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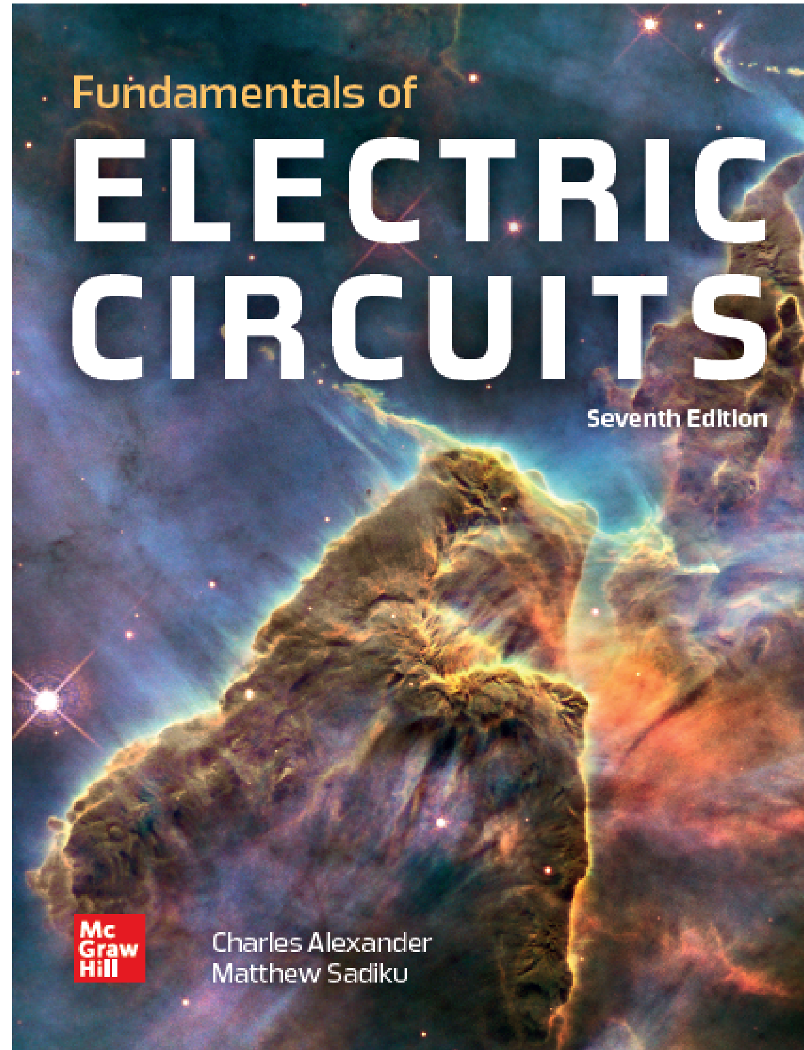
Fundamentals of Electric Circuits

Chapter 6

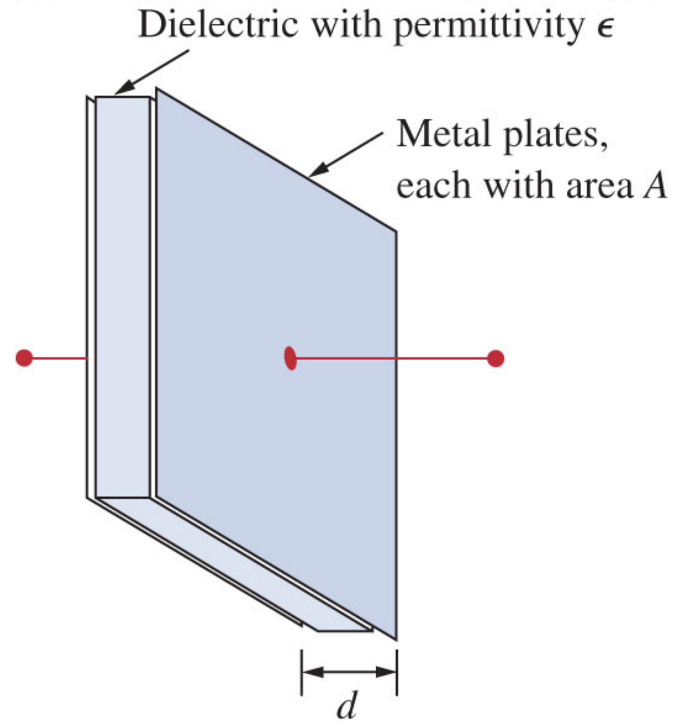


Overview

- This chapter will introduce two new linear circuit elements:
- The capacitor.
- The inductor.
- Unlike resistors, these elements do not dissipate energy.
- They instead store energy.
- We will also look at how to analyze them in a circuit.

Capacitors [1]

- A capacitor is a passive element that stores energy in its electric field.
- It consists of two conducting plates separated by an insulator (or dielectric).
- The plates are typically aluminum foil.
- The dielectric is often air, ceramic, paper, plastic, or mica.



Capacitors 2

- When a voltage source *v* is connected to the capacitor, the source deposits a positive charge *q* on one plate and a negative charge –*q* on the other.
- The charges will be equal in magnitude.
- The amount of charge is proportional to the voltage:

$$q = Cv$$

- Where *C* is the capacitance.

Capacitors 3

The unit of capacitance is the Farad (F)

One Farad is 1 Coulomb/Volt

Most capacitors are rated in picofarad (pF) and microfarad (μF)

Capacitance is determined by the geometery of the capacitor:

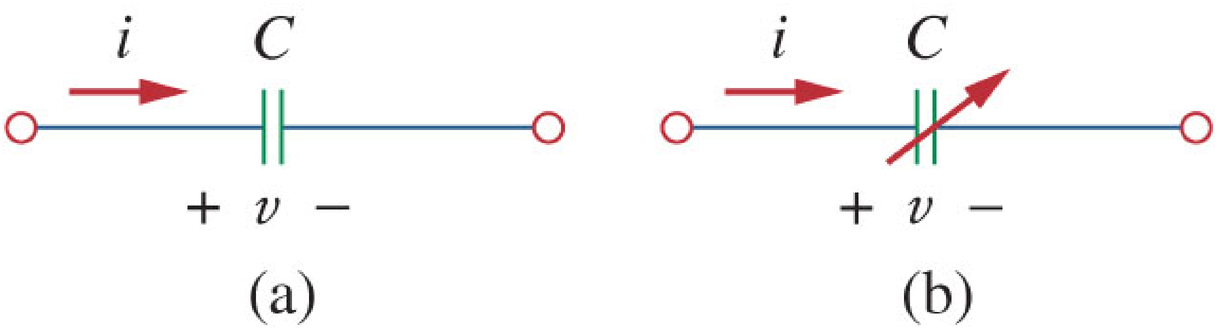
- Proportional to the area of the plates (A).
- Inversely proportional to the space between them (d).

$$C = \frac{\varepsilon A}{d}$$

- ε is the permittivity of the dielectric.

Types of Capacitors

- The most common types of capacitors are film capacitors with polyester, polystyrene, or mica.
- To save space, these are often rolled up before being housed in metal or plastic films.
- Electrolytic caps produce a very high capacitance.
- Trimmer caps have a range of values that they can be set to.
- Variable air caps can be adjusted by turning a shaft attached to a set of moveable plates.



Applications for Capacitors

Capacitors have a wide range of applications, some of which are:

- Blocking DC.
- Passing AC.
- Shift phase.
- Store energy.
- Suppress noise.
- Start motors.

(a) (b) (c)

Courtesy of Tech America

Current Voltage Relationship

- Using the formula for the charge stored in a capacitor, we can find the current voltage relationship.
- Take the first derivative with respect to time gives:

$$i = C\frac{dv}{dt}$$

- This assumes the passive sign convention.

Stored Charge

- Similarly, the voltage current relationship is:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

- This shows the capacitor has a memory, which is often exploited in circuits.
- The instantaneous power delivered to the capacitor is.

$$p = vi = Cv\frac{dv}{dt}$$

- The energy stored in a capacitor is:

$$w = \frac{1}{2}Cv^2$$

Properties of Capacitors 1

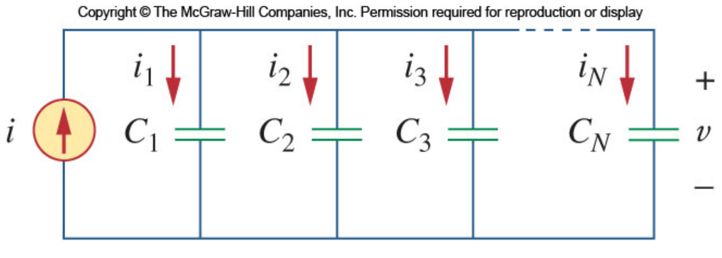
- Ideal capacitors all have these characteristics:
- When the voltage is not changing, the current through the cap is zero.
- This means that with DC applied to the terminals no current will flow.
- Except, the voltage on the capacitor’s plates can’t change instantaneously.
- An abrupt change in voltage would require an infinite current!
- This means if the voltage on the cap does not equal the applied voltage, charge will flow and the voltage will finally reach the applied voltage.

Properties of Capacitors 2

- An ideal capacitor does not dissipate energy, meaning stored energy may be retrieved later.
- A real capacitor has a parallel-model leakage resistance, leading to a slow loss of the stored energy internally.
- This resistance is typically very high, on the order of 100 MΩ and thus can be ignored for many circuit applications.

Parallel Capacitors 1

- We learned with resistors that applying the equivalent series and parallel combinations can simply many circuits.
- Starting with *N* parallel capacitors, one can note that the voltages on all the caps are the same.
- Applying KCL:



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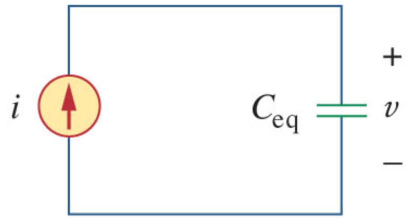


(b)

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

Parallel Capacitors 2

- Taking into consideration the current voltage relationship of each capacitor:

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

- Where

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

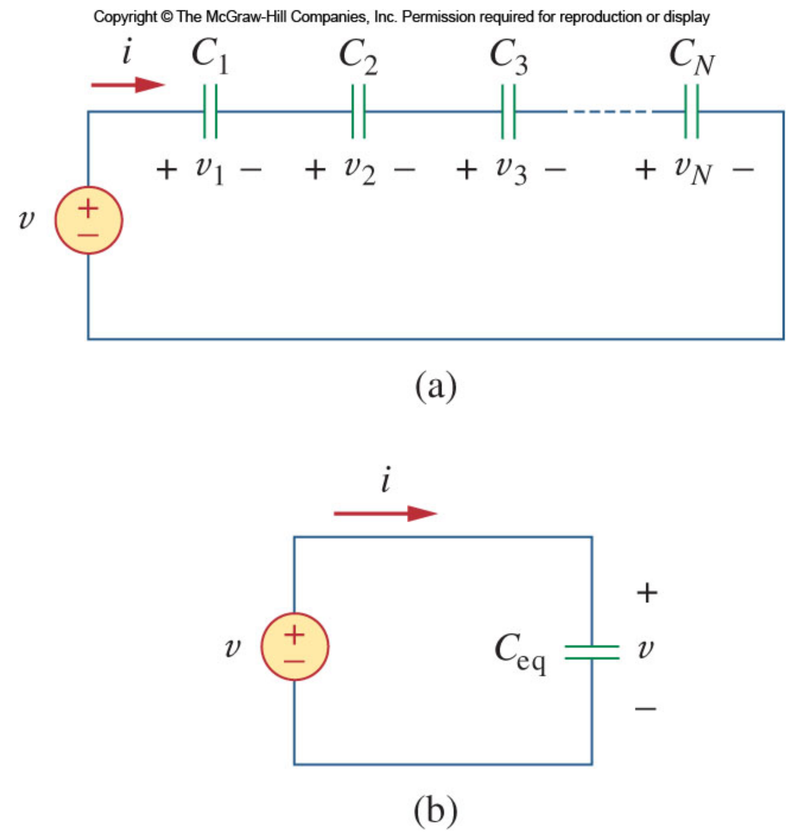
- From this we find that parallel capacitors combine as the sum of all capacitance.

Series Capacitors [1]

- Turning our attention to a series arrangement of capacitors:
- Here each capacitor shares the same current.
- Applying KVL to the loop:

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

- Now apply the voltage current relationship.



Series Capacitors 2

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)\,d\tau + v_2(t_0) + \frac{1}{C_3}\int_{t_0}^{t} i(\tau)\,d\tau + v_3(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)\,d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + v_2(t_0) + v_3(t_0) + \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

- Where

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

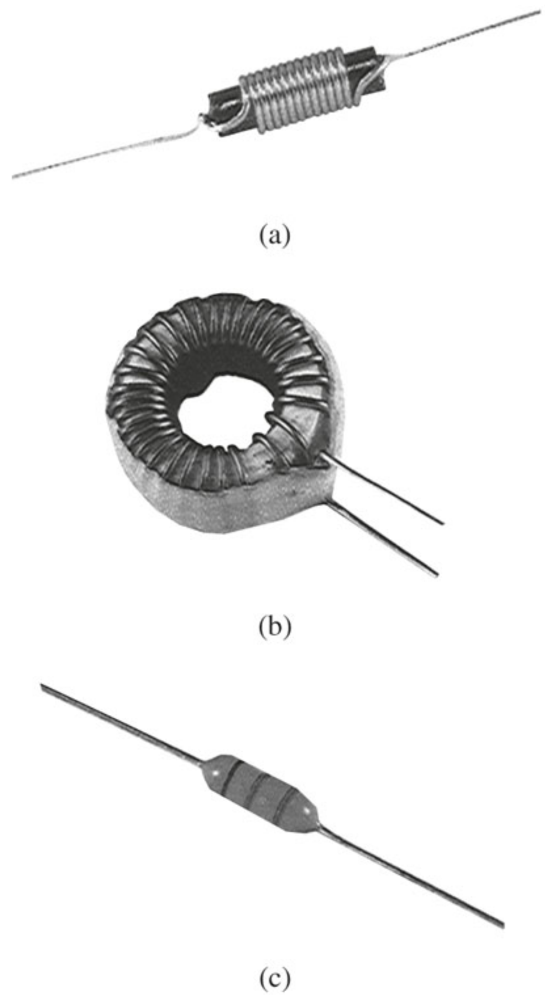
- From this we see that the series combination of capacitors resembles the parallel combination of resistors.

Series and Parallel Caps

- Another way to think about the combinations of capacitors is this:
- Combining capacitors in parallel is equivalent to increasing the surface area of the capacitors:
- This would lead to an increased overall capacitance (as is observed).
- A series combination can be seen as increasing the total plate separation.
- This would result in a decrease in capacitance (as is observed).

Inductors 1

- An inductor is a passive element that stores energy in its magnetic field.
- They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up.



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Inductors 2

- If a current is passed through an inductor, the voltage across it is directly proportional to the time rate of change in current.

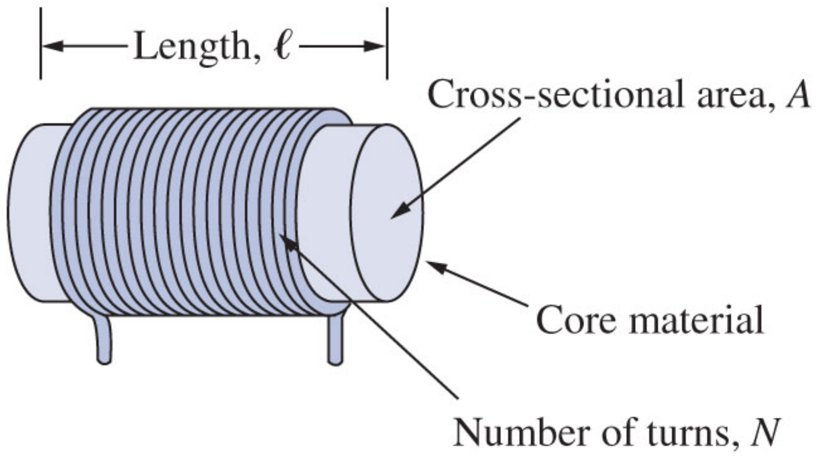
$$v = L\frac{di}{dt}$$

- Where, *L*, is the unit of inductance, measured in Henries, H.
- On Henry is 1 volt-second per ampere.
- The voltage developed tends to oppose a changing flow of current.

Inductors 3

- Calculating the inductance depends on the geometry:
- For example, for a solenoid the inductance is:

$$L = \frac{N^2 \mu A}{l}$$



- Where N is the number of turns of the wire around the core of cross sectional area A and length *l*.
- The material used for the core has a magnetic property called the permeability, *μ*.

Current in an Inductor

- The current voltage relationship for an inductor is:

$$I = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$$

- The power delivered to the inductor is:

$$p = vi = \left(L\frac{di}{dt}\right)i$$

- The energy stored is:

$$w = \frac{1}{2}Li^2$$

Properties of Inductors 1

- If the current through an inductor is constant, the voltage across it is zero.
- Thus an inductor acts like a short for DC.
- The current through an inductor cannot change instantaneously.
- If this did happen, the voltage across the inductor would be infinity!
- This is an important consideration if an inductor is to be turned off abruptly; it will produce a high voltage.

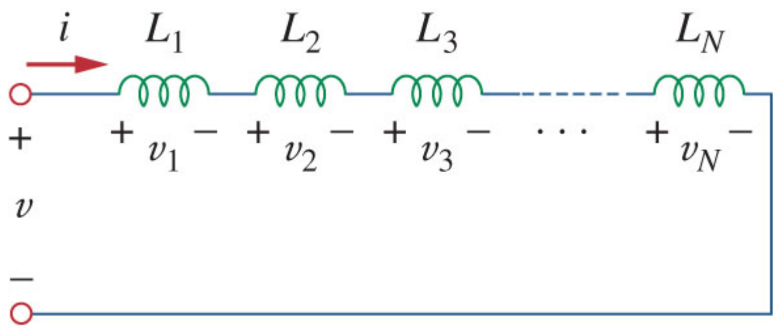
Properties of Inductors 2

- Like the ideal capacitor, the ideal inductor does not dissipate energy stored in it.
- Energy stored will be returned to the circuit later.
- In reality, inductors do have internal resistance due to the wiring used to make them.
- A real inductor thus has a winding resistance in series with it.
- There is also a small winding capacitance due to the closeness of the windings.
- These two characteristics are typically small, though at high frequencies, the capacitance may matter.

Series Inductors [1]

- We now need to extend the series parallel combinations to inductors.
- First, let's consider a series combination of inductors.
- Applying KVL to the loop:

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$



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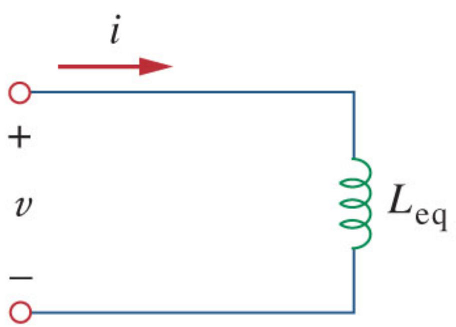

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Series Inductors 2

- Factoring in the voltage current relationship.

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \cdots + L_N \frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k \right) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

- Where

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

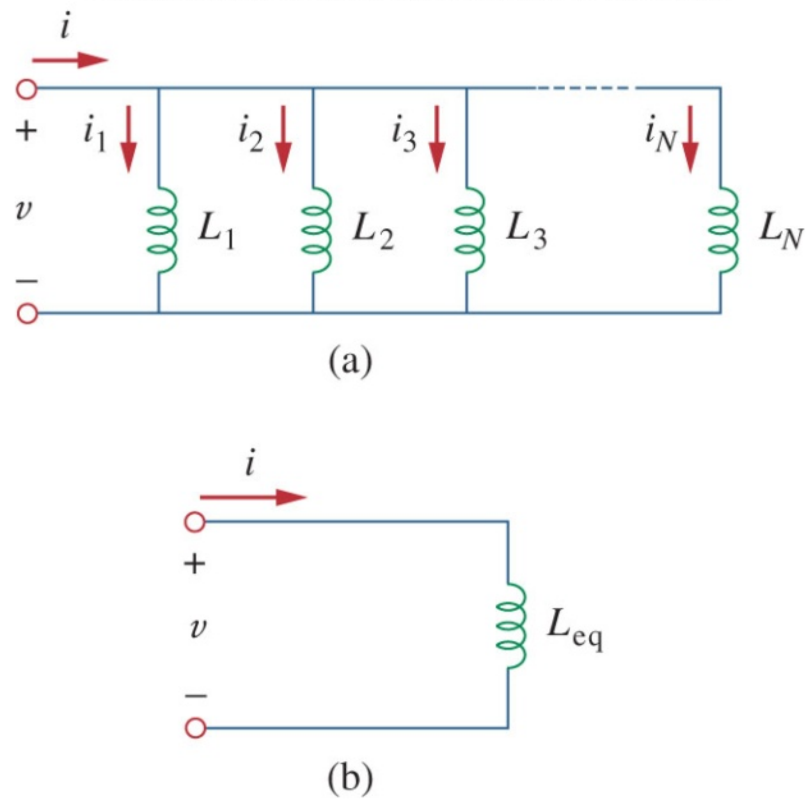
- Here we can see that the inductors have the same behavior as resistors.

Parallel Inductors 1

- Now consider a parallel combination of inductors:
- Applying KCL to the circuit:

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

- When the current voltage relationship is considered, we have:



$$i = \left(\sum_{k=1}^{N} \frac{1}{L_k} \right) \int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}} \int_{t_0}^{t} v\,dt + i(t_0)$$

Parallel Inductors 2

- The equivalent inductance is thus:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

- Once again, the parallel combination resembles that of resistors.
- On a related note, the Delta-Wye transformation can also be applied to inductors and capacitors in a similar manner, as long as all elements are the same type.

Summary of Capacitors and Inductors

TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

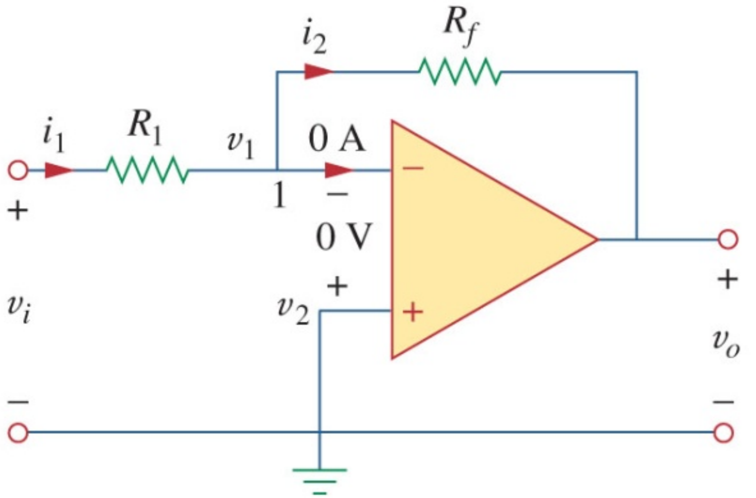
Applications

- Due to their bulky size, inductors are less frequently used as compared to capacitors, however they have some applications where they are best suited.
- They can be used to create a large amount of current or voltage for a short period of time.
- Their resistance to sudden changes in current can be used for spark suppression.
- Along with capacitors, they can be used for frequency discrimination.

Integrator

- Capacitors, in combination with op-amps can be made to perform advanced mathematical functions.
- One such function is the integrator.
- By replacing the feedback resistor with a capacitor, the output voltage from the op-amp is:

$$v_0 = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$



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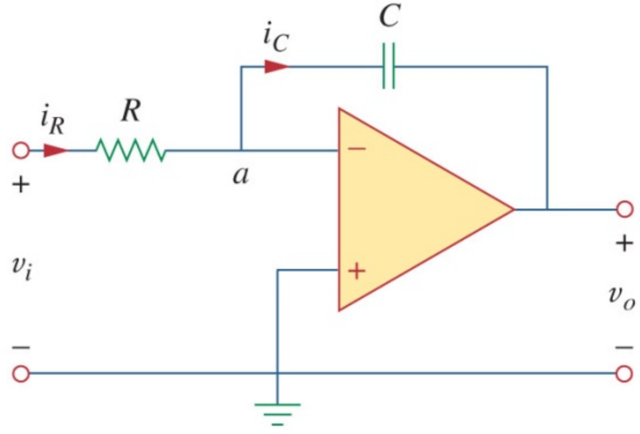
Differentiator

- The previous circuit functions as an integrator with time.
- If the capacitor is used in place of the input resistor instead of the feedback resistor, there will only be current flowing if the voltage is changing.
- The output voltage in this case will be:



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$$v_o = -RC\frac{dv_i}{dt}$$

- From this it is clear this circuit performs differentiation with time.

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