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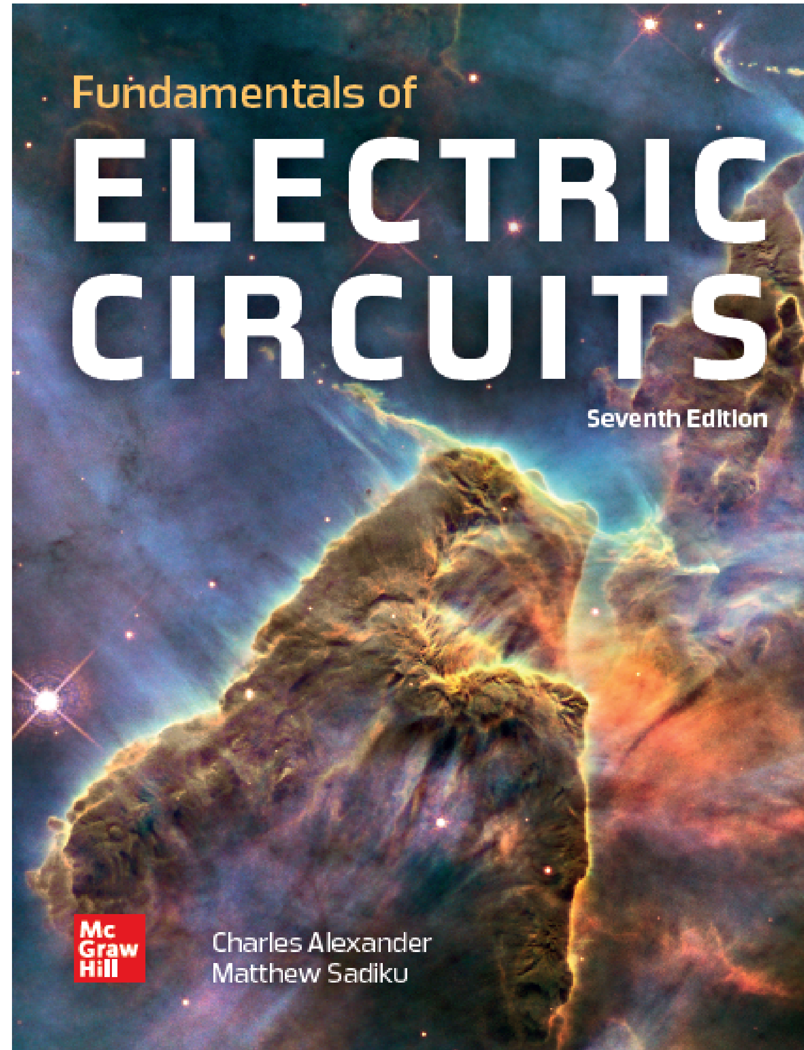
# Fundamentals of Electric Circuits

Chapter 7



# Overview

- This chapter examines RC and LC circuits' reaction to switched sources.
- The circuits are referred to as first order circuits.
- Three special functions, the unit step, unit impulse, and unit ramp function are also introduced.
- Both source free and switched sources are examined.

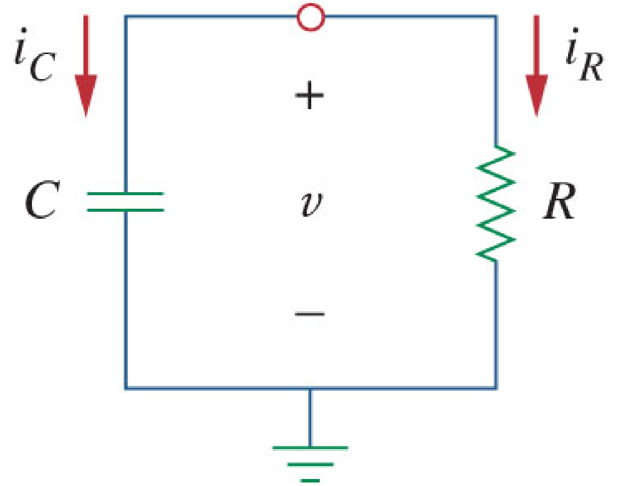
# First Order Circuits

- A first order circuit is characterized by a first order differential equation.
- There are two types of first order circuits:
- Resistive capacitive, called RC.
- Resistive inductive, called RL.
- There are also two ways to excite the circuits:
- Initial conditions.
- Independent sources.

# Source Free RC Circuit 1

- A source free RC circuit occurs when its dc source is suddenly disconnected.
- The energy stored in the capacitor is released to the resistors.
- Consider a series combination of a resistor and a initially charged capacitor as shown:





# Source Free RC Circuit 2

- Since the capacitor was initially charged, we can assume at t=0 the initial voltages is:

$$v(0) = V_0$$

- Applying KCL at the top node:

$$i_C + i_R = 0$$

- Or

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

- This is a first order differential equation.

# Source Free RC Circuit 3

- Rearranging the equation and solving both sides yields:

$$\ln v = -\frac{t}{RC} + \ln A$$

- Where *A* is the integration constant.
- Taking powers of *e* produces Taking powers of *e* produces.

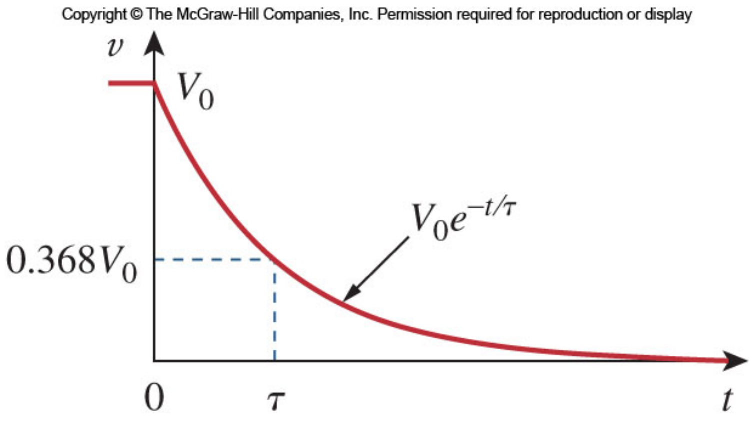
$$v(t) = Ae^{-t/RC}$$

- With the initial conditions:

$$v(t) = V_0 e^{-t/RC}$$

# Natural Response

- The result shows that the voltage response of the R C circuit is an exponential decay of the initial voltage.
- Since this is the response of the circuit without any external applied voltage or current, the response is called the natural response.



# Time Constant [1]

- The speed at which the voltage decays can be characterized by how long it takes the voltage to drop to $1/e$ of the initial voltage.
- This is called the time constant and is represented by $\tau$.
- By selecting $1/e$ as the reference voltage:

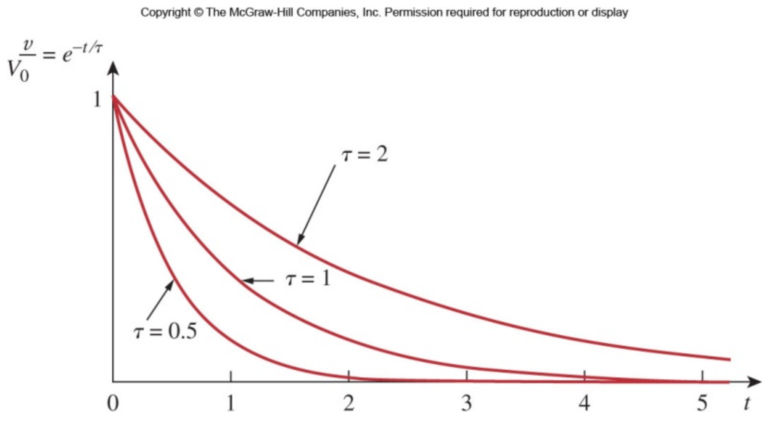
$$\tau = RC$$

- The voltage can thus be expressed as:

$$v(t) = V_0 e^{-t/\tau}$$

# Time Constant 2

- After five time constants the voltage on the capacitor is less than one percent.
- After five time constants a capacitor is considered to be either fully discharged or charged.
- A circuit with a small time constant has a fast response and vice versa.



# RC Discharge

- With the voltage known, we can find the current:

$$i_R(t) = \frac{V_0}{R} e^{-t/\tau}$$

- The power dissipated in the resistor is:

$$p(t) = \frac{V_0^2}{R} e^{-2t/\tau}$$

- The energy absorbed by the resistor is:

$$w_R(t) = \frac{1}{2} C V_0^2 \left(1 - e^{-2t/\tau}\right)$$

# Source Free RC Circuit Summary

- The key to working with this type of situation is:
- Start with the initial voltage across the capacitor and the time constant.
- With these two items, the voltage as a function of time can be known.
- From the voltage, the current can be known by using the resistance and Ohm's law.
- The resistance of the circuit is often the Thevenin equivalent resistance.

# Source Free RL Circuit 1

- Now lets consider the series connection of a resistor and inductor.
- In this case, the value of interest is the current through the inductor.
- Since the current cannot change instantaneously, we can determine its value as a function of time.
- Once again, we will start with an initial current passing through the inductor.

# Source Free RL Circuit 2

- We will take the initial current to be:

$$i(0) = I_0$$

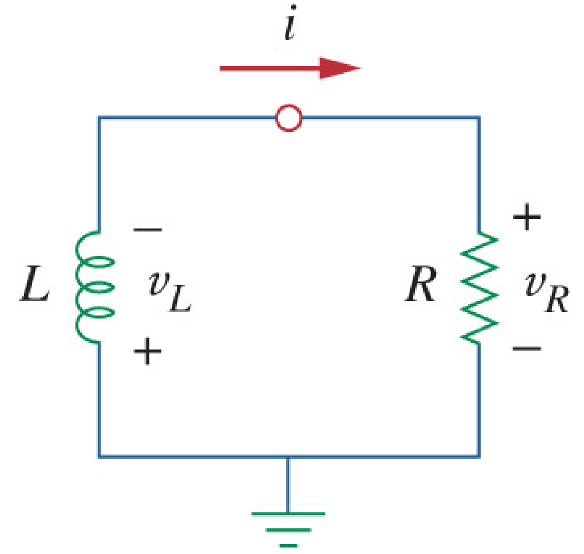
- Applying KVL around the loop:

$$v_L + v_R = 0$$

- Or:

$$L\frac{di}{dt} + Ri = 0$$





# Source Free RL Circuit 3

- After integration:

$$i(t) = I_0 e^{-Rt/L}$$

- Once again, the natural response is an exponentially decaying current.
- The time constant in this case is:

$$\tau = \frac{L}{R}$$

- The same principles as the RC circuit apply here.

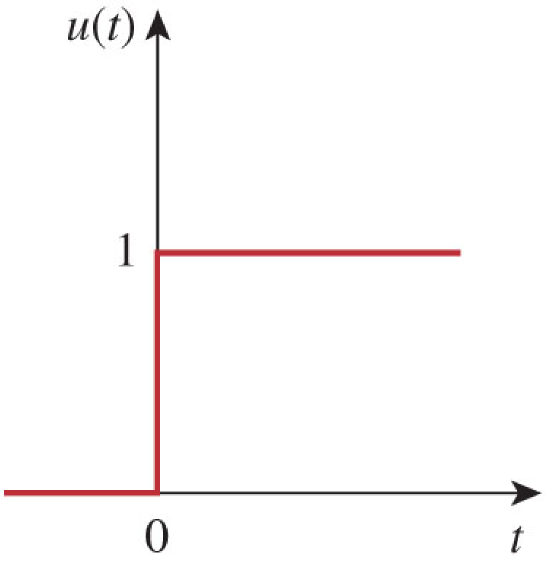
# Singularity Functions

- Before we consider the response of a circuit to an external voltage, we need to cover some important mathematical functions.
- Singularity functions serve as good approximations to switching on or off a voltage.
- The three most common singularity functions are the unit step, unit impulse, and unit ramp.

# The Unit Step [1]

- A step function is one that maintains a constant value before a certain time and then changes to another constant afterwards.
- The prototypical form is zero before *t=0* and one afterwards.
- See the graph for an illustration.





# The Unit Step 2

- Mathematically, the unit step is expressed as:

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

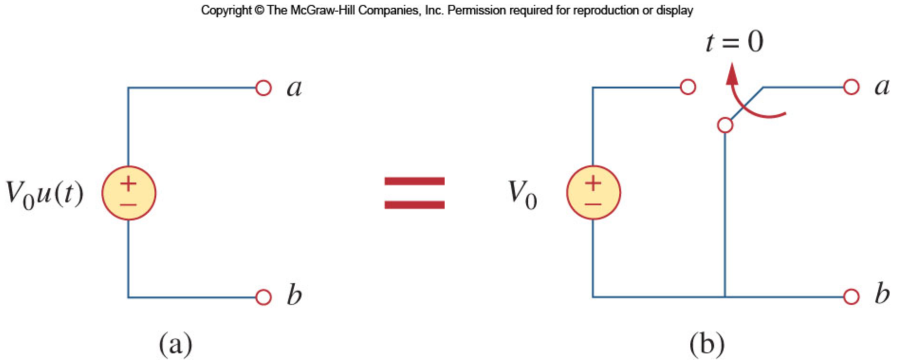
- The switching time may be shifted to $t=t_0$ by:

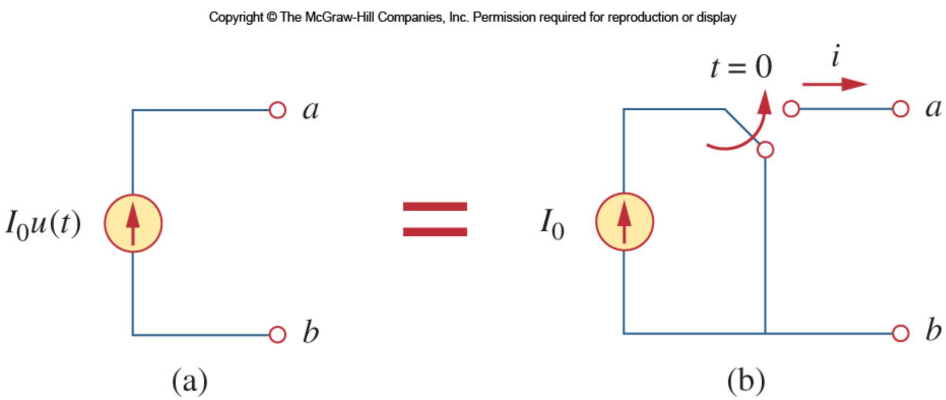
$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

- Note that this results in a *delay* in the switch.
- The unit step function is written as *u(t)*

# Equivalent Circuit

- The unit step function has an equivalent circuit to represent when it is used to switch on a source.
- The equivalent circuits for a voltage and current source are shown.



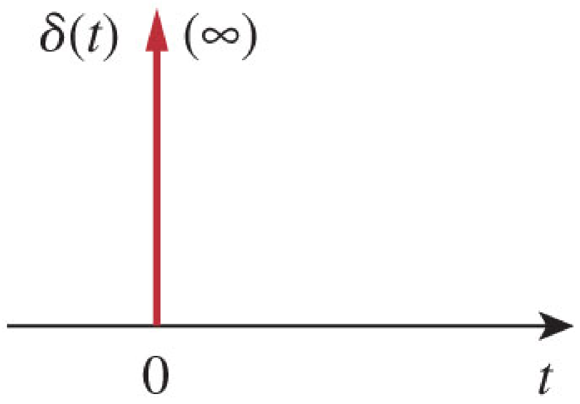



# The Unit Impulse Function

- The derivative of the unit step function is the unit impulse function.
- This is expressed as:

$$\delta(t) = \begin{cases} 0 & t < 0 \\ \text{Undefined} & t = 0 \\ 0 & t > 0 \end{cases}$$






- Voltages of this form can occur during switching operations.

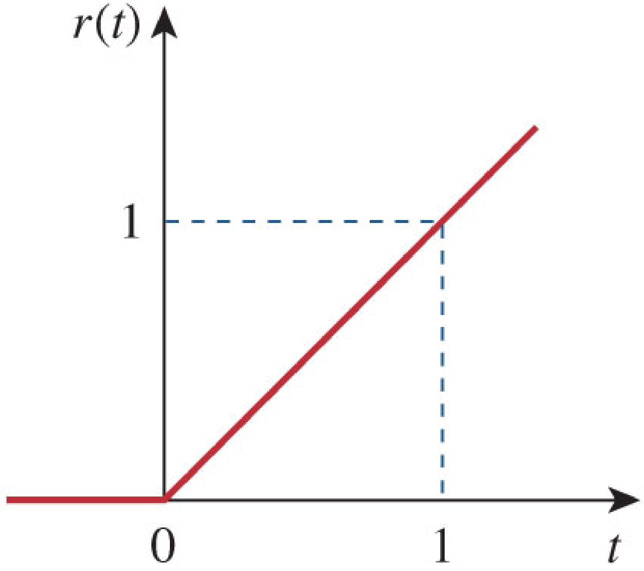
# The Unit Ramp Function

- Integration of the unit step function results in the unit ramp function:

$$r(t) = \begin{cases} 0, & t \le 0 \\ t, & t \ge 0 \end{cases}$$

- Much like the other functions, the onset of the ramp may be adjusted.

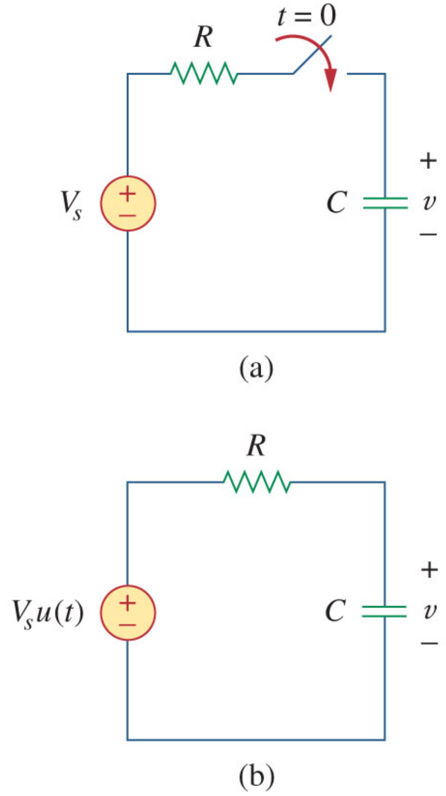




# Step Response of RC Circuit 1

- When a DC source is suddenly applied to a RC circuit, the source can be modeled as a step function.
- The circuit response is known as the step response.
- Let's consider the circuit shown here.
- We can find the voltage on the capacitor as a function of time.





# Step Response of RC Circuit 2

- We assume an initial voltage of $V_0$ on the capacitor.
- Applying KCL:

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC} u(t)$$

- For $t>0$ this becomes:

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}$$

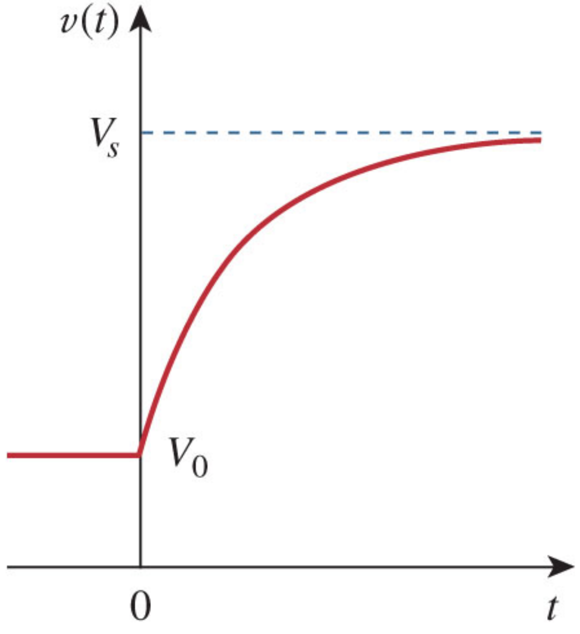
- Integrating both sides and introducing initial conditions finally yields:

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s) e^{-t/\tau} & t > 0 \end{cases}$$

# Step Response of RC Circuit [3]

- This is known as the complete response, or total response.
- We can consider the response to be broken into two separate responses:
- The natural response of the capacitor or inductor due to the energy stored in it.
- The second part is the forced response.





# Forced Response

- The complete response can be written as:

$$v = v_n + v_f$$

- Where the nature response is:

$$v_n = V_0 e^{-t/\tau}$$

- And the forced response is:

$$v_f = V_s\left(1 - e^{-t/\tau}\right)$$

- Note that the eventual response of the circuit is to reach $V_s$ after the natural response decays to zero.

# Another Perspective

- Another way to look at the response is to break it up into the transient response and the steady state response:

$$v = v_t + v_{ss}$$

- Where the transient is:

$$v_t = (V_0 - V_s)e^{-t/\tau}$$

- And the steady state is:

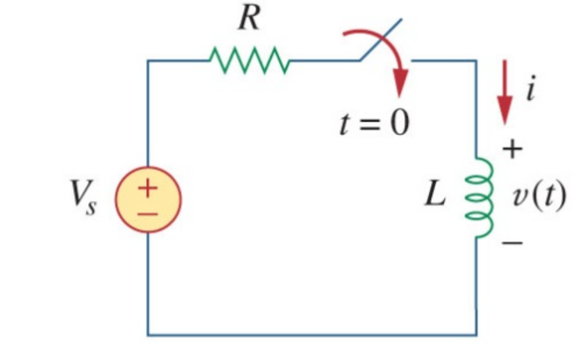
$$v_{ss} = V_s$$

# Step Response of RL Circuit 1

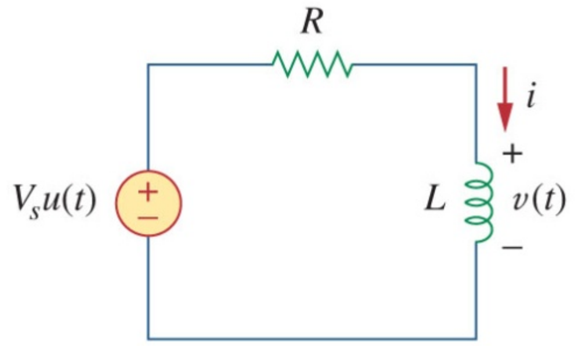
- Now we can look at the step response of a RL circuit.
- We will use the transient and steady state response approach.
- We know that the transient response will be an exponential:

$$i_t = Ae^{-t/\tau}$$





(a)



(b)

# Step Response of RL Circuit 2

- After a sufficiently long time, the current will reach he steady state:

$$i_{ss} = \frac{V_s}{R}$$

- This yields an overall response of:

$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

- To determine the value of *A* we need to keep in mind that the current cannot change instantaneously.

$$i\left(0^+\right) = i\left(0^-\right) = I_0$$

# Step Response of RL Circuit 3

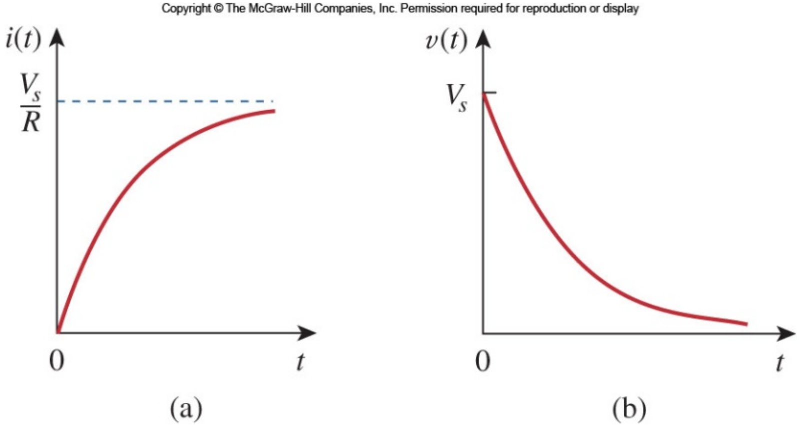
- Thus we can use the *t=0* time to establish *A.*

$$A = I_0 - \frac{V_s}{R}$$

- The complete response of the circuit is thus:

$$i(t) = \frac{V_s}{R} + \left( I_0 - \frac{V_s}{R} \right) e^{-t/\tau}$$

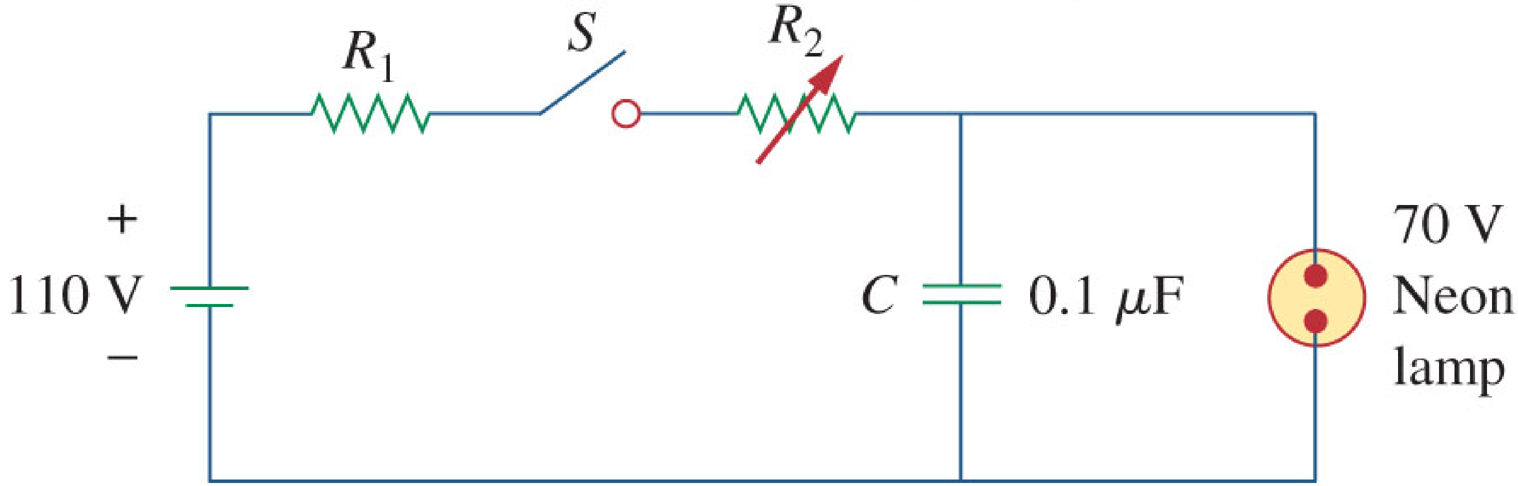
- Without an initial current, the circuit response is shown here.



# Application: Delay Circuit

- The RC circuit can be used to delay the turn on of a connected device.
- For example, a neon lamp which only triggers when a voltage exceeds a specific value can be delayed using such a circuit.





# Delay Circuit 2

- When the switch is closed, the capacitor charges.
- The voltage will rise at a rate determined by:

$$\tau = (R_1 + R_2) C$$

- Once the voltage reaches 70 volts, the lamp triggers.
- Once on, the lamp has low resistance and discharges the capacitor.
- This shuts off the capacitor and starts the cycle over again.

End of Main Content

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