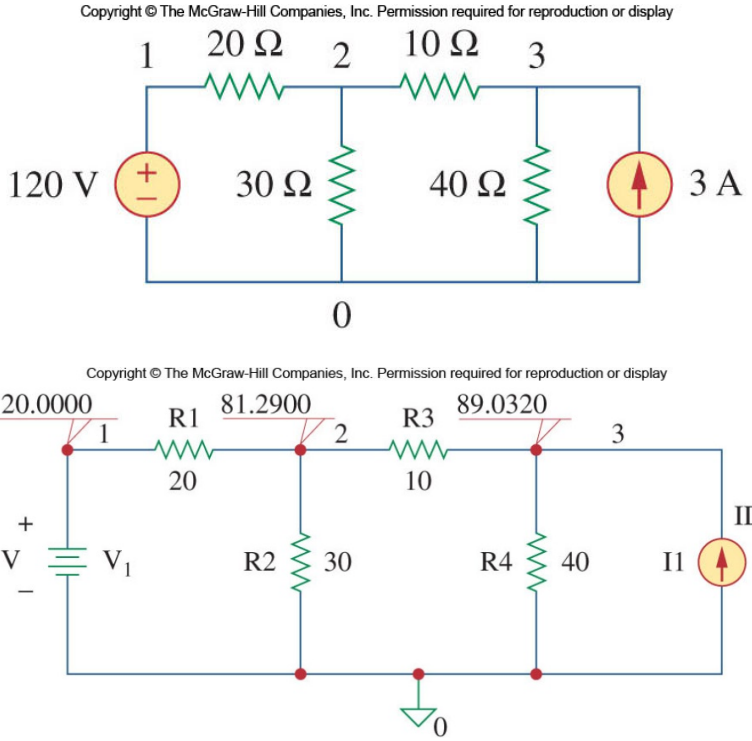
Non-planar circuits can only be solved using nodal analysis.

This format is easier to solve by computer.

Circuit Analysis with PSpice

- PSpice is a common program used for circuit analysis.
- It is capable of determining all of the branch voltages and currents if the numerical values for all circuit components are known.
- Analysis using PSpice begins with drawing a schematic view of the circuit.
- The node voltages of interest can be indicated during the schematic setup using ‘VIEWPOINTS.’
- The values are obtained by running ‘Analysis/Simulate.’

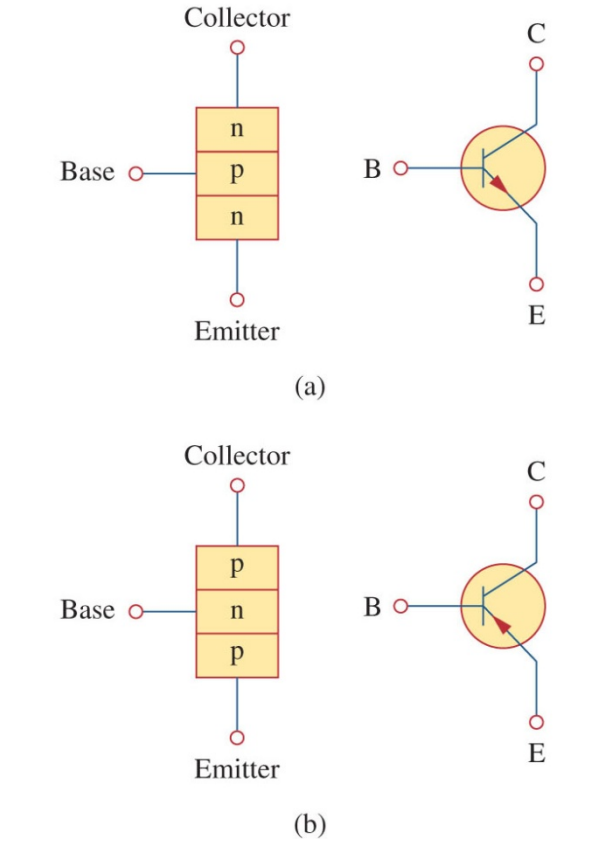
Rendering a circuit in PSpice



Converting a standard schematic (top) to a PSpice schematic (bottom) is fairly straightforward. Note the explicit definition of the reference node by using the ground symbol labeled with a ‘0’ and the nodal voltages of interest displayed with the ‘viewpoints’

Application: DC transistor circuit

- Here we will use the approaches learned in this chapter to analyze a transistor circuit.
- In general, there are two types of transistors commonly used: Field Effect (FET) and Bipolar Junction (BJT).
- This problem will use a BJT.
- A BJT is a three terminal device, where the input current into one terminal (the base) affects the current flowing out of a second terminal (the collector).
- The third terminal (the emitter) is the common terminal for both currents.



KCL and KVL for a BJT

- The currents from each terminal can be related to each other as follows:

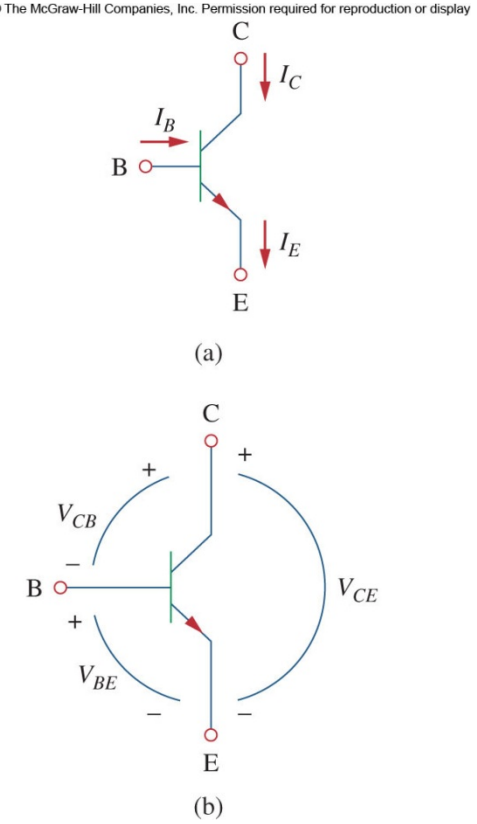
$$I_E = I_B + I_C$$

- The base and collector current can be related to each other by the parameter β, which can range from 50-1000.

$$I_C = \beta I_B$$

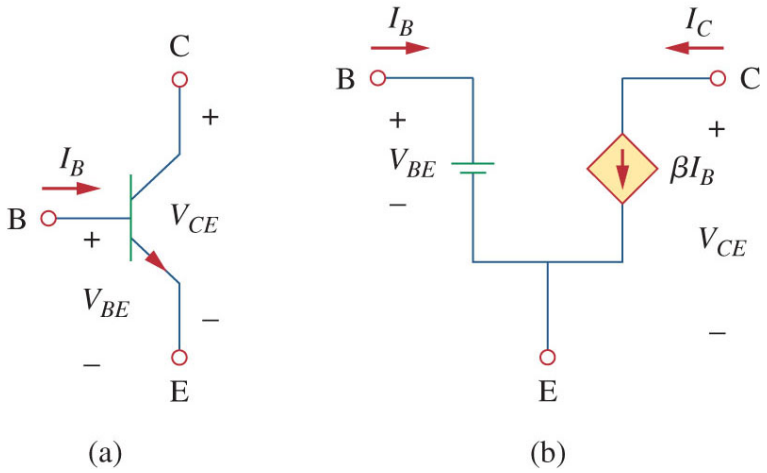
- Applying KVL to the BJT gives:

$$V_{CE} + V_{EB} + V_{BC} = 0$$



DC model of a BJT

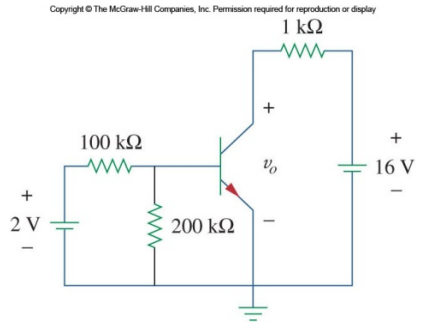
- A transistor has a few operating modes depending on the applied voltages/currents. In this problem, we will be interested in the operation in “active mode.”
- This is the mode used for amplifying signals.
- The figure below shows the equivalent DC model for a BJT in active mode.



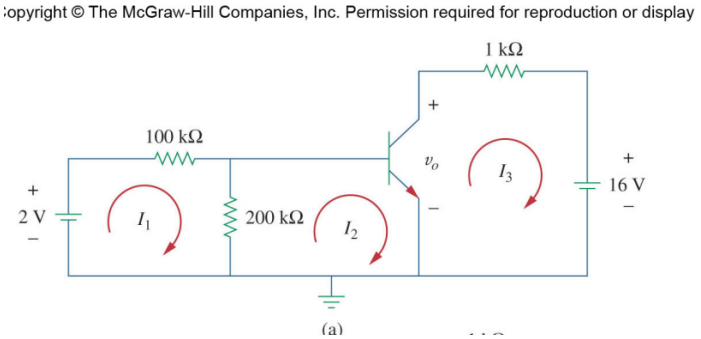
Note that nodal analysis can only be applied to the BJT after using this model

Setting up a BJT circuit

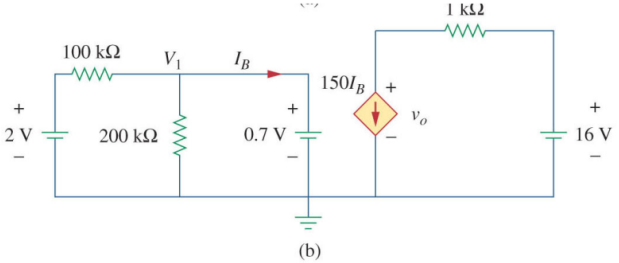
Below are three approaches to solving a transistor circuit. Note when the equivalent model is used and when it is not.



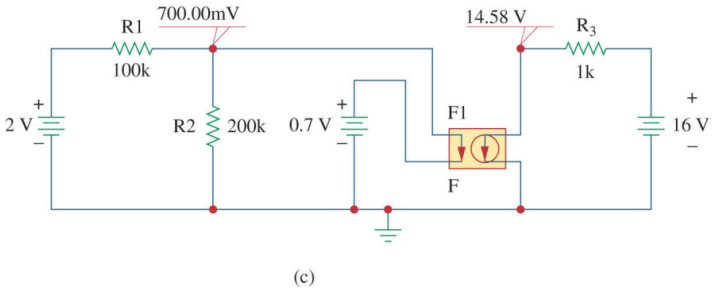
Original circuit



Mesh analysis



Nodal analysis



PSpice analysis

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